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PART I

ARTICLE I — *175 Parabolic Orbits and Other Results Deduced from over 6,200 Meteors*
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ARTICLE I.

175 PARABOLIC ORBITS AND OTHER RESULTS DEDUCED FROM OVER 6,200 METEORS.*

(Read April 21, 1911.)

By Chas. P. Olivier.

The first observations on which the results in this paper depend were made on November 14, 1898. This does not include a few records found in some old books, which had been made many years previously, but never apparently used. No year from 1898 to the present has passed without the addition of quite a number of meteor observations until, up to the end of 1910, about 6,000† had been recorded. From lack of experience, both in meteor observing and other lines of astronomical work, the three Leonid, one Perseid, and the Beilid radiants deduced from the 1898 and 1899 observations, can not be considered as accurate.

Even the paths of meteors plotted in 1900 are probably not so good as those since obtained. However, from 1901 on, while naturally each year should improve the methods and accuracy slightly, yet there is reason to feel almost equal confidence in the results.

I am under deep obligation to Director Ormond Stone of the Leander McCormick Observatory, for continued encouragement and advice in this work from its very beginning to the present.

In scarcely less degree am I under obligation to Director Campbell of the Lick Observatory, for encouragement in this work and allowing me time to carry most of the computations to a conclusion while I was a member of the staff there during 1909 and 1910.

It must also be stated that about 1,900 meteors, or nearly one third of the total, were observed by me while at the Lick Observatory. All of the remainder, except about 300, were observed at or near the Leander McCormick Observatory, University of Virginia.

* Presented to the Faculty of the University of Virginia in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

† So far as I have been able to find from records Corder, Denning, Heis, and Zezioli are the only four observers who have each observed over 5000 meteors.

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I am further indebted to the following observers, who at various times have either assisted in recording or made separate observations at other places under the same general plan. Messrs. T. B. Lyons, G. F. Paddock, K. S. Patton, J. B. Smith and J. P. Smith, of the University of Virginia, Dr. S. Albrecht, Evanden, Miss E. Glancy, Messrs. P. W. Merrill, K. Lows, R. Young, of the Lick Observatory. Occasional or remarkable meteors have been reported by others.

The methods of observing have evolved with increasing experience, but from 1900 on they have not changed greatly. At present a meteor is observed as follows: Maps are prepared of the region of the sky that is to be especially observed on a given night, care being taken to choose that map whose projection is best for the region in question. In a large recording book a number of columns are ruled, headed as follows (1) Time, (2) Number, (3) Class, (4) Color, (5) Magnitude, (6) Length of Path, (7) Duration in tenths of seconds, (8) Duration of Train in tenths of seconds, (9) Remarks, (10) Serial Number, (11) Accuracy. The designations are mostly self explanatory. (2) gives the number of the meteor for the night, (10) the serial number for the year—filled in later, (11) the accuracy, on a scale of 3, with which the meteor was observed. Beside the plotted path of the meteor on the map is placed the number for the night and later the serial number in ink. The serial numbers are so arranged that the first figure itself gives the year during which the meteor was seen. Thus 1—117 shows that the meteor was seen in 1901, 9—1,136 in 1909, etc. The methods used to obtain the most accurate plot of a meteor's path are as follows: The greatest care was taken to obtain the direction and any one point over which the meteor passed. Often, of course, a meteor's beginning and ending points fall exactly at or very near a convenient star, or at such a distance between two near stars that it is easy to estimate the distance proportionally and accurately. In such a case the direction, determined nearly always by holding up a straight rod so that it appeared to lie parallel to the meteor's path in the sky, served mainly as a convenient check.

But in most cases a meteor neither begins nor ends at a point which is easy to determine. Then by glancing backwards and forwards along the rod the eye can always pick up a star in the same great circle. Also there is scarcely ever any difficulty in finding some one point actually in the path itself. As the eye readily estimates the length of path of a meteor with fair accuracy, the parts in front and behind the chosen point can be estimated instantly, and by means of the other reference point entirely outside the path, the meteor's position can be gotten with great accuracy and speed, compared with other methods. By choosing some point behind rather than in front of the path we also in this method eliminate to a great extent the effects of poor projection, which may be troublesome near the edges of almost any map. However

I wish to state that meteors beginning at a greater distance than 30° from the radiant were never given much weight, and whenever there was reason to suspect that a plotted path was distorted by poor projection the meteor was either given extremely little or no weight at all.

For the strong streams such as the Leonids and Perseids, the radiants could frequently be determined by meteors within 10°.

Other meteors further out but well observed were always used, but the resulting point would generally have been practically the same, had they been omitted.

It is obvious that the short paths near the radiant are most useful in its determination, both because of their nearness and also their low apparent velocity, which permits of the most accurate plotting. They nearly always have trains also, a most material assistance.

If for any reason certain meteors could not be at once plotted their paths were described with such detail that afterwards when put upon maps the results were quite comparable with these plotted at the moment. As far as possible, however, each meteor was plotted where observed.* The usual plan was to work up the result partially the next day so that details could be added, when necessary, while the recollection was fresh. The observations made by others, who have assisted or observed elsewhere for me, were made in the same general way, only the results were left to me to work up completely and the responsibility for the latter rests upon me. As confirming the results of previous observers the following points may be noted as of general interest with regard to the work and results. Most of the meteors were observed after midnight and to the east of the meridian. I have no reason to believe that the south-east or north-east quadrants differed appreciably in the number of meteors seen within them. Those seen before midnight differed much, as a rule, in apparent velocity from those seen after, being much slower. Nearly always there was a marked falling off in numbers before the least trace of twilight appeared. Very slow meteors leave trains, nearly without exception. Meteors with curved or sinuous paths are rare (see tables). In very many cases two or even three meteors travel the same apparent paths within a few seconds. In showers like the Perseids, two or more meteors frequently appear at the same instant. In most cases radiants cover large areas only because of poor observations, since the paths of well observed meteors generally intersect nearly in a point or within a small area. The most notable exception to this rule was on August 10 and 11, 1910.

The physical appearance of meteors, such as color, apparent velocity, etc., while all very useful in assigning a given meteor to the proper radiant, can scarcely ever

* On an average a single meteor was plotted and a full record made in about 40 seconds. However, about 60 per hour would be the most possible to observe fully under usual conditions.

be held as conclusive evidence that it does belong to any given radiant. Many cases could be cited, especially in August and October, in which meteors have every physical characteristic exactly like numbers of the main stream but come from distant radiants.

Also numerous meteors of the principal streams differ very much from the average member of that stream. This point is to be especially noted in view of the statement often found that a meteor belongs to a given stream because it looks like the average member of it, though its direction was frequently very poorly determined or perhaps not at all.

Radiants have been found by projecting the plotted paths backwards. In regard to those for which parabolic elements have been computed this rule was followed. At least three meteor paths on projection must meet within a circle not more than 0°.5 in diameter.

Any other well-observed meteor whose projected path comes within 1° of the center is accepted and given due weight. Any other meteor whose projected path comes within 2° of the point may be used, but would be given little weight. The radiant when finally accepted lies at the weighted center of gravity, as it may be called, of the area enclosed by the projections. The weight given each projection depends on how near the meteor was and how well observed.

An absolute rule has been made that under no conditions have meteors observed on more than one night been used to determine any radiant.

It is my firm conviction that not following this rule has led many previous observers to catalogue hundreds of fictitious radiants whose presence in our catalogues only hampers the future growth of meteoric astronomy.

Only in the case of a radiant known to be stationary could the combination of meteor paths observed on different nights be justifiable, and stationary radiants must be rare phenomena,* since the meteoric apex moves, on account of the orbital motion of the earth, about 1° per day through the sky and each stream has also its own motion in space.

Since such combinations, in effect, presuppose the existence of stationary radiants, they appear to prove that on the assumption of which they largely owe their own apparent existence. Had I been willing to combine several nights' work there is little doubt that the number of my radiants could have easily been doubled. Owing to this precaution largely, I presume, my own work gives little indication of stationary radiation.

Most of the meteors observed came during July, August, October and November.

* Refer to *Mon. Not. R. A. S.*, Vol. XXXVIII, p. 115; *Astr. Nach.*, Vol. XCIII, p. 209; *Bulletin Astr.*, Vol. XI, p. 409–10.

Quite a number have been recorded in January, April and May, and a few in December. The other four months have furnished no results, observations never having been made during them.

It should be stated that only a small fraction of my time given to observing could be devoted to meteor observations, which fact explains why the total number is not larger.

The formulæ and methods for computing the parabolic orbits which follow in the tables were taken from "Die Bahnbestimmung der Himmelskorper" by Julius Bauchinger, which has proved invaluable. In the thirty-fifth chapter of this work the equations will be found in full.

Believing that in some cases the radiants were known more closely than to whole degrees the measurements on the maps were made to tenths of degrees. The transformations from right ascension and declination to longitude and latitude and all the actual computations for the elements were made with 4-place logarithms, the angles being taken out to the nearest minute. However, in the tables of results they were again reduced to the nearest tenth degree, that being quite as accurate as the observations could give.

All obtainable works on meteoric astronomy have been freely consulted and, as will appear later, certain conclusions on the question of stationary radiation have been partially reached by discussion of the work of other observers.

Finally it may be stated that it is hoped at some future time to study the data on which these result are based with a view to the solution of other problems, not taken up or only mentioned in this paper.

The Aquarid Meteors of May.

These meteors were never seen in sufficient numbers before 1910 to deduce a radiant for them. Indeed, cloudy weather or moonlight had never permitted any of the former attempts to be successful. However, in 1910 good radiants were secured on May 4 and May 11. On May 4, at the Lick Observatory, numerous meteors were seen in the southeast by Dr. Curtis and later by myself. During the half hour before dawn, at intervals when my assistance could be dispensed with, I was able to observe 9 meteors, 6 coming from the Aquarid radiant. This was therefore well determined, as the observations were good. During about the same interval Dr. Curtis saw 4 more Aquarids. The hourly rate must certainly have been quite high. The average length of path for the Aquarids was 10°.5 and average duration $0^s.64$.

On May 5, my undivided attention was given to meteors from $13^h\ 23^m$ to $14^h\ 58^m$,

during which interval 13 meteors were observed. Not more than 2 could have been Aquarids. On May 11, during an intermittent watch of 35 minutes while in the Crossley dome, just before dawn, 8 meteors, of which 5 or possibly 6 were Aquarids, were observed. A gold radiant was obtained from them. The average length of path was 7°.8, and the average duration 0ˢ.55.

On May 13, from 14^h7^m to 15^h20^m, 14 meteors were seen. Not more than 2 could have been Aquarids.

On May 14, information was given me that 14 meteors had been seen by one person from 14^h45^m to 15^h30^m. Whether any Aquarids were among them was not stated.

On May 15, one Aquarid was seen.

On May 19, during a continuous watch of one hour before dawn, and an intermittent watch earlier, no Aquarids were seen.

The vicinity of the radiant of possible meteors in the tail of Halley's Comet was examined several times for short periods with the 12-inch refractor and low power giving a large field of view. No meteors were seen with this instrument.

The following table gives the elements for Comet Halley and the parabolic elements deduced from the observations on the three dates.

	G.M.T.	α	δ	ι	Ω	π	$\pi-\Omega$	q
	1910	°	°	° ′	° ′	° ′	° ′	
Comet Halley				163 12	57 16	168 58	111 42	0.5869
Aquarids	May 4.97	334.0	−3.4	166 15	44 4	155 6	111 2	0.6770
Aquarids*	May 6.93	337.7	−0.6	163 9	45 58	148 17	102 19	0.6067
Aquarids	May 11.99	342.0	−0.6	166 41	50 51	155 7	104 16	0.6297

The connection of the meteors with the comet is quite obvious. The probable connection of these meteors with Comet Halley was pointed out long ago by Professor A. Herschel†, but the data on which his conclusions were based was not extensive.

However, in 1910 the most interesting point is the enormous size indicated for this meteroic current. On May 4.97 Halley was about 63 million miles from the earth, which was at the same time about 6 million miles from Halley's orbit. On May 11.99 these figures had changed to 34 million and 13 million miles respectively. In other words the space at least partially filled by meteors connected with this comet was presumably a cylinder of 13 million miles radius. Nothing could illustrate better the extreme complexity of some of the principal meteor currents than this example of their possible size. It is to be noted also that some of these meteors

* Computed from data given in *Popular Astronomy*, November, 1910, p. 538. Observed at Vieques, P. R., by George Hurtnell. I projected the meteors on the printed chart accompanying the article and found η Aquarii at the radiant. The computation was made on this assumption which must be nearly true.

† *Monthly Notices R.A.S.*, XXXVIII, p. 379.

preceded the comet by 63 million miles and therefore it is probable greater numbers followed it, but of course no data can be gotten on this point. It is of interest to note the eastward movement of the radiant between the dates given, and further that May 6 seems the latest that these meteors have previously been seen.

There is a curious coincidence between some of the elements of the Aquarids and the main Orionid stream. The mean elements are tabulated here for comparison.*

	Mean Date 1900+	L	ι	q	π	Ω	$\pi-\Omega$
		°	°		°	°	°
Aquarids (3).....	10 May 7.96	316.3	165.4	0.638	152.8	47.5	105.3
Orionids (9).....	1900 to 1908	116.6	161.4	0.536	113.4	25.6	87.8

One can see how closely the inclinations agree, and also that the perihelion distances do not differ very greatly. The longitudes of perihelia differ about 40°.

The July and August Meteors.

Both July and August are months during which meteors are very numerous and, besides, the great Perseid stream offers exceptional opportunities.

My first observations of the Perseids were made in 1899, and every year since has added considerable data. The radiants deduced show unmistakably the regular shift of the radiant from day to day in the direction of increasing right ascension, and, to a smaller degree, also in declination.

But as elliptical orbits have several times been computed for the Perseids it was thought useless to compute either a single new elliptical one or parabolic orbits for the separate dates. However, in the table the residuals from Denning's ephemeris† are given.

In this table are given in order named: (1) G.M.T. of the middle of observations, (2) L, (3) α, (4) δ, (5) numbers of meteors used to find radiant point, (6)‡ Ephemeris—Observed values in α, (7) Ephemeris—Observed values in δ. A stationary meteor is considered to give a separate radiant, and the two cases of this kind are so tabulated. Altogether 37 radiants are given. The next table gives: (1) Date, (2) time of beginning, (3) time of ending, (4) time actually occupied in observing and recording, (5) total number of meteors, (6) rate per hour, (7) factor of rate,§ (8) corrected rate, (9) number of Perseids, (10) rate of Perseids, (11) corrected rate of Perseids. Approxi-

* See also *Popular Astronomy*, August-September, 1910, p. 422, for another mention of these meteors.

† See his "General Catalogue of Meteor Radiants," p. 210.

‡ Denning's positions are assumed as being at Greenwich Mean Midnight. However as his are given by dates and not by L, therefore the positions are not strictly comparable.

§ This factor is taken as 1.0 when the night is clear and free from moonlight, when observing conditions are good and when the horizon is unobstructed. If any unfavorable conditions arise, the factor is lowered in the proportion that it is believed the number of meteors seen was diminished.

mately 3,100 meteors were observed during July and August and results from these from the basis for these tables and results.

G.M.T. 1900+	L	a	δ	s $\rightarrow$	Δa $E-O$	$\Delta\delta$ $E-O$	Notes.
	°	°	°		°	°	
1899 Aug. 10.75	47.9	39.5	+56.3	32	+5.8	+0.7	
1900 " 10.82	48.8	45.6	55.0	24	−0.3	+2.0	
1901 " 8.79	46.5	37.7	57.4	9	+5.3	−0.8	
1901 " 9.77	47.5	42.0	54.8	28	+2.2	+2.1	
1902 " 10.82	48.2	40.0	57.0	24	+5.4	+0.1	
1902 " 11.8	49.1	46.4	57.0	26	+0.9	+0.4	
1902 " 11.8	49.1	47.0	56.8	1	−0.3	+0.5	
1903 July 21.76	28.5	23.6	50.0	5	−1.4	+1.7	
1903 " 23.84	30.4	26.0	51.2	6	−1.6	+1.1	
1903 " 28.7	35.4	36.0	55.9	4	−6.0	−2.2	[G.F.P.]
1903 " 28.79	35.4	32.4	54.7	3	−2.4	−1.0	
1903 Aug. 11.77	48.9	44.0	56.2	17	+2.6	+.11	
1904 " 10.79	48.8	43.1	56.0	5	+2.3	−1.1	
1904 " 10.83	48.8	41.0	56.2	11	+4.4	−0.9	[J.B.S.]
1904 " 11.77	49.6	45.2	58.0	6	+1.4	−0.7	
1904 " 11.84	49.7	46.7	57.0	25	+0.0	−0.3	[J.B.S.]
1904 " 12.74	50.7	50.8	57.4	5	−3.0	−0.2	
1904 " 14.82	52.7	54.9	63.4	7	−4.3	−5.3	Perseids??
1904 " 16.74	54.5	53.6	61.4	5	−0.6	−2.8	
1905 " 9.57	47.3	41.1	57.2	10	+3.0	+1.6	
1905 " 11.63	49.4	44.7	55.3	24	+1.8	+1.9	
1906 " 10.75	48.2	41.2	57.0	9	+4.1	+0.0	
1906 " 11.75	49.2	43.4	56.0	5	+3.2	+1.3	
1907 " 4.75	42.2	35.0	55.1	4	+3.2	+0.5	
1907 " 11.74	49.0	45.7	56.2	10	+0.9	+1.1	
1908 " 1.74	40.0	28.6	50.4	5	+6.1	+4.4	
1908 " 1.74	40.0	29.2	52.5	1	+5.5	+2.3	No. 8–049
1909 July 23.91	31.2	24.4	50.9	4	+0.0	+1.4	
1909 Aug. 9.88	47.6	39.1	56.8		+5.3	+0.1	
1909 " 10.93	48.6	40.0	57.0	11	+5.5	+0.1	
1909 " 11.87	49.6	42.4	57.5	44	+4.4	+0.2	
1909 " 13.87	51.5	45.6	57.7	17	+3.7	+0.2	
1910 " 1.93	39.7	31.9	58.6	4	+3.0	−3.8	
1910 " 4.95	42.6	37.6	56.6	4	−0.8	−1.0	
1910 " 6.9	44.5	37.6	55.7	12	+3.1	+0.4	
1910 " 10.93	48.4	41.4	56.4	12	+4.1	+0.7	
1910 " 11.86	49.3	44.3	57.9	31	+2.4	−0.6	

While these tables give the principal results some remarks may be added. The richness of the stream varied greatly from year to year, not only in numbers but in the brightness of the meteors, especially near maximum.

So far as numbers go, 1909 August 11 furnished the finest shower, 338 meteors being seen, of which 223 were Perseids. The radiant areas were larger than usual in 1909. (For a fuller description of my observation of this return see the *Lick Observatory Bulletin*, No. 166.)

In 1901 on August 9, meteors were numerous but on former dates scarce. 1902 gave a good display at maximum. In 1903 Perseids were numerous late in July but moonlight spoiled the first half of August. 1904 gave a good display. The maximum of 1905, as observed at Daroca, Spain, was not a conspicuous one though weather conditions were good. During 1906 and 1908 the maximum came in bad weather, while that of 1907 was not a rich one. On an average a Perseid meteor seen before

Date.	Beginning.	Ending.	Total.	$s \rightarrow$ Total.	Rate.	Factor.	Rate Cor.	$s \rightarrow$ Perseid	Rate.	Rate Cor	
	h. m.	h. m.	m.								
1899 Aug. 10	10 40	16 18	83	35	25.3	0.9	28.1	32	23.1	25.7	
1900 July 30	12 20	16 0		20		0.4					Intermittent watch.
Aug. 10	12 0	16 50	290	42	8.7	0.4	21.8	40	8.3	20.8	
15	8 0	11 40		49		0.9		8			Intermittent watch.
17	8 40	10 40	120	25	12.5	0.9	13.4	3	1.5	1.7	
18	8 0	10 55		53		1.0					Intermittent watch.
19	9	11	120	20	10.0	1.0	10.0				
1901 July 28	13 58	16 10	132	23	10.5	0.7	14.4	1	0.5	0.7	
Aug. 7	11 23	12 50	87	10	6.9	0.5	13.8	4	2.8	5.6	
8	11 46	15 36	180	32	10.7	1.0	10.7	16	5.3	5.3	
9	10 31	15 58	327	89	16.3	1.0	16.3	55	10.1	10.1	
1902 July 28	13 12	15 34	99	18	10.8	1.0	10.8	2	1.2	1.2	
Aug. 10	9 21	16 9	285	72	14.9	0.8	18.6	44	9.3	11.6	
11	13 40	16 10	150	96	38.4	1.0	38.4	76	30.4	30.4	
1903 July 20	11 52	13 52	120	9	4.5	0.8	5.6	1	0.5	0.6	
21	11 51	14 10	139	19	8.2	1.0	8.2	11	4.7	4.7	
23	14 0	16 0	120	25	12.5	0.7	17.9	6	3.0	4.3	
24	13 0	15 35	155	31	12.0	0.7	17.1	5	3.9	5.6	
27	11 40	15 34	234	80	20.5	0.5	41.0	6	1.5	3.0	
28	11 50	15 55	234	47	12.1	0.7	17.3	3	0.8	1.1	
28			205	33	9.7	0.7	13.9	5	1.5	2.1	[G.F.P.]
Aug. 11	11 10	15 47	174	22	7.6	0.4	19.0	21	7.2	18.0	
11	12 0	14 24	150	18	7.2	0.5	14.4	2	0.8	1.6	[G.F.P.]
1904 Aug. 5	15 10	16 21	71	9	7.6			7	5.9		
10	12 50	14 50	120	21	10.5	0.3	35.0	19	9.5	31.7	
10	12 22	15 22	180	102	34.0						[J.B.S.] Charlevoix, Mich.
11	9 50	16 26	300	95	19.0	0.5	38.0	69	13.8	27.6	
11	13 56	14 40	34	17	30.0			16	28.2		[J.B.S.] Charlevoix, Mich.
12	10 10	14 55	210	26	7.4	0.5	14.8	17	4.9	9.7	
14	12 45	16 5	200	46	13.8	0.8	17.2	15	4.5	5.6	
16	12 20	14 40	80	10	7.5	0.7	10.7	6	4.6	6.6	
1905 Aug. 9	10 10	15 45	250	77	18.5	1.0	18.5	41	9.8	9.8	at Daroca, Spain.
10	13 40	15 0	80	17	12.8	0.5	25.6	16	12.3	24.6	" " "
11	14 0	16 0	120	96	48.0	1.0	48.0	84	42.0	42.0	" " "
1906 Aug. 10	9 40	15 40	194	42	13.0	0.5	26.0	28	8.7	17.4	
11	12 20	13 20	60	16	16.0	0.5	32.0	14	14.0	28.0	
1907 Aug. 1	11 40	15 15	115	20	10.5	0.7	15.0	7	3.7	5.3	
4	12 18	15 21	180	40	13.3	0.7	19.0	15	5.0	7.1	
11	9 32	15 21	181	81	26.9	0.6	44.8	57	18.9	30.2	
1908 Aug. 1	11 29	14 20	171	43	15.1			16	5.6		
1909 July 21	13 12	14 42	90	34	22.7	1.0	22.7	6	4.0	4.0	
23	12 49	14 19	90	20	13.3	0.8	16.6	2	1.3	1.6	
26	12 42	14 42	120	43	28.7	1.0	28.7	2	1.0	1.0	
27	12 56	14 41	105	37	21.1	1.0	21.1	3	1.8	1.8	
Aug. 9	11 52	13 57	120	47	23.5	0.8	29.4	25	12.5	15.6	
10	12 52	15 38	155	122	47.2	0.9	52.4	102	39.5	43.9	22 more by [E.G.]
11	9 21	16 10	395	338	51.3	1.0	51.3	223	35.4	35.4	
12	9 38	9 49	9	8	53.3	1.0	53.3	8	53.3	53.3	
13	11 20	14 30	190	79	24.9	1.0	24.9	42	13.3	13.3	
11	11 17	11 49	32	64	120						[S.A.] counting only.
11	11 15	12 15	60	134	134						[E.] " "
11	12 3	12 26	23	40	104						[P.W.M.] " "
11	13 49	14 13	24	30	75						[P.W.M.] " "
11	14 13	14 41	28	70	150						[P.W.M.] " "
1910 July 28	11 0	11 58		11		0.7					Intermittent watch.
29	10 34	11 27		7		0.8		2			" "
31	12 50	15 50		24		0.8		7			" "
Aug. 1	12 38	15 38		51		1.0		9			" "
4	13 28	16 1		19		0.9		10			" "
6			190	59	18.6	1.0	18.6	29	9.2	9.2	
8	13 56	15 56		18		0.8		8			" "
10	12 53	15 33	160	100	37.5	0.8	46.9	61	22.9	27.9	
11	9 24	15 47	383	263	41.2	0.8	51.5	204	32.0	40.0	
11	12 36	12 56	20	35	105.0	1.0	105.0				[P.W.M.] counting only.

midnight remains visible $0^s.525$ and after midnight $0^s.385$. These figures are deduced from 393 Perseids observed 1903–1909 inclusive for which the durations were tabulated. They usually leave good trains if the meteor is as bright as the third magnitude. Their prevailing color is red or yellow, few blue or green ones being seen. Other radiants in the neighborhood often furnish meteors precisely like the Perseids themselves, and great care has to be used to keep from misidentification, especially in the case of meteors from near β Persei, which come in some numbers about August 10. This trouble is more serious earlier when the Perseids themselves are no more plentiful than some of the other radiants in contemporaneous activity. On any clear night after July 20, one can be fairly certain of seeing enough meteors to well repay observing, and often enough Perseids to obtain a good radiant for them.

The October Meteor Streams.

During this month many rich streams, whose radiants are situated in and near Orion, are in activity. This group was observed with great care because several of the best meteor observers have referred to it as the typical case in which a radiant remains in a practically constant position for quite a long period.

The paper dealing most at length with observations of the Orionids, so far as I have been able to find, is that by W. F. Denning in the *Monthly Notices R. A. S.*, Vol. 56, 74–79. He also treats briefly of them in Vol. 50 of the same publication and in his "General Catalogue of Meteor Radiants."

In all these papers it is stated that the radiant is stationary. Later these papers will be referred to at length.

Date.	Began.	Ended.	Total.	Meteors.	Rate.	Factor.	Rate Cor.	Remarks.
	h. m	h. m.	m.					
1900 Oct. 19	11 39	17 13	334	117	21.0	0.9	23.3	
26	8 39	10 44	125−	16	7.7+	1.0	7.7+	
01 18	12 27	16 9	220	63	17.2	0.9	19.1	
19	11 24	16 37	313	83	15.9	1.0	15.9	
02 19	12 13	16 50	225	16	3.5	0.2±	17.5±	
03 18	13 13	16 16	183	54	17.7	0.8	22.1	
19	11 24	17 38	360	144	18.8	1.0	18.8	31 by [J.P.S.]
04 14	13 22	15 5	100	23	13.8	0.8	17.2	
16	12 23	15 29	160	39	14.6	0.9	16.2	
18	11 8	17 16	360	75	12.5	0.6	20.8	
18	12 0	16 30	270	60	13.3	0.7	19.0	[J.B.S.]
18	12 0	16 40	280	55	11.8	0.7	16.9	[J.P.S.]
05 20	14 16	17 16	180	34	11.3	0.6	18.8	
23	14 25	16 15	110	28	15.3	0 9	17.0	
06 12	12 2	12 58	56	13	13.9	1.0	13.9	
25	12 36	17 6	270	76	16.9	1.0	16.9	
26	14 43	16 13	150	33	13.2	1.0	13.2	
07 15	15 18	16 46	88	20	13.7	0.9	15.2	
08 18	11 12	13 45	150	24	9.6	0.8	12.0	
09 12	14 55	17 5	120	27	13.5	1.0	13.5	
13	13 17	16 52	215	47	13.1	0.8	16.4	
15	11 30	16 0	270	88	16.7	0.8	20.9	
19	13 39	15 19	100	20	12.0	0.6±	20.0±	
22	12 0	15 50	230	69	18.0	0.9	20.0	
10 8	15 7	16 32		7				Intermittent watch.
13	14 5	15 10		8				" "
25	11	13		11				" "

My observations of this most important group of radiants began in 1900 and, during every October since, some data have been collected bearing upon them. The following table gives the number of meteors observed in October for the year 1900 to 1910 inclusive, on nights when regular observations were made. The columns give from left to right :(1) Date, (2) time of beginning, (3) time of ending, (4) number of minutes actually spent in observing, (5) number of meteors, (6) rate, (7) factor depending on sky, etc., (8) corrected rate. The rates are for one observer.

Seven other observers have assisted in this work. Their assistance was especially valuable in 1904, when J. B. Smith and J. P. Smith on October 18 observed 115 meteors at a station 7 miles southwest from the Charlottesville, Va., station.

The other observers only assisted, as a rule, in counting and recording meteors. Of the 1,279 meteors seen in this month I personally observed 1,075 ±. On working over the maps, on which are the paths of such meteors as were well enough observed to be worth plotting, 64 radiants were obtained, which were considered sufficiently accurately determined to have parabolic orbits calculated for them. The elements will be found in the general table of orbits. In the table of poorly determined or uncertain radiants will be found 7 more.

Of the good radiants, the 55 which fall within the region of the sky shown are plotted in Fig. 1. This figure is purposely drawn on a very large scale so that the radiants could be accurately plotted to tenths of a degree. The 11 radiants that belong to the main stream and all of which were observed on either Oct. 18 or Oct. 19 (when L was between 115°.7 and 117°.4) fall within a quadrilateral bounded by $\alpha = 90°.0$ and $\alpha = 92°.1$, and $\delta = +13°.6$ and $\delta = 16°.6$. If No. 112 and No. 113 which were observed by J. B. Smith and J. P. Smith are omitted, leaving the 9 observed by myself, the limits reduce to $\alpha = 90°.1$ and $\alpha = 92°.1$, $\delta = +13°.6$ and $\delta = +15°.9$.

In other words the greatest possible deviation from the mean when all are considered is $\Delta\alpha = \pm 1.°05$, $\Delta\delta = \pm 1°.5$. When the 9 observed by myself are considered this falls to $\Delta\alpha \pm 1°.0$, $\Delta\delta = \pm 1°.15$. These greatest possible residuals give evidence of the probable error of any single radiant determined and how nearly the positions can be relied on.

No. 121 should have been combined at one third weight with No. 122 before the orbits were calculated, but, since that was not done, the positions of the two are so combined and plotted on the map as one point. No. 123 and No. 124 were weighted equally and treated in the same manner. In both of these cases they are undoubtedly identical, having been observed on the same night, however, as two maps were used, two positions were obtained, and to be on the safe side orbits were calculated for each position.

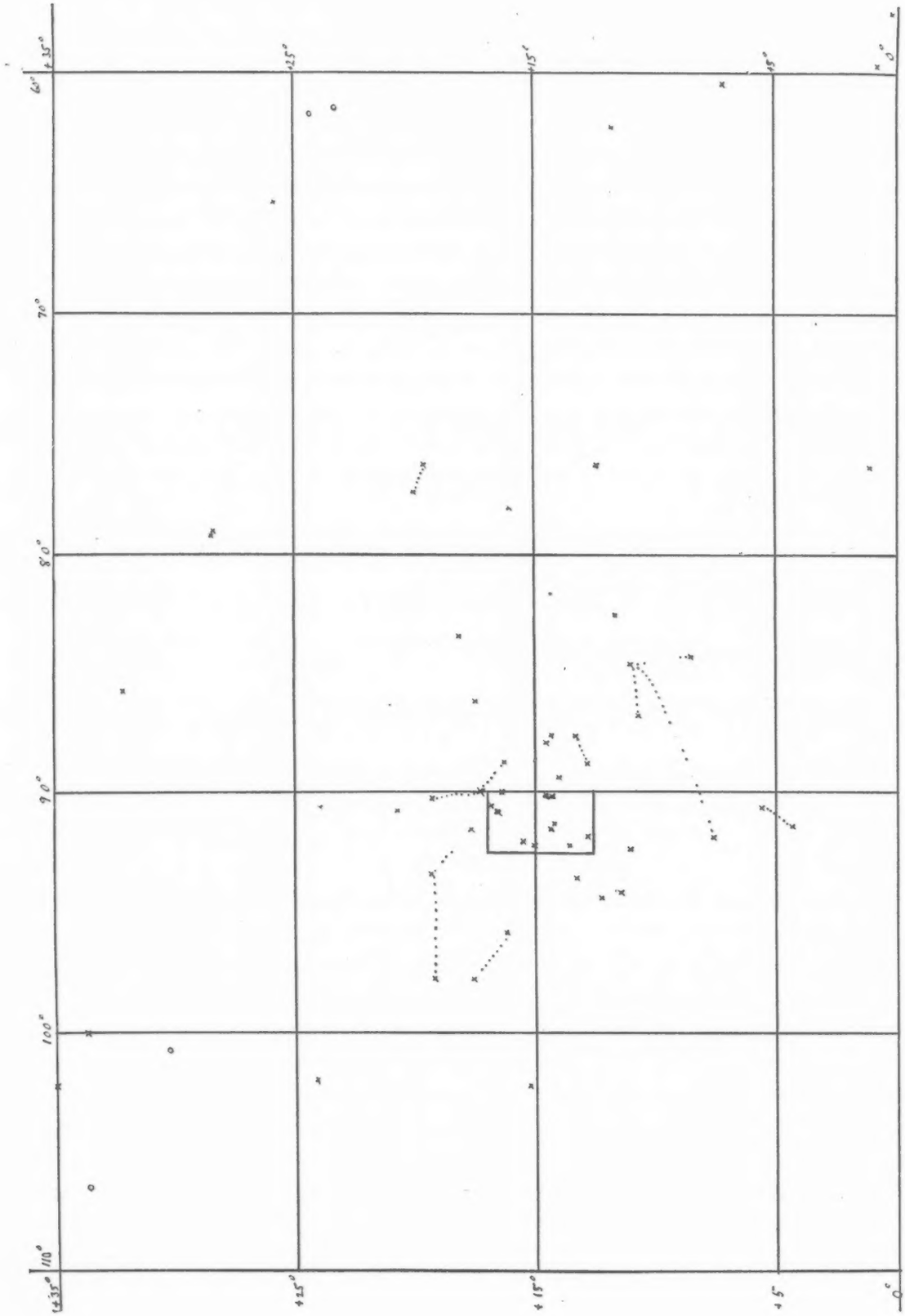

FIG. 1.

The following table groups these 9 positions with regard to the year and L.

No.	Year.	L	α	ϱ	ι	η	π	Ω	q
		°	°	°	°	°	°	°	
105	1901	116.2	91.2	+14.2	160.5	130.4	106.0	25.2	0.578
100	1903	115.7	92.1	13.6	159.9	128.0	110.8	24.8	0.618
112	1904	116.4	90.0	16.4	164.6	133.2	111.9	25.5	0.529
113	1904	116.4	90.8	16.6	165.1	132.3	110.2	25.5	0.544
114	1904	116.4	92.0	15.5	161.0	137.4	120.4	25.5	0.456
117	1908	116.5	90.2	14.3	160.2	132.4	110.3	25.4	0.542
Mean.		116.3	91.0	+15.1	161.9	132.3	113.3	25.3	0.544
123, 124	1900	117.4	91.4	+15.4	162.7	132.1	110.2	26.4	0.548
121, 122	1901	117.2	90.7	13.9	159.4	132.7	111.6	26.2	0.541
118	1903	116.6	91.5	14.4	158.7	136.5	118.7	25.7	0.472
Mean.		117.1	91.2	+14.6	160.3	133.8	113.5	26.1	0.520

As the extreme range in L is only 1°.7, and as the difference of the mean α's and δ's fall within the possible errors, it may be permitted to combine the above in ratio 2 to 1, to determine the best parabolic elements for the mean L. Whence we obtain:

1900.0 $L = 116.6°$ $\alpha = 91.1°$ $\delta = +14.9$

Elements of Orionids:
$\iota = 161°.4$
$\eta = 132°.8$
$\pi = 113°.4$
$\Omega = 25°.6$
$q = 0.536$

The following combinations are also suggested:

No.	Year.	L	α	δ	ι	η	q	π	Ω
		°	°	°	°	°		°	°
97	1904	114.4	91.6	+17.7	170.3	127.7	0.628	98.9	23.5
115	1904	116.4	93.4	+19.4	171.8	128.4	0.612	102.2	25.5
126	1905	118.2	97.8	+19.2	172.0	124.4	0.678	96.0	27.3
		116.3			171.4		0.639	99.0	26.1
125	1905	118.2	88.7	+16.2	163.0	138.2	0.443	123.6	27.3
128, 129	1909	120.3	90.0	+17.4	165.3	139.7	0.414	128.8	29.4
130	1905	121.1	90.3	+19.3	169.8	140.0	0.394	132.1	30.3
		119.9			166.0		0.417	128.2	29.0
80	1904	112.4	95.9	+16.1	166.4	118.1	0.776	77.7	21.5
95	1909	113.3	97.9	+17.7	169.8	115.7	0.809	73.4	22.4
		112.8			168.2		0.792	75.6	22.0
87	1909	113.3	76.3	+19.7	170.8	149.8	0.252	132.0	22.4
96	1904	114.4	77.4	+20.0	169.2	156.1	0.163	135.8	23.5
		113.8			170.0		0.208	133.9	23.0
103, 104	1901	116.2	85.7	+10.9	150.4	138.0	0.446	121.2	25.2
98	1903	115.7	84.2	+ 8.6	144.6	138.7	0.434	122.2	24.8
134	1906	122.9	91.9	+ 7.7	143.1	137.3	0.457	126.6	32.0
		118.3			146.0		0.446	123.3	27.3
74	1909	110.4	91.3	+ 4.4	144.1	118.4	0.772	76.3	19.5
78	1909	111.4	90.7	+ 5.7	145.8	121.3	0.729	83.0	20.4
		110.9			145.0		0.750	79.6	20.0
99	1903	115.7	87.6	+13.3	157.3	135.0	0.498	114.8	24.8
111	1904	116.4	88.9	+12.9	156.6	134.1	0.513	113.8	25.5
		116.0			157.0		0.506	114.3	25.2
73	1909	110.4	42.0	+10.5	17.5	163.3	0.103	166.5	20.4
83	1909	113.3	47.5	+13.1	19.5	165.6	0.062	173.5	22.4
101	1901	116.1	47.8	+11.8	18.8	161.3	0.104	167.8	25.2
		113.3			18.6		0.090	169.3	22.7

However, there still remain within the area bounded by $\alpha = 75°$ to $100°$, $\delta = +5°$ to $+25°$, 13 radiants which apparently do not have any near connection with any group. No less than 8 of these fall within the area bounded by $\alpha = 85°$ to $95°$, $\delta = +10°$ to $+20°$. These 8 were observed on October 13, 15, 18, 19 and 25 of various years.

As stated before the radiants of the main stream which appears on October 18 and 19, all fall within an area bounded by $\alpha = 90°.0$ to $92°.1$, $\delta = +13°.6$ to $+16°.6$. Of the 8 radiants spoken of above only the two seen 1910, October 15, are near enough to the principal radiant for errors of observation to throw them within this area. For the other 6 this possibility hardly exists. Indeed, 3 of these 6 were observed on October 18 and 19, and of the last 3, No. 79 was uncertain, having been gotten from only 3 meteors. To show that the distribution is not entirely without order, even for these isolated cases, it should be noted that no radiant observed after October 19 lies south of $+15°$, except No. 134 and No. 137. No. 137 is too far to the west to enter into the discussion and 134 is evidently connected with No. 103, 104 and 98 in a small system, separate from the main current. I feel quite satisfied that the positions of the radiants given in the tables represent their real places within about 1°, sometimes less.

Two curious examples of the recurrence of radiants in the same places are given by No. 108 at $\alpha = 79°.2$, $\delta = +28°.6$ on 1904, October 18.81, and No. 132 at $\alpha = 79°.0$, $\delta = +28°.5$ on 1906, October 25.84; also by No. 77 at $\alpha = 87°.9$, $\delta = +14.6$ on 1909, October 13.94 and No. 110 at $\alpha = 87°.5$, $\delta = 14°.4$ on 1904, October 18.81.

The general conclusions drawn from my October meteor observations are as follows:

Within an area bounded by $\alpha = 79°$ to $103°$, $\delta = +4°$ to $+25°$, from October 12 to 26 inclusive, are found a great number of distinct radiants, which in general furnish similar meteors.

That on October 18 or 19 the maxima occur, the principal radiant being always within less than 2° of $\alpha = 91°$, $\delta = +15°$.

That minor branches or streams appear which give evidence of an eastward movement in longitude with increase of date.

That these minor streams sometimes appear only during the same October or may reappear in following years.

That many isolated radiants are given which do not seem to have any connection with others either in position or elements.

That since for these radiants an error of as much as 2° in the given position seems unreasonable, from a study of the maps and records, they can not belong to one radiant, considered stationary.

Lastly that the suggested explanation is that most of the meteor currents had a common origin, but with the lapse of time have been separated into many minor branches, besides the great central stream.

These minor streams come irregularly, in most cases, and it is by no means necessary to suppose that any given one should appear every year. Indeed it may well be that a small number of meteors give a radiant one year which could never again be observed, because without doubt in the immense extent of such a general system or family of currents many small isolated groups are present which from their small size would never again cut the earth's orbit in future returns to the sun.

In Vol. 56 of the *Monthly Notices R. A. S.*, 74–79, there is an article by W. F. Denning on the Orionids, in which he gives his grounds for concluding that their radiant remains stationary. However, in reviewing it, it is found that in the first table of 19 radiants, 6 are useless for this discussion.* In the second table of 30, 17 are also of no value, the reason being that observations of different nights and years were combined. The 11 available ones in the first table, all observed by Denning himself, fall within an area 3° by 3½°. The 13 in the second fall within 6° by 7°. Therefore what he then called a stationary radiant covered at least 6° by 7°, or were we to discuss all the 49 radiants given by him 8° by 9°. Later in his "General Catalogue" are found 57 radiants assigned to this shower, scattered over an area of 8° by 8½°. When the 25 radiants not observed on a single night are thrown out, the rest lie within 8° by 6°. Nos. 51 and 52 require mention. Deduced from the same observations of Zezioli by Schiaparelli and later Denning, the results differ 2° in R.A. and 3° in decl. Further most or all of the 19 radiants given as Group LXXIX appear to have as much right to be included under the Orionids as many given as such. Group LXXV of 10 radiants, 8 gotten by combining two or more nights' observations, also shows the same coordinates as the Orionids. Were these two groups combined, the Orionids could be made to appear throughout the whole year. Indeed, radiants are very numerous in this region of the sky and stationary radiants can be made to appear by loose combinations of observations and uncritical selection of material. A close study of Denning's own "General Catalogue" from 244 to 250 inclusive will show that many other equally logical conclusions could be reached by merely regrouping. Therefore it is very unsafe to conclude from this data that the Orionids really have a stationary radiant, as stated by him.

The Leonid Meteors.

Observations on the Leonids were first made on November 14, 1898. Afterwards they were continued in 1899, 1900, 1901, 1903, 1904, 1907 and 1909.

*See Denning's own words p. 78, lines 7 to 10.

Cloudy weather or moonlight prevented observations in 1902, 1905 and 1908. However, in all the other years mentioned meteors were observed, the richness of the showers varying very greatly.

Three tables are given for the Leonids, two quite similar to those already explained for the Perseids. The third is one giving the estimated durations for the Leonids in tenths of seconds. These estimates of course cannot be very accurate, but it is believed the means represent the truth fairly well. Practically all these meteors were observed after midnight. The mean duration for the 257 given in the table is $0^s.39$.

Referring to the table of radiants it will be seen that no certain movement from date to date is indicated. But since L changes only a little over 3° for the extreme dates, this is hardly to be wondered at.

Altogether about 1,030 + meteors have been observed between November 12–16 of the years enumerated above.

1901, November 14, furnished the finest shower; 1898, November 14, the second best; while in 1903 and 1904 considerable numbers appeared, with many very bright meteors, particularly in 1903.

Many Leonids give exceptionally long apparent paths and leave splendid trains which remain visible from 1^s to 5^s, often longer. They also furnish bright meteors of several different colors, which would seem to indicate the preponderance of different elements in individual meteors.

Date.	Beginning.		Ending.		Total.	$\overset{s}{\rightarrow}$ Total.	Rate.	Factor.	Rate cor.	$\overset{s}{\rightarrow}$ Leonid.	Rate.	Rate cor.	Remarks.
	h.	m.	h.	m.	m.								
1898 Nov. 14	13	15	17	40	240 ±	120	30.0	0.4	75.0	100	25.0	62.5	
1899 " 14	14	11	14	53	42	7	10.0	0.2	50.0	7	10.0	50.0	
1899 " 14	12	30	18	30	360	20	3.3	0.4	8.2	14	2.3	5.7	
1900 " 12	14	13	16	50	157	11	4.2	0.6	7.0	2	0.8	1.3	
1900 " 13	16	40	17	50	70	14	12.0	0.6	20.0	9	7.7	12.8	
1900 " 14	12	12	17	12	150	25	5.0	0.4	12.5	15	6.0	15.0	
1900 " 15	12	20	17	30	235	30	7.7	0.7	11.0	13	3.3	4.7	
1901 " 13	12	30	17	45	300	57	11.4	1.0	11.4	15	3.0	3.0	
1901 " 14	16	50	18	18	88	82	55.9	0.5	111.8	75	51.1	102.2	
1901 " 15	13	10	17	55	265	74	16.8	1.0	16.8	41	9.3	9.3	
1903 " 12	14	35	16	18	103	14	8.2	0.6	13.7	1	0.6	1.0	
1903 " 14	12	28	17	28	275	80	17.5	0.5	35.0	55	12.0	24.0	2 observers.
1903 " 15	12	39	16	58	233	92	23.7	0.4	59.2	78	20.0	50.0	
1903 " 18	10	50	13	20	140	17	7.3	0.6	12.2	1	0.4	0.7	
1904 " 14	12	33	17	23	275	93	20.3	0.9	22.6	65	14.2	15.8	
1904 " 16	14	55	17	16	141	28	11.9	0.6	19.8	10	4.3	7.2	
1906 " 16	11	50	17	21	306	105	17.8	0.8	22.2	52	10.2	12.8	
1907 " 13	14	18	16	18	120	18	9.0	1.0	9.0	1	0.5	0.5	
1907 " 14	14	38	15	58	140	48	20.6	1.0	20.6	3	1.1	1.1	
1909 " 14	12	32	15	52	180	44	14.7	0.9	16.3	14	4.7	5.2	
1909 " 15	14	12	17	36	160	54	20.2	0.8	25.2	10	3.7	4.6	

G.M.T.	L	α	δ	$\overset{s}{\rightarrow}$	
	°	°	°		
1898 Nov. 14.86	142.7	153.0	+20.8	29	
1899 " 14.82	142.4	151.4	+19.3	3	
1899 " 15.86	143.5	151.2	+20.6	8	
1900 " 14.82	141.3	151.	+21.		
1900 " 15.84	144.2	151.	+21.		
1901 " 13.86	142.0	150.8	+22.5	13	
1901 " 14.95	143.1	151.6	+21.8	49	
1901 " 15.87	144.0	150.6	+22.4	37	
1903 " 14.84	142.4	151.2	+21.7	20	
1903 " 15.83	143.5	150.8	+22.0	51	
1903 " 15.83	143.5	150.8	+22.1	1	No. 3–562
1904 " 14.88	143.3	150.6	+21.8	34	
1904 " 16.85	145.3	151.6	+22.1	6	
1906 " 16.82	144.8	151.3	+22.4	36	
1909 " 14.84	143.1	149.3	+21.6	7	
1909 " 16.00	144.2	150.6	+23.1	10	

DURATIONS OF LEONIDS.

Date.	1.2	1.0	0.6	0.5	0.4	0.3	0.2	0.1
1903 Nov. 12						1		
" 14				2	9	20	2	
" 15				7	46	19		
" 18					1			
1904 " 14			6	12	30	9	4	1
" 16			1	2	7			
1906 " 16	1	2	2	4	26	13	2	1
1907 " 13							1	
" 14					1	1		1
1909 " 14			1	2	5	2		
" 15				1	6	6	1	
Totals	1	2	10	30	131	70	10	3

The following November radiants are considered to be connected and are given with their elements. The means are given for each group.

No.	Year.	L	α	δ	ι	η	q	π	Ω
		°	°	°	°	°		°	°
150	1904	143.3	138.0	+49.2	122.6	113.3	0.834	99.4	232.5
158	1900	144.2	138.8	+47.8	124.6	113.7	0.829	100.9	233.5
160	1906	144.8	137.	+49.	122.2	116.3	0.797	106.6	234.1
					123.1		0.820	102.3	233.4
176	1907	142.6	139.5	+35.0	147.6	107.4	0.900	86.7	231.9
147	1909	143.1	139.9	+34.8	147.6	107.6	0.899	87.5	232.3
					147.6		0.900	87.1	232.1
146	1909	143.1	128.6	− 2.0	140.0	120.6	0.733	113.6	52.3
163	1906	144.8	130.7	− 1.2	143.0	120.0	0.741	114.2	54.1
					141.5		0.737	113.9	53.2
152	1903	143.5	72.5	+40.5	43.2	153.0	0.203	178.9	232.8
155	1901	144.0	70.2	+41.5	40.2	150.3	0.242	173.9	233.2
					41.7		0.222	176.4	233.0
142	1901	142.0	159.2	+52.6	112.6	94.3	0.984	59.8	231.3
148	1909	143.1	155.0	+58.0	107.0	94.4	0.993	61.2	232.3
157	1901	144.0	155.4	+50.9	116.3	98.8	0.966	70.8	233.3
					112.0		0.981	63.9	232.3

EXISTENCE OF STATIONARY RADIANTS.

As a typical case of a stationary radiant the group No. XLIII in Denning's "General Catalogue of Radiant Points of Meteoric Showers" was chosen for study. So far as the theoretical impossibility of all the radiants put within this group being really connected is concerned, it is most clearly proved by Th. Bredichin in his memoir "Sur L'Origine Des Étoiles Filantes," pp. 39–44. However, further observational data will be quite useful.

Therefore 68 maps have been examined which contain meteors recorded from 1900 to 1909 inclusive, and on which meteors coming from the region $\alpha - \beta$ Persei would be plotted. These maps were used in January, April, July, August, October and November, all being months during which this shower is supposed to be visible.

On 24 of the maps not a single meteor can be found whose projected path would come within 5° of the point $\alpha = 47°.3$, $\delta = +45°.0$. On the 18 others one meteor might fall within these wide limits but is considered to belong to some other radiant for good reasons. Of the remaining 26 maps, 12 have one meteor each which would satisfy conditions. On the 14 several are found, but in most cases these were used about August 11, and these meteors clearly belonged to the main Perseid stream.

However, three radiants are actually found in this region. No. 33, $\alpha = 42°.4$, $\delta = +49°.2$, No. 38, $\alpha = 43°.8$, $\delta = +39°.6$ and No. 54, $\alpha = 46°.0$, $\delta = +45°.3$. For completeness No. 65, $\alpha = 44°.1$, $\delta = +52°.0$ might be added. The dates on which these were observed are as follows: 1901 August 8 and 9, 1903 August 11, 1904 August 11. For group No. XLIII the limits given are $\alpha = 42°$ to 51°, $\delta = +39°$ to $+49°$. Therefore three of the above fall within them. In no other month have I been able to confirm the existence of a radiant within these limits.

A little analysis of the data in the "General Catalogue" will be helpful in understanding how such results were gotten. No less than 59 positions are there given. Of these only 15 were obtained from observations of a single night, therefore the remaining 44 are nearly worthless for the discussion of stationary radiation. For example (1) depends on 8 meteors seen within 24 days, (13) on 3 meteors within 9 day, (22) on 11 meteors in 8 days, (50) on 34 meteors during all October and November, etc. Equally bad or worse examples could be quoted, these being taken at random. It should be plain to any observer of meteors or to any person familiar with their theory that such combinations are unsafe and generally misleading. Fortunately out of the 15 positions properly determined, 4 were observed by Zezioli and the resulting orbits calculated by Schiaparelli. His results follow:

	Date.	L	α	δ	ι	Ω	π	q
D Z+S								
(2) = (4)	Jan. 11	201°	47°	+40°	9°D	291°	131°	0.970
(19) = (137)	Aug. 7	45	42	+48°	53 R	135	341	0.949
(26) = (142)	Aug. 11	49	47	+43	43 R	139	338	0.973
(4) = (150)	Sept. 18	86	51	+39	41 R	176	273	0.561

For purposes of comparison my own four orbits follow:

Ol.	Year.								
(33)	1901	Aug. 8	46°.5	42°.4	+49°.2	126°.5	136°.0	339°.0	0.973
(38)	1901	" 9	47 .5	43 .8	+39°.6	142 .6	136 .9	298 .9	0.989
(54)	1903	" 11	48 .9	46 .0	+45 .3	134 .0	138 .4	295 .4	0.973
(65)	1904	" 11	49 .7	44 .1	+52 .0	122 .9	139 .2	339 .1	0.983

Referring to the four radiants of Zezioli above, we are forced to believe that one of the best of meteor observers plotted 4 radiants over limits of $9° \times 9°$, when they should have been near the mean position, if we try to obtain a stationary radiant here. But if doubt remains a mere glance at either of the above tables of elements must banish it completely.

That the radiants (137) and (142) of Zezioli are perhaps the same with its position shifted in the 4-day interval is indeed probable. Also my orbits (33) and (65), and (38) and (54) are probably the same, their positions being slightly shifted between the dates. But that all 8 orbits can refer to the same stream is an obvious and mathematical impossibility.

As for the rest of the 15 radiants we had under discussion, (6) seems determined by Denning from a fire-ball path, (7), (34), (38), are found by the same observer from 4 to 6 meteors each. For (14) and (43) no numbers are given. (6), (18), (20), (21), (23) are from duplicate observations of one meteor, and so are not very valuable for the question under consideration.

Therefore of the whole 15 reliable ones, which include (6), (18), (20), (21), (23) also, 7 fall in August. It then follows that we find 8 radiants for all the other months scattered over the area 9° to 10° and it is on these mainly that the claim for an observed stationary radiant here must rest. It is of further interest to remark that these radiants were observed all the way from 1867 to 1897 inclusive.

TABLE OF RADIANTS.

No.	G.M.T.1900+	L	α	δ	l'	b'	l	b	ι	η	q	π	Ω	$\frac{s}{\rightarrow}$	
		°	°	°	°	°	°	°	°	°	°	°	°		
1	05 Jan. 1.80	191.3	199.4	+47.6	172.2	+50.0	112.0	+74.4	92.8	105.3	0.915	131.9	281.3	3	
2	09 18.83	208.9	174.4	+ 8.4	171.6	+ 5.4	145.6	+ 8.0	162.8	152.0	0.217	142.6	298.6	5	
3	09 18.83	208.9	211.1	+21.7	200.2	+32.0	189.8	+53.6	124.9	101.1	0.948	140.8	298.6	3	D., CLXI, 3, Z15?
4	06 Apr. 18.78	297.4	274.0	+33.2	276.0	+56.5	2.0	+75.1	96.7	76.6	0.951	181.5	28.3	3	Lyrids.
5	04 18.75	297.9	277.0	+36.7	281.2	+59.9	342.5	+76.7	99.7	99.2	0.989	227.1	27.8	4	Lyrids.
6	03 July 21.76	28.5	271.8	+41.2	273.2	+64.6	224.4	+32.5	33.6	103.6	0.960	325.4	118.3	4	
7	09 21.87	29.2	4.6	+31.4	17.6	+26.8	6.3	+44.6	133.2	105.9	0.940	330.7	118.9	4	Compare 10?
8	09 21.87	29.2	348.9	+ 0.9	350.9	− 5.2	323.7	− 7.5	162.6	154.1	0.193	67.2	298.9	4	
9	03 23.84	30.4	0.2	+24.9	10.6	+22.6	353.4	+36.5	137.3	119.7	0.782	359.5	120.1		
10	03 24.81	31.4	2.4	+29.4	14.7	+25.8	359.0	+42.2	133.1	113.2	0.858	347.4	121.1	5	Compare 7?
11	03 24.81	31.4	15.8	+34.8	28.7	+25.8	26.2	+42.9	137.0	93.6	1.012	301.2	121.1	4	Z 110.
12	03 27.79	34.4	2.0	+41.4	21.0	+36.6	2.2	+59.6	116.5	105.5	0.943	335.0	124.0	4	
13	03 27.79	34.4	5.1	+51.3	30.8	+43.9	20.2	+72.6	106.9	94.1	1.010	312.2	124.0	3	D., VII, 10, Z101?
14	03 27.79	34.4	299.9	+49.0	325.9	+66.7	241.8	+49.0	52.4	107.8	0.921	339.6	124.0	7	
15	03 27.79	34.4	307.9	+56.7	349.1	+69.9	238.8	+57.9	60.3	102.9	0.965	329.8	124.0	4	Z 114.
16	03 27.79	34.4	337.7	−16.6	333.2	− 6.7	294.9	− 7.6	40.1	168.2	0.042	100.4	304.0	5	δ Aquarids.
17	03 July 27.79	34.4	339.9	+60.9	23.0	+60.2	250.7	+79.2	81.3	96.4	1.003	316.8	124.0	11	D., CCLXIV, 3*
18	03 27.79	34.4	350.4	+48.2	16.5	+46.7	332.6	+71.8	98.9	105.9	0.939	335.9	124.0	4	Compare 24.
19	09 27.91	35.0	15.4	+15.4	20.1	+ 8.2	9.4	+13.5	165.1	114.5	0.841	353.6	124.6	3	
20	03 28.79	35.4	22.2	+59.5	47.8	+45.6	79.0	+72.9	102.5	78.2	0.973	281.4	125.0	3	D, XV, 19* [G.F.P.]
21	03 28.79	35.4	297.0	+ 7.3	300.6	+27.9	253.3	+18.1	22.6	126.1	0.663	17.2	125.0	5	[G.F.P.]
22	03 28.79	35.4	337.7	−13.7	334.3	− 4.0	296.3	− 4.6	27.9	170.2	0.030	105.4	305.0	12	δ Aquarids.
23	03 28.79	35.4	352.9	+21.4	357.5	+22.4	327.1	+31.9	121.1	141.8	0.388	47.6	125.0	6	[G.F.P.
24	03 28.79	35.4	357.6	+47.7	21.8	+43.6	353.3	+69.8	105.4	103.3	0.962	331.6	125.0	4	Compare 18.
25	02 28.82	35.5	337.6	−15.4	333.4	− 5.6	295.1	− 6.2	32.0	168.2	0.042	101.5	305.1	9	δ Aquarids.
26	01 28.85	35.9	10.3	+45.6	30.1	+37.3	21.4	+62.1	117.2	95.6	1.002	318.6	125.5	5	
27	07 Aug. 1.73	39.5	17.8	+36.8	31.3	+26.8	23.1	+45.0	134.0	101.0	0.978	340.8	128.8	3	
28	07 1.73	39.5	26.9	+53.2	46.7	+38.8	58.4	+64.4	114.3	81.6	0.993	292.0	128.8	4	
29	07 4.75	42.1	18.8	+45.8	36.7	+34.6	29.4	+57.9	121.5	96.5	1.001	324.6	131.7	3	
30	04 5.88	43.9	22.4	+56.5	45.8	+43.0	51.5	+71.4	71.6	87.4	1.012	308.3	133.4	5	
31	01 8.79	46.5	31.2	+31.8	40.2	+18.0	35.0	+30.3	149.3	99.5	0.986	334.8	136.0	2	Compare 37.
32	01 8.79	46.5	32.2	+20.8	37.1	+ 7.3	30.4	+12.3	167.3	105.2	0.944	346.4	136.0	4	
33	01 Aug. 8.79	46.5	42.4	+49.2	55.6	+31.2	27.0	+52.0	126.5	101.5	0.973	339.0	136.0	3	A. P. 14; 21, Z 137.
34	01 9.77	47.5	10.4	+49.0	32 4	+40.2	7.1	+64.6	110.1	106.0	0.937	348.8	136.9	6	D., VII, 20*, 21*
35	01 9.77	47.5	14.9	+27.9	24.8	+19.8	6.3	+31.6	141.0	56.3	0.770	249.6	136.9	3	D, XIII, 5, 4*
36	01 9.77	47.5	40.6	+52.6	55.8	+34.9	66.8	+57.9	120.5	79.6	0.980	296.0	136.9	20	A. P. 14; 26 and 27?
37	01 9.77	47.5	31.8	+32.7	41.0	+18.6	35.7	+31.4	148.0	100.6	0.980	337.9	136.9	3	Compare 31.
38	01 9.77	47.5	43.8	+39.6	53.2	+21.9	58.2	+36.9	142.6	81.0	0.989	298.9	136.9	3	D., XXXV, 3.
39	01 9.77	47.5	349.5	+27.3	2.2	+29.1	323.0	+37.9	97.8	38.4	0.390	213.6	136.9	4	
40	05 9.57	47.3	29.0	+38.5	41.0	+24.8	35.2	+41.8	137.6	81.4	0.991	299.5	136.8	4	D, XXV, 12*, 13*
41	09 9.88	47.6	35.5	+34.9	44.9	+19.6	42.6	+33.2	146.8	93.8	1.009	324.6	137.1	4	D, XXX, 11.
42	09 9.88	47.6	41.4	+26.3	47.0	+ 9.8	46.5	+16.7	163.3	90.8	1.013	318.6	137.1		
43	09 9.88	47.6	42.4	+60.1	60.6	+41.4	84.3	+66.9	108.8	76.3	0.957	289.7	137.1	4	Z 139.
44	02 10.82	48.2	1.1	+60.1	35.4	+52.3	331.1	+80.0	92.4	99.3	0.985	336.2	137.7	4	
45	02 10.82	48.2	327.9	+29.0	342.5	+39.0	288.5	+37.7	57.7	133.7	0.529	45.1	137.7	3	
46	09 10.93	48.6	5.2	+18.1	12.1	+14.5	345.4	+21.2	139.8	145.9	0.319	69.8	138.1	3	
47	09 10.93	48.6	35.0	+56.5	54.0	+39.7	63.3	+66.0	113.2	83.9	1.002	305.8	138.1	10	A. P. 10; 8.
48	04 10.83	48.8	10.2	+55.2	36.8	+45.5	6.8	+73.1	102.9	101.1	0.976	340.3	138.2	3	D, VII, 23.
49	04 Aug. 10.83	48.8	28.2	+31.2	37.4	+18.4	28.2	+30.6	147.8	107.1	0.926	352.4	138.2	5	Compare 31? 37?
51	03 11.77	48.8	5.7	+58.5	37.6	+49.0	21.2	+70.3	107.7	98.8	0.989	335.9	138.2	3	Compare 173, 57.
52	03 11.77	48.9	23.0	+52.4	43.6	+39.2	34.5	+65.2	114.2	95.8	1.002	329.9	138.4	3	D, XV, 34. [G.F.P.]
53	03 11.77	48.9	32.7	+56.2	52.3	+40.0	58.4	+66.8	112.9	94.0	1.009	326.2	138.4	5	A. P. 10; 15. P. 14, 33. Compare 175. [G.F.P.]
54	03 11.77	48.9	46.0	+45.3	56.8	+26.8	64.6	+44.9	134.0	78.5	0.973	295.4	138.4	3	D, XLIII, 23, 26, Z 142. [G.F.P.]
55	02 11.84	49.1	31.6	+35.1	41.8	+20.9	35.5	+35.2	144.1	100.6	0.979	359.8	138.5	4	A. P. 10; 6.

TABLE OF RADIANTS.—*Continued.*

No.	G.M.T.1900+	L	α	δ	l'	b'	l	b	ι	η	q	π	Ω	$\frac{s}{\rightarrow}$	
		°	°	°	°	°	°	°	°	°	°	°	°		
56	09 11.87	49.6	0.2	+26.6	11.4	+24.2	339.8	+34.3	117.5	140.6	0.409	60.1	139.0	4	
57	09 11.87	49.6	7.5	+55.4	35.1	+46.5	357.5	+73.3	100.6	103.0	0.962	345.1	139.0	4	D, VII, 23. Compare 173, 51.
58	09 11.87	49.6	18.4	+25.9	26.9	+16.8	9.3	+26.8	146.7	124.8	0.684	28.5	139.0	3	
59	09 11.87	49.6	43.6	+28.4	49.4	+11.2	49.3	+19.0	171.0	180.0	1.013	139.0	139.0	3	
60	09 11.87	49.6	58.3	+74.3	76.6	+52.3	90.4	+61.8	111.9	71.8	0.914	282.7	139.0	3	D. XLV, 12.
61	09 11.87	49.6	335.6	+68.4	34.6	+66.0	246.9	+68.9	69.8	96.4	1.001	331.7	139.0	4	
62	09 11.87	49.6	339.3	+52.4	11.6	+54.4	293.3	+63.8	78.0	113.4	0.853	5.8	139.0	9	
63	09 11.87	49.6	353.4	+76.4	57.2	+64.5	63.6	+75.4	104.2	86.4	1.011	311.7	139.0	3	D, CCLXXVII, 7*, 8*
64	04 11.84	49.7	42.1	+30.4	48.8	+13.5	48.2	+23.0	23.0	89.1	1.013	317.3	139.2	3	[J.B.S.]
65	04 Aug. 11.84	49.7	44.1	+52.0	58.0	+33.5	31.2	+55.8	122.9	100.0	0.983	339.1	139.2	5	Z. 139, [J.B.S.]
66	09 13.87	51.5	15.5	+43.5	32.9	+33.7	10.6	+53.9	119.1	112.4	0.865	5.4	140.9	5	
67	09 13.87	51.5	21.8	+45.7	38.9	+33.6	23.2	+55.2	121.6	105.4	0.942	351.7	140.9	5	
68	04 14.82	52.6	23.2	+21.2	29.3	+10.7	12.4	+17.1	158.2	127.6	0.636	37.2	142.0	3	
69	04 14.82	52.6	35.0	+51.8	51.6	+35.4	50.0	+59.4	120.6	91.0	1.012	324.0	142.0	3	A. P. 10; 13.
70	04 14.82	52.6	46.2	+64.4	65.1	+44.6	11.0	+71.8	103.9	101.8	0.970	345.6	142.0		
71	04 14.82	52.6	54.9	+63.4	69.6	+42.4	105.3	+69.5	102.6	73.7	0.932	289.4	142.0		
72	04 14.82	52.6	73.0	+43.2	76.8	+20.5	97.5	+32.3	137.6	52.3	0.635	245.9	142.0	3	
73	09 Oct. 13.01	110.4	42.0	+10.5	42.7	− 5.4	1.6	− 5.5	17.5	163.3	0.103	166.5	20.4	5	Compare 101, 83.
74	09 13.01	110.4	91.3	+ 4.4	90.3	−19.0	75.8	−31.0	144.1	118.4	0.772	76.3	19.5	3	Compare 78.
75	09 13.94	111.4	62.2	+11.8	62.5	− 9.2	38.7	−11.5	125.2	165.8	0.059	172.2	20.4	5	
76	09 13.94	111.4	87.4	− 3.7	86.7	−27.1	63.1	−42.5	126.5	122.9	0.704	86.2	20.4	4	
77	09 13.94	111.4	87.9	+14.6	88.0	− 8.6	71.1	−14.1	162.0	128.0	0.620	96.4	20.4	5	
78	09 13.94	111.4	90.7	+ 5.7	90.7	−17.8	74.2	−28.8	145.8	121.3	0.729	83.0	20.4	4	Compare 74.
79	09 13.94	111.4	94.4	+12.2	94.4	−11.1	81.8	−18.4	159.2	117.1	0.790	74.6	20.4	3	
80	04 14.81	112.4	95.9	+16.1	95.7	− 7.2	83.7	−12.0	166.4	118.1	0.776	77.7	21.5	4	Compare 95.
81	07 Oct. 15.89	112.8	90.8	+20.9	90.8	− 2.6	75.4	− 4.1	169.0	157.8	0.143	157.4	21.9	4	
82	09 15.90	113.3	46.4	− 1.4	43.6	−18.1	359.4	−17.6	139.1	151.3	0.230	144.9	22.4	4	
83	09 15.90	113.3	47.5	+13.1	48.7	− 4.5	8.7	− 4.8	19.5	165.6	0.062	173.5	22.4	5	Compare 73, 101.
84	09 15.90	113.3	49.5	+22.6	53.1	+ 4.2	14.9	+ 4.8	175.2	180.2	0.974	202.9	202.4	5	
85	09 15.90	113.3	59.8	+ 0.8	57.8	−19.3	18.4	−23.0	80.7	156.7	0.156	159.8	22.4	7	
86	09 15.90	113.3	60.4	+ 7.2	59.8	−13.2	23.8	−16.2	95.0	173.7	0.987	189.8	22.4	6	
87	09 15.90	113.3	76.3	+19.7	77.1	− 3.1	52.3	− 4.6	170.8	149.8	0.252	132.0	22.4	5	Compare 96, Z 156?
88	09 15.90	113.3	76.4	+ 1.1	75.4	−21.7	44.8	−31.0	122.4	142.4	0.372	127.1	22.4	4	
89	09 15.90	113.3	78.0	+16.0	78.4	− 7.0	54.1	−10.5	160.6	146.8	0.300	115.9	22.4	6	
90	09 15.90	113.3	80.4	− 2.7	79.3	−25.8	49.2	−37.8	120.1	134.8	0.501	112.0	22.4	4	
91	09 15.90	113.3	85.9	+32.2	86.0	+ 8.8	66.5	+13.9	160.4	134.2	0.513	110.7	202.4	4	
92	09 15.90	113.3	89,4	+14.0	89.4	− 9.4	72.0	−15.0	158.9	133.9	0.518	110.2	22.4	5	
93	09 15.90	113.3	90.6	+24.0	90.5	+ 0.6	74.6	+ 1.0	178.8	125.8	0.656	94.0	202.4	4	
94	09 15.90	113.3	93.6	+13.3	93.6	−10.1	79.2	−16.5	160.5	122.5	0.721	87.4	22.4	6	
95	09 15.90	113.3	97.9	+17.7	97.6	− 5.5	86.3	− 9.2	169.8	115.7	0.809	73.4	22.4	3	Compare 80.
96	04 16.80	114.4	77.4	+20.0	78.2	− 2.9	53.3	− 4.3	169.2	156.1	0.163	135.8	23.5	3	Compare 87.
97	04 Oct. 16.80	114.4	91.6	+17.7	91.5	− 5.7	75.4	− 7.6	170.3	127.7	0.628	98.9	23.5	7	Compare 115, 126.
98	03 18.83	115.7	84.2	+ 8.6	84.1	−14.8	60.4	−22.5	144.6	138.7	0.434	122.2	24.8	4	Compare 103+104, 134.
99	03 18.83	115.7	87.6	+13.3	87.7	−10.1	67.4	−15.8	157.3	135.0	0.498	114.8	24.8	5	Compare 111.
100	03 18.83	115.7	92.1	+13.6	92.1	− 9.8	75.0	−15.7	159.9	128.0	0.618	110.8	24.8	13	
101	01 18.81	116.1	47.8	+11.8	48.6	− 5.8	7.4	− 6.0	18.8	161.3	0.104	167.8	25.2	6	Compare 73, 83.
102	01 18.81	116.1	57.5	+ 0.2	55.3	−19.4	13.6	−21.5	62.9	155.7	0.169	156.5	25.2	5	
103	01 18.81	116.1	86.8	+10.8	86.8	−12.7	65.2	−19.6	151.0	136.2	0.476	117.7	25.2	5	} Combine.
104	01 18.83	116.2	84.6	+11.0	84.5	−12.4	61.4	−18.9	149.9	139.8	0.415	124.8	25.2	4	} Combine.
105	01 18.83	116.2	91.2	+14.2	91.2	− 9.2	73.2	−14.7	160.5	130.4	0.578	106.0	25.2	5	
106	01 18.83	116.2	94.2	+11.4	94.2	−12.0	77.9	−19.4	156.0	124.8	0.671	94.9	25.2	7	
107	04 18.81	116.4	65.4	+25.9	68.0	+ 4.3	35.8	+ 5.7	150.8	168.3	0.047	182.1	205.5	6	
108	04 18.81	116.4	79.2	+28.6	80.4	+ 5.4	72.1	+ 8.6	168.2	132.8	0.537	111.0	205.5	4	A. P. 11; 2 and 4.
109	04 18.81	116.4	83.5	+18.2	83.8	− 5.3	61.1	− 8.0	166.4	143.6	0.351	132.6	25.5	4	[J.B.S.]
110	04 18.81	116.4	87.5	+14.4	87.6	− 9.1	66.9	−14.1	159.2	136.6	0.470	118.8	25.5	7	

TABLE OF RADIANTS.—*Continued.*

No.	G.M.T.1900+	L	α	δ	l'	b'	l	b	ι	η	q	π	Ω	$\overset{s}{\rightarrow}$	
		°	°	°	°	°	°	°	°	°		°	°		
111	04 18.82	116.4	88.9	+12.9	88.9	−10.5	69.0	−16.5	156.6	134.1	0.513	113.8	25.5	4	Compare 99. [J.B.S.]
112	04 18.81	116.4	90.0	+16.4	90.0	− 7.0	71.2	−11.2	164.6	133.2	0.529	111.9	25.5	6	[J.B.S.]
113	04 Oct. 18.82	116.4	90.8	+16.6	90.8	− 6.8	72.2	−11.0	165.1	132.3	0.544	110.2	25.5	5	
114	04 18.81	116.4	92.0	+15.5	92.0	− 7.9	74.5	−12.7	161.0	137.4	0.456	120.4	25.5	16	
115	04 18.81	116.4	93.4	+19.4	93.2	− 4.0	76.8	− 6.4	171.8	128.4	0.612	102.2	25.5	5	A.P. 11; 1 and 3. Compare 97, 126. [J.P.S.]
116	04 18.81	116.4	94.1	+25.0	93.7	+ 1.6	77.8	+ 2.6	176.7	127.6	0.624	100.8	205.5	5	
117	08 18.73	116.5	90.2	+14.3	90.2	− 9.2	71.2	−14.5	160.2	132.4	0.542	110.3	25.4	6	
118	03 19.82	116.6	91.5	+14.4	91.4	− 9.1	73.2	−14.5	158.7	136.5	0.472	118.7	25.7	23	
119	03 19.82	116.6	92.3	+11.0	92.3	−12.4	74.2	−19.7	154.4	128.6	0.608	102.9	25.4	4	
120	01 19.81	117.2	82.6	+11.7	82.6	−11.6	57.8	−17.2	149.4	134.5	0.335	135.2	26.2	5	
121	01 19.81	117.2	90.1	+14.4	90.1	− 9.0	70.7	−14.2	160.2	133.8	0.519	113.8	26.2	13	
122	01 19.81	117.2	91.9	+12.9	91.9	−10.6	73.5	−16.8	157.7	130.5	0.575	107.2	26.2	30+	
123	00 19.82	117.4	90.6	+15.9	90.6	− 7.6	71.5	−11.9	163.4	133.7	0.520	113.0	26.4	9+	
124	00 19.82	117.4	92.1	+15.0	92.4	− 8.5	74.5	−13.6	162.0	130.5	0.575	107.4	26.4	29+	
125	05 20.88	118.2	88.7	+16.2	88.7	− 7.2	67.9	−11.2	163.0	138.2	0.443	123.6	27.3	10	Compare 128+129, 130.
126	05 20.88	118.2	97.8	+19.2	97.4	− 4.1	82.7	− 6.6	172.0	124.4	0.678	96.0	27.3	5	Compare 97, 115.
127	09 22.92	120.3	76.3	+12.4	76.4	−10.4	45.9	−14.3	138.2	158.3	0.137	165.9	29.4	4	D, LXIV, 13*
128	09 22.92	120.3	89.4	+16.6	89.4	− 6.8	67.8	−10.6	163.3	140.4	0.404	130.2	29.4	7	} Combine. Compare
129	09 Oct. 22.92	120.3	90.5	+18.1	90.5	− 5.4	69.7	− 8.3	167.3	139.0	0.428	127.3	29.4	8	} 125, 130.
130	05 23.86	121.1	90.3	+19.3	90.3	− 4.2	68.8	− 6.4	169.8	140.0	0.394	132.1	30.3	4	Compare 125, 128+129.
131	06 25.84	122.9	56.0	+12.6	56.6	− 7.0	15.8	− 7.3	24.6	162.2	0.926	186.5	32.0	4	
132	06 25.84	122.9	79.0	+28.5	80.3	+ 5.3	51.4	+ 7.4	158.4	159.3	0.124	170.6	212 0	5	
133	06 25.84	122.9	86.2	+17.6	86.3	− 5.8	61.0	− 8.6	162.7	149.9	0.251	151.7	32.0	6	
134	06 25.84	122.9	91.9	+ 7.7	92.0	−15.7	68.4	−24.0	143.1	137.3	0.457	126.6	32.0	6	Compare 98, 103 + 104,
135	06 25.84	122.9	100.0	+33.7	98.4	+10.5	80.6	+16.8	158.0	129.3	0.596	110.5	32.0	5	
136	06 25.84	122.9	102.0	+24.1	101.0	+ 1.1	85.6	+ 1.8	177.8	143.6	0.643	139.2	212.0	6	
137	06 26.86	123.9	102.4	+15.3	102.1	− 7.6	86.5	−12.3	164.8	54.3	0.657	321.6	33.0	5	
138	06 26.86	123.9	113.8	− 6.8	117.0	−28.0	68.8	−42.9	122.2	126.4	0.643	105.9	33.0	4	
139	01 Nov. 13.84	142.0	102.3	+35.0	100.3	+12.0	60.6	+16.8	118.3	160.8	0.107	192.7	231.3	4	
140	01 13.84	142.0	109.0	+67.6	100.0	+44.8	86.5	+50.8	115.2	121.1	0.725	113.5	231.3	4	
141	01 13.84	142.0	110.6	+ 2.8	111.8	−19.1	81.7	−29.3	132.1	138.8	0.430	148.8	51.7	6	
142	01 13.84	142.0	159.2	+52.6	137.8	+40.0	130.3	+67.0	112.6	94.3	0.984	59.8	231.3	4	Compare 148, 157.
143	01 13.84	142.0	169.4	+22.7	161.2	+16.6	355.4	+27.2	31.7	119.5	0.749	110.3	231.3	4	
144	03 14.84	142.4	76 2	+31.0	78.1	+ 8.1	37.7	+ 8.7	32.1	163.5	0.080	198.7	231.8	5	
145	03 14.84	142.4	94.4	+ 8.0	94.5	−15.4	60.0	−20.9	110.9	22.5	0.144	276.5	51.6	4	
146	09 14.84	143.1	128.6	− 2.0	131.6	−20.0	121.8	−33.6	140.0	120.6	0.733	113.6	52.3	2	Compare 163.
147	09 14.84	143.1	139.9	+34.8	131.4	+18.3	121.8	+30.7	147.6	107.6	0.899	87.5	232.3	4	Compare 176, Z167?
148	09 14.84	143.1	155 0	+58.0	138.5	+43.5	127.6	+72.4	107.0	94.4	0.993	61.2	232.3	4	Compare 142, 157.
149	04 14.88	143.3	137.8	+18.2	134.8	+ 1.9	128.7	+ 3.2	176.7	103.9	0.932	80.3	232.6	5	
150	04 14.88	143.3	138.0	+49.2	124 7	+31.4	103.9	+50.7	122.6	113.3	0.834	99 4	232.5	5	Compare 158, 160.
151	04 14.88	143.3	147.2	+60.8	124.8	+44.1	86.8	+68.6	102.4	162.4	0.898	207.4	232.6	5	
152	03 15.83	143.5	72.5	+40.5	76.1	+17.9	32.4	+18.1	43.2	153.0	0.203	178.9	232.8	3	Compare 155.
153	03 15.83	143.5	87.0	+22.8	87.2	− 0.6	51.2	− 0.7	24.9	178.2	0.001	229.2	52 8	4	D, LXXV*
154	03 15.8	143.5	112.0	+10.8	112.0	−11.0	89.2	−16.8	153.0	140.4	0.402	53.6	52.8	4	
155	01 15.87	144.0	70.2	+41.5	74.4	+19.1	29.7	+18.6	40.2	150.3	0.242	173.9	233.2	4	Compare 152.
156	01 15.87	144.0	137.3	+19.2	134.0	+ 2.4	127.0	+ 4.1	174.9	126.8	0.633	127.0	233.3	2+	
157	01 15.87	144.0	155.4	+50.9	136.2	+37.4	124.2	+62.4	116.3	98.8	0.966	70.8	233.3	5	Compare 142, 148.
158	00 15,84	144.2	138.8	+47.8	125.4	+30.2	105.8	+48.9	124.6	113.7	0.829	100.9	233.5	2	Compare 150, 160.
159	09 16.00	144.2	147.0	+36.5	136.6	+21.8	129.8	+36.9	142.3	100.9	0.953	75.3	233.5	5	
160	06 16.82	144.8	137.0	+49.0	124.0	+31.0	111.2	+49.4	122.2	116.3	0.797	106.6	234.1	3	Compare 150, 158.
161	06 Nov. 16.83	144.8	78.6	+29.0	80.0	+ 6.0	249.9	+ 6.4	22.2	17.0	0.850	268.2	234.1	7	D, LXVII, 26*
162	06 16.82	144.8	101.0	+34.9	99.2	+11.8	67.1	+15.9	128.3	159.6	0.121	193.2	234.1	5	
163	06 16.83	144.8	130.7	− 1.2	133.5	−18.7	124.1	−31.4	143.0	120.0	0.741	114.2	54.1	4	Compare 146.
164	06 16.82	144.8	140.0	+26.7	130.1	+10.6	119.2	+17.7	142.8	149.9	0.249	173.8	234.1	4	
165	04 18.67	146.5	95.4	+ 10.9	95.4	−12.4	60.2	−15.8	105.8	16.5	0.079	268.4	235.6	4	

TABLE OF RADIANTS.—*Continued.*

No.	G.M.T.1900+	L	α	δ	l'	b'	l	b	ι	η	q	π	Ω	$\overset{s}{\rightarrow}$	
		°	°	°	°	°	°	°	°	°	°	°	°		
166	10 May 4.97	313.3	334.0	− 3.4	334.6	+ 6.9	349.8	+11.2	166.2	55.0	0.677	155.1	44.1	6+	η Aquarids.
167	10 May 6.93	315.4	337.7	− 0.6	339.2	+ 8.2	356.0	+13.0	163.2	51.2	0.607	148.3	46.0	25	η Aquarids. [G.H.]
168	10 May 11.99	320.1	342.0	− 0.6	343.2	+ 6.5	359.5	+10.5	166.7	52.1	0.630	155.1	50.8	5+	η Aquarids.
169	10 Aug. 1.93	39.7	14.6	+16.3	19.8	+ 9.3	5.9	+14.9	162.3	122.0	0.935	13.3	129.2	3	
170	10 1.93	39.7	16 9	+ 8.0	18.6	+ 0.8	4.5	+ 1.2	178.5	124.7	0.686	18.5	129.2	6	
171	10 6.9	44.5	10.1	+19.4	17.0	+13.8	356.6	+21.6	149.7	133.2	0.538	30.4	134.0	3	
172	10 6.9	44.5	30.4	+56.4	51.0	+44.7	62.8	+67.4	111.4	82.9	0.998	299.8	134.0	5	D, XXXII, 5. Compare 175.
173	10 11.86	49.3	7.2	+58.9	38.0	+49.4	9.3	+75.3	101.4	99.3	0.987	337.3	138.8	2	Compare 51, 57.
174	10 11.86	49.3	7.6	+68.6	48.9	+56.6	230.4	+75.8	75.8	90.4	1.013	319.6	138.8	3	
175	10 11.86	49.3	35.8	+56.6	54.6	+39.6	63.6	+65.9	113.4	84.0	1.003	306.7	138.8	4	Compare 53, 175.
176	07 Nov. 14.88	142.6	139.5	+35.0	131.0	+18.3	121.5	+30.8	147.6	107.4	0.900	86.7	231.9	3	Compare 147, Z 173 ? ?

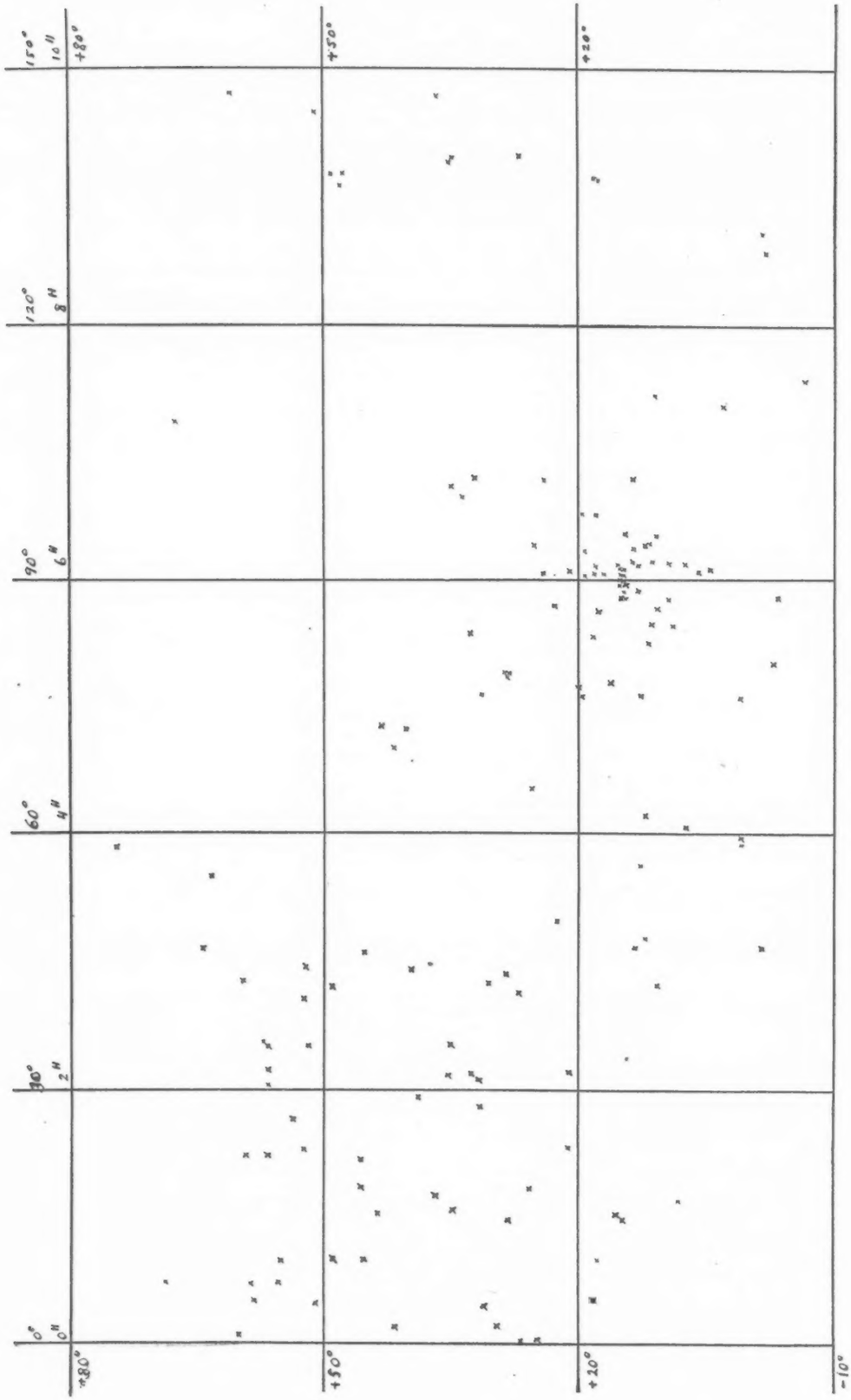

FIG. 2.

UNCERTAIN RADIANTS.

1900+	L	α	δ	$\overset{s}{\rightarrow}$	Notes.
	°	°	°		
05 Jan. 1.80	191.3	175.9	+37.5	4	D CXXXVIII ?
09 " 18.83	208.9	198.2	− 4.0	4	D CLVIII ?
04 Apr. 18.75	297.4	279.1	+30.8	3	
04 " 18.75	297.4	293.6	+58.0		D CCXXIV ?
01 Aug. 8.79	46.5	19.4	+15.6	3	2 on Aug. 8.
01 " 9.77	47.5	10.2	+26.6	5	
08 Oct. 18.73	116.5	43.3	+ 3.8	3	D XXXVIII, 6, 7
08 " 18.73	116.5	106.5	+33.8	2	Z 161?
04 " 18.81	116.4	100.8	+30.3	2	D LXXXIX.
03 " 18.83	115.7	61.7	+24.4	2	D LXIII, 13*
03 " 19.82	116.6	48.2	− 4.7	3	D XLI, 5.
03 19.82	116.6	61.5	+23.3	2	D LXIII, 13*
06 " 25.84	122.9	64.1	−16.3	3	
00 Nov. 13.95	142.3	93.8	+44.0	3	
01 " 13.84	142.0	97.8	+16.5	3	
01 " 13.84	142.0	135.4	+ 6.2	3	
03 " 14.84	142.4	118.4	+55.8	3	
03 " 14.84	142.4	147.7	+53.6	2	
01 " 15.87	144.0	135.7	− 4.7		
01 " 15.87	144.0	145.2	+70.6	2	
00 " 15.84	144.2	92.2	+24.0	4	Nov. 13, 1 seen, Nov. 15, 1 seen.
00 " 15.84	144.2	93.8	+44.0	3	
00 " 15.84	144.2	158.4	+54.3	3	Nov. 13, 1 seen.
09 " 16.00	144.2	120.9	+11.8	3	
09 " 16.00	144.2	130.9	+ 8.5	3	
06 " 16.83	144.8	114.2	− 9.0	3	
1899 Nov. 24.		20.2	+37.0	8	Beilids. 75 meteors seen in all between 8^h10^m and 10^h10^m.
		23.6	+29.9	6	
		25.2	+40.8	5	

TABLE OF INCLINATIONS AND PERIHELIA.

Inclinations of Orbits.	0° to 10°	10° 20°	20° 30°	30° 40°	40° 50°	50° 60°	60° 70°	70° 80°	80° 90°	90° 100°	100° 110°	110° 120°	120° 130°	130° 140°	140° 150°	150° 160°	160° 170°	170° to 180°	T'l
Before combination	0	3	6	5	3	2	3	3	2	8	14	17	16	12	20	16	33	12	175
After combination.	0	1	5	5	1	2	3	3	2	7	11	15	14	11	14	14	22	9	139
Zezioli's.....	4	22	9	17	15	18	22	12	3	23	7	13	8	5	3	3	2	0	189

Longitudes of Perihelia.	0° to 10°	10 20	20 30	30 40	40 50	50 60	60 70	70 80	80 90	90 100	100 110	110 120	120 130	130 140	140 150	150 160	160 170	170 180	180 190	190 200	200 210	210 220	220 230	230 240	240 250	250 260	260 270	270 280	280 290	290 300	300 310	310 320	320 330	330 340	340 350	350° to 360°	
Before combination	2	3	1	2	2	2	4	6	6	6	13	20	4	6	4	5	3	5	4	3	2	1	2	0	2	0	2	1	4	6	4	7	8	13	7	5	175
After combination.	2	3	1	2	2	1	4	4	4	4	6	12	4	6	4	5	3	5	4	3	2	1	2	0	2	0	2	1	4	6	4	7	8	10	6	5	139
Zezioli's.....	2	3	2	3	2	2	1	1	5	2	4	2	2	3	4	4	5	4	6	3	8	7	8	3	6	10	5	10	11	5	12	10	10	8	9	7	189

MAGNITUDES OF METEORS.

Year.	>0	0	1	2	3	4	5	6	<6	—	Total
1898		1								119	120
1899	0	1	10	12	4	7	1	1	0	101	137
1900	21	2	41	54	59	39	31	7	0	182	436
1901	10	14	46	96	128	116	51	15	0	50	526
1902	2	11	19	44	91	51	12	2	0	12	244
1903	28	33	71	122	268	166	34	2	0	7	731
1904	12	24	101	125	273	170	48	7	0	27	787
1905	8	10	27	45	123	106	33	4	0	13	369
1906	15	6	27	52	111	107	30	22	2	3	375
1907	8	8	15	27	64	95	31	8	0	2	258
1908	5	2	10	20	27	34	14	1	0	13	126
1909	15	16	37	117	363	410	197	27	5	30	1213
1910	10	5	16	71	181	266	104	16	0	7	676
1900 } 1908	109	110	357	585	1,142	884	284	68	2	309	3,852
1909 } 1910	25	21	53	188	544	676	301	43	5	37	1,889
Others }	0	0	3	14	14	12	12	4	10	47 }	373
1898–9 }	0	2	10	12	4	7	1	1	0	220 }	
Total											6,114

MAGNITUDES OF METEORS. PERCENTAGE.

Year.	> 0	0	1	2	3	4	5	6	< 6
1900 } 1908	03.1	03.0	10.3	16.5	32.3	25.0	08.0	01.9	00.1
1909 } 1910	01.3	01.1	02.9	10.2	29.4	36.5	16.3	02.3	00.3

COLORS OF METEORS.

Year.	Red.	Orange.	Yellow.	Green.	Blue.	Purple.	White.	Total.
1898				1				1
1899	14		1	18	1		1	35
1900	81		13	53	5		10	162
1901	96	67	15	77	2		27	284
1902	38	40	1	37	4			120
1903	108	178	17	89	6	4	67	469
1904	90	27	58	24	3	7	30	239
1905	32	2	28	11	2	1	1	77
1906	42	15	17	17		2	17	110
1907	46	5	12	7	3		8	81
1908	20	6	2	2	1		6	37
1909	120	3	96	36	4	1	16	276
1910	51	3	50	11	1	3		119
Total	738	346	310	383	32	18	183	2,010
Per cent.	36.7	17.2	15.4	19.1	01.6	00.9	09.1	100.0

DURATIONS OF FLIGHT WITH REGARD TO COLOR.

Seconds	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.5	3.0	3.5	4.0	5.0	
Red.....	4	11	60	101	75	34	12	12	4	11		5	1		5			1	3	4	1			1	
Orange ..	2		57	103	40	25	5	7		8		2			3	1		3	2	1	1		1		
Yellow...		2	19	58	18	12	1	1		2		1						1			1				
Green ...		2	24	54	38	28	6	2	1	8		1			3		1	1	5			1	1		
Blue.....			2	1	3	1		2		1					2	1				1	1				
Purple...		1	1	4	1	3			1		1	1						1							
White ...		1	8	33	17	10	1	3	1	2		3													

1902–1908

Seconds																									
Red.....	3	3	43	60	22	12	3	3		2		2			1			1	1	1	1				
Orange ..				3	1	1			1	1															
Yellow...		1	32	56	30	7		2				1							1	2					
Green ...		3	5	15	5	3		2		4		2		1	2			1		1					
Blue.....					1	1									1				1					1	
Purple...				2								1							1						
White ...				4	5	1						1													

1909+1910

Means	All 1902–'08	No.	All 1909+'10	No.	<1ˢ.1 1902+'08	No.	<1ˢ.1 1909+'10	No.	All 1902–'10	No.	<1ˢ.1 1902–'10	No.	Δ
	s		s		s		s		s		s		s
Red.....	0.557	345	0.479	158	0.468	324	0.414	151	0.532	503	0.451	475	0.091
Orange ..	0.529	261	0.600	7	0.448	247	0.600	7	0.531	268	0.452	254	0.079
Yellow...	0.474	116	0.464	132	0.434	113	0.414	128	0.469	248	0.423	241	0.046
Green ...	0.605	176	0.661	44	0.489	163	0.486	37	0.616	220	0.488	200	0.128
Blue.....	1.053	15	1.920	5	0.570	10	0.550	2	1.245	20	0.567	12	0.678
Purlpe...	0.671	14	1.000	4	0.482	11	0.400	2	0.745	18	0.469	13	0.276
White ...	0.506	79	0.569	13	0.478	76	0.517	12	0.514	92	0.484	88	0.030
———	0.404	1319			0.398	1313							

EXCEPTIONAL METEORS.

Year.	Sta. Meteors.	Var. Magn.	Irr. Path.	Curved Path.	Rem. Trains.	Hazy.	Trains 2ˢ or over.
1898	0	0	0	—	0	0	—
1899	0	—	—	—	—	—	—
1900	0	1	3	4		2	7
1901	1	2	3	5	2	1	20
1902	1	—	—	1	—	1	10
1903	1	1	1	2	2	4	50
1904	1	4	5	1	—	—	51
1905	0	—	2	—	—	—	21
1906	0	—	—	1	2	1	24
1907	0	1	—	—	1	2	12
1908	1	1	4	—	—	1	4
1909	0	3	—	5	3	4	66
1910	0	2	1	—	1	2	32
Total	5		15	19	19	11	18	297

EXPLANATION OF TABLES.

Table of Radiants.—This table contains the parabolic elements calculated for the 175 radiants, which were considered good enough to justify the computations, and which did not belong to the Perseids or Leonids. The columns give from left to right: (1) Serial number of radiant, (2) Greenwich Mean Time of observations, (3) longitude of the meteoric apex, (4) right ascension of radiant, (5) declination, (6) apparent longitude, (7) apparent latitude, (8) true longitude, (9) true latitude, (10) inclination to the ecliptic of parabolic orbit, (11) the angle between the true position

of the radiant point and the sun, (12) perihelion distance in terms of earth's distance from sun, (13) longitude of perihelion, (14) longitude of ascending node, (15) number of meteors from which radiant was deduced, (16) notes and references.

In (16), when the radiant was observed by anyone other than myself, the observer's initials are enclosed in square brackets. Other references are as follows: A. P. — — refers to *Annales de l'Observatoire d'Athénes*, Vol. III, with page and number of the radiant on page.

D, — — refers to W. F. Denning's "General Catalogue of Meteor Radiants," giving the group and number in group. An asterisk following the number means that several nights' observations were used by observers referred to.

Z, — — refers to Schiaparelli's corrected orbits, deduced from Zezioli's observations, giving the number of the orbit in his work.

Uncertain Radiants.—This table gives 26 uncertain radiants. The columns give from left to right: (1) Greenwich Mean Time, (2) longitude of meteoric apex, (3) right ascension of radiant, (4) declination, (5) number of meteors from which radiant was deduced, (6) notes and references.

Table of Inclinations and Perihelia.—The first of these tables gives for each 10° of inclinations three series of results. The first row contains the inclinations of all 175 orbits, just as taken from Table of Radiants. The second row gives the distribution after allowance has been made for the same meteor stream reappearing one or more times. The third row gives the distribution as taken from Schiaparelli's orbits, based on Zezioli's observations.

The second table gives the longitudes of perihelia, in three exactly similar rows.

Table of Magnitudes of Meteors.—This table gives the number of meteors of each magnitude observed in every year. Below are three combinations, first all meteors seen from 1900 to 1908 inclusive, second all meteors seen in 1909 and 1910, third for al other years.

Percentage.—This table gives the percentage of each magnitude in the first and second combination just mentioned. Of course no account is taken of meteors whose magnitudes are not given, in this table.

Table of Colors of Meteors.—This table is self-explanatory.

Table of Duration of Flight, etc.—Two tables are given under this head. The first is divided into two periods, namely 1902 to 1908 inclusive, 1909 and 1910. In each portion the numbers of meteors of each color are divided up to show how many of a given duration of visibility were seen.

The second table gives the mean visibility for each color divided as follows: (1) All meteors, 1902–08, (2) number of meteors, (3) all meteors 1909+1910, (4) num-

ber of meteors, (5) meteors whose length of visibility was not over 1ˢ.0 in 1902–1908, (6) number of these meteors, (7) same for 1909+1910, (8) number of these meteors, (9) all meteors 1902 to 1910 inclusive, (10) their number, (11) meteors not over 1ˢ.0, (12) their number, (13) difference between (9) and (11).

Table of Exceptional Meteors.—This gives the peculiar meteors seen in each year under the following headings: (1) Stationary meteors, (2) meteors whose magnitude varied, (3) meteors with irregular paths, (4) meteors with curved paths, (5) very remarkable trains left, (6) meteors which certainly had hazy nuclei,(7) numbers of trains recorded as being visible at least 2.0 seconds.

This table is only partially correct because undoubtedly not all peculiar meteors seen were so recorded.

Deductions from Tables.

When my inclinations and perihelia are examined, they at once show certain marked maxima. As to the inclinations very many more are retrograde than direct with a strong maximum at 160°–170°. This is worthy of attention when we remember that most of the short period comets move with direct motion. However, Zezioli's orbits show more of direct than retrograde motion. I can, at present, only explain this difference in our results by pointing out that while his observations were made throughout the entire year, most of these were made between July 20 and November 20, and therefore are not well enough distributed to be strictly comparable with his. These were also generally made after midnight. As to the perihelia two maxima are shown, one about 110° and the other at 320°. This last agrees fairly well with Zezioli, but no maximum near the first position is shown by his orbits.

In the results from the tables of magnitudes the most important is that which shows that during 1900 to 1908 inclusive, when observations were made mostly in Virginia, at the Leander McCormick Observatory, meteors of the third magnitude were the most numerous by far. However, during 1909 and 1910, when observations were made in California, at the Lick Observatory, meteors of the fourth magnitude were greatly in the majority. This proves conclusively, since the numbers are quite comparable, that there is every reason to believe we would find meteors of the fifth magnitude, and so on indefinitely, most numerous could we get sky as much clearer than the Lick, as the Lick is clearer than the Virginia sky.

The series representing meteor magnitudes is similar to that for stars, only the factor seems less than 2.5.

The question of the length of visibility of meteors, with regard to their color appeared a sufficiently interesting question to investigate. Consequently the two tables on p. 31 were formed for this purpose.

It was not until 1902 that this particular datum was observed, which explains why meteors seen previously are not included.

To understand the results some remarks are necessary, In the first place few meteors whose magnitude was below the third had color recorded for them. Yellow and orange were used loosely, and could often have been interchanged. Blue, purple and to a lesser extent green meteors are difficult to distinguish from white meteors, unless bright or slow. Indeed very few blue or purple meteors were seen. Finally, white practically means no color could be detected, and as a rule was not entered on the observing book, which will explain why there are so few white meteors in the lists.

The first part of the table merely gives the data for forming the second. In this latter the meteors are studied in three classes, each with two subdivisions. All those observed 1902 to 1908 inclusive form one class, those observed 1909 and 1910 at the Lick Observatory form the second, and these are combined to form the third class and give the definitive results.

Each class is subdivided to give the means for all meteors and then the means for those whose visibility did not exceed 1.0 second. This last is the column which should be studied to obtain results, because any meteor whose visibility is over 1.0 second is an unusual one. The results are rather surprising, though well marked.

Yellow meteors have the shortest, orange and red the next and equal times of visibility, finally green and white almost equal and the longest.

Blue and purple are also visible a longer time, but the results depend on too few meteors to have much weight. It is noteworthy that the other columns, in general, bear out the results deduced from the last. Taking as a first assumption that meteors of all colors enter the atmosphere with the same mean velocity and become visible at the same mean height, then we must conclude that yellow meteors are composed of materials which are more inflammable than red and orange meteors, and these in turn more so than white or green. It may be objected that the error of observation is so great that the differences in the column referred to mean little or nothing. While it is true enough that for any single meteor the error is large, yet when the means of several hundred are taken, as in this case, I believe the accidental errors are largely eliminated.

Finally the mean values for meteors whose color was unrecorded in 1902–1908 are given, merely for comparison. As most of these were of the fainter magnitudes than the third, it is quite obvious that they should in general be seen a less time than brighter meteors, as indeed the table proves.

The exceptional meteors call for no further comment here than to say that my

observations give fewer per thousand than is usually the case. As there were 297 trains of 2.0 seconds duration, or over, it seems that about one meteor in 20 leaves such an one. A full description of all these exceptional phenomena will probably be published later in another paper.

CONDENSED SUMMARY OF RESULTS.

This summary is intended to give in a very few words what appear to be the main results deduced in this paper.

1. Stationary radiants appear to be rare if they exist at all.

2. Proof that Halley's Comet and the η Aquarids are intimately connected.

3. The change in position of the Perseid radiant, from day to day, is fully confirmed.

4. The Orionids do not seem to have a stationary radiant.

5. The radiant of the Leonids shows no apprecialbe change of position from day to day.

6. The existence of the so-called $\alpha-\beta$ Perseids, except in August, is not confirmed.

7. By observing in a clearer atmosphere, meteors of the fourth magnitude are in the majority, while formerly more of the third magnitude were seen.

8. Yellow meteors have the shortest time of visibility, red and orange somewhat longer times, while green and white are seen longest.

9. Peculiar meteors are not so common as thought.

LEANDER MCCORMICK OBSERVATORY,
UNIVERSITY OF VIRGINIA.

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P59

TRANSACTIONS

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HELD AT PHILADELPHIA

FOR PROMOTING USEFUL KNOWLEDGE

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PART 2

The Secular Variations of the Elements of the Orbits of the Four Inner Planets Computed for the Epoch 1850.0 G. M. T. By ERIC DOOLITTLE.

Philadelphia:
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104 SOUTH FIFTH STREET
1912

ARTICLE II.

THE SECULAR VARIATIONS OF THE ELEMENTS OF THE ORBITS OF THE FOUR INNER PLANETS COMPUTED FOR THE EPOCH 1850.0 G. M. T.

By ERIC DOOLITTLE.

(Read March 1, 1912.)

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To my Father,

PROFESSOR CHARLES L. DOOLITTLE,

this work

is Inscribed.

1. INTRODUCTION.

The usual method of determining the secular variations of the elements of any planet is the well-known one based upon the development of the perturbing function into an infinite series whose successive terms involve continually higher powers of the eccentricities and the mutual inclination. This method possesses two advantages. The first is that when an extreme degree of accuracy is not required, so that higher terms of the development may be disregarded, it is the simplest method available; and, in the second place, since the coefficients of all terms are general literal expressions, the change produced in the value of any variation by a change in the assumed values of one or more of the elements can readily be ascertained by a simple substitution of the more accurate values. On the other hand, this method possesses the disadvantage that the complexity of the expansion grows rapidly greater as the order of the included terms is increased, so that a slight increase in the desired accuracy greatly increases the labor of the computation.

The integral methods, founded upon the celebrated theorem of GAUSS[1],* are wholly free from this latter disadvantage, for if it is desired to include all terms to the twenty fourth order this can be done by a computation which is less than twice as long as that required when the approximation is stopped at terms of the eleventh order. But the integral method, though thus extremely accurate, leads only to the numerical values of the variations dependent upon the values of the elements assumed; if they are desired for some other epoch at which the various elements possess different values from those adopted, or if an improved value of any of the elements becomes known, they can only be found by an entire repetition of the computation.

The only determinations of the secular perturbations of the four inner planets which are in any sense modern ones are the classic investigation of LE VERRIER[7] and the computation of NEWCOMB[15]. The latter furnishes the most accurate values of these variations so far determined; the series were extended to terms of the eighth order, only those terms of this order being included, however, which seemed likely to be most important, and in some cases terms of the tenth order were included, though usually by induction merely.

In both of the above computations the usual expansion into an infinite series was employed. As the GAUSSIAN method is so extremely accurate, and as its formulas throughout are wholly different from those hitherto employed, it seemed that an

* These symbols wherever they occur refer to the list of titles at the end of the present paper.

application of it to a re-determination of these variations based upon the most accurate values of the several elements now obtainable would be of value. The results of this work will be found in the following pages; the final comparison with the earlier results is given in Article 11, and the comparison with the results of observation in Articles 12 and 13. The epoch throughout is 1850.0, G. M. T.

In the four following articles an attempt is made to state briefly the essential features of the various methods of computing secular variations which are founded on GAUSS's theorem, but for a detailed account of the long and often complex transformations which are involved, the original papers must be consulted.

2. THE METHOD OF GAUSS.

The equations which express the complete variations of the elements of the orbit of any body revolving about the sun when it is disturbed in its motion by the presence of a third body, may, as is well known, be put in a variety of different forms; the form selected as the basis for all developments founded on GAUSS's method[1] is that in which three rectangular components of the disturbing force enter into the expressions for the differential coefficients. Thus, if R denote the component lying in the direction of the radius vector of the disturbed body, positive outward from the sun; S, the component lying in the plane of the orbit of the disturbed body and perpendicular to the radius vector, positive in the direction of motion; and W, the component perpendicular to this plane and positive northward, we will have for the variation of the eccentricity of the orbit of the disturbed body,

$$\frac{de}{dt} = \frac{a^2 n \cos\varphi}{k^2(1+m)}[\sin v \cdot R + (\cos v + \cos E)S],$$

with similar expressions for the variations of the six remaining elements.*

In the original memoir of GAUSS the determination of the secular terms of these expressions was given a geometrical aspect. Thus, since each variation may obviously be expressed in terms of the two single variables M and M', the secular term in question will be that given by the equation,

$$\left[\frac{de}{dt}\right]_{00} = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\frac{de}{dt}\,dM\,dM',$$

* The usual notation is adopted throughout. Thus a, $e = \sin\varphi$, i, Ω, π, n, and L are respectively the half major axis, the eccentricity, the inclination, the longitude of the ascending node, the longitude of perihelion, the mean motion and the longitude at the epoch of the disturbed body; M, E, v and r are respectively its mean, eccentric and true anomalies and its radius vector, m_0 is its mass, k^2 is the mass of the sun, and $m_0 = mk^2$. The same letters with accents refer to the disturbing body.

Watson, Theoretical Astronomy, pp. 516–523; Oppolzer, Lehrbuch zur Bahnbestimmung, Vol. II, p. 213; Tisserand, Mecanique Celeste, Vol. I, pp. 431–433, etc. The final forms of the equations expressing the other variations may be inferred from those stated at the end of Article 7.

and this is the same as,

$$\left[\frac{de}{dt}\right]_{00} = \frac{a^2 n \cos\varphi}{2\pi k^2(1+m)} \int_0^{2\pi} \left[\sin v \cdot \frac{1}{2\pi}\int_0^{2\pi} R dM' + (\cos v + \cos E)\cdot \frac{1}{2\pi}\int_0^{2\pi} S dM' \right] dM,$$

since the variable of the first integration enters the expression only through R and S. In the equation as thus written R and S are supposed to contain the mass, m_0', as a factor so that if R_i and S_i are the corresponding values produced by a unit mass, $R = m_0' R_i$ and $S = m_0' S_i$.

If we now imagine an infinitely thin elliptic ring which coincides with the orbit of m', whose total mass is equal to the mass m_0', and the density of any portion of which is proportional to the time occupied by m' in describing that portion of its orbit, we will have for the three components of the attraction exerted by any portion dm_0',

$$R_i dm_0', \qquad S_i dm'_0, \qquad \text{and} \qquad W_i dm_0',$$

and integrating about the entire ring, we find for the complete components,

$$\int_0^{2\pi} R_i dm_0', \qquad \int_0^{2\pi} S_i dm_0', \qquad \text{and} \qquad \int_0^{2\pi} W_i dm_0'.$$

But by the conditions,

$$\frac{dm_0'}{m_0'} = \frac{dt}{T} = \frac{dM'}{2\pi},$$

and hence the components are,

$$\frac{1}{2\pi}\int_0^{2\pi} m_0' R_i dM', \qquad \frac{1}{2\pi}\int_0^{2\pi} m_0' S_i dM' \qquad \text{and} \qquad \frac{1}{2\pi}\int_0^{2\pi} m_0' W_i dM',$$

which are identical with

$$\frac{1}{2\pi}\int_0^{2\pi} R dM', \qquad \frac{1}{2\pi}\int_0^{2\pi} S dM' \qquad \text{and} \qquad \frac{1}{2\pi}\int_0^{2\pi} W dM'.$$

Thus the expressions giving the secular variations are seen to be the same whether these are derived from the moving planet or from the elliptic ring.*

The work of GAUSS contains no application to the determination of secular variations nor are all the formulas necessary for this purpose there developed; the first integration alone is effected, and it is shown that by changing first to the variable E' and afterward introducing a new variable, T, each of the complicated integrals may be made to depend upon elliptic integrals whose values GAUSS obtained by the introduction of a new algorithm called by him the Arithmetico-geometrical mean.

* Other interesting geometrical aspects of the problem are treated by Bour [5], Hill [37], [38], and Halphen [28], but for brevity a detailed account of these is here omitted.

The first application of GAUSS's method was made by NICOLAI[2], who determined by it the secular variations of the Earth's orbit, but the results only were published.* The first development of the method is by CLAUSEN[3] who also applied it to a determination of the perturbations of Tuttle's Comet produced by the action of Jupiter[4], dividing the disturbed orbit into 120 parts with reference to the true anomaly. It was next, in 1867, applied by ADAMS[6] to the orbit of the November meteors with a special view to ascertaining the cause of the steady progression of the node of the orbit, but in this investigation certain small terms were neglected by ADAMS and the solution of a fundamental cubic equation which occurs in the original method was in this manner avoided.

No further applications of GAUSS's method seem to have been made until after the publication of HILL's extensive development[8] and modifications of it in 1882.

3. HILL'S FIRST MODIFICATION OF GAUSS'S METHOD.

Although the first of the above integrations may be rigorously effected, the value of the second must be approximated to by a mechanical quadrature about the orbit of m, a greater or less number of terms being employed in the quadrature according as the disturbed orbit is more or less eccentric. Since either the true, eccentric, or mean anomalies may be selected as the variables, it becomes of importance to decide which of these must be chosen in order to render the quadrature most accurate. It is readily proved† that the inequalities of distribution of a series of points on an elliptic orbit corresponding to a series of equidistant values of the eccentric anomaly are of the order of the square of the eccentricity while for the other two anomalies they are of the order of the first power of this quantity, and therefore HILL has employed the eccentric anomalies throughout his development, although SEELIGER[9] showed that a still higher accuracy will be obtained if the true anomalies are chosen.

If, therefore, we decide to make the integrations with reference to the eccentric anomalies, we will obtain, since

$$dM = \frac{r}{a}dE, \qquad dM' = \frac{r'}{a'}dE', \qquad \text{and} \qquad r' = a'(1 - e'\cos E'),$$

$$\left[\frac{de}{dt}\right]_{\infty} = \frac{n\cos\varphi}{k^2(1+m)} \cdot \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} [\sin v \cdot Rar(1 - e'\cos E') + (\cos v + \cos E) \cdot Sar(1 - e'\cos E')]dEdE'dt,$$

* See Article 11.

† See Tisserand's Mecanique Celeste, Vol. I, page 442.

and writing,

$$R_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{ar}{m_0'} R(1 - e' \cos E')dE', \qquad S_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{ar}{m_0'} S(1 - e' \cos E')dE',$$

$$W_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{ar}{m_0'} W(1 - e' \cos E')dE',$$

the expression for the secular variation will become,

$$\left[\frac{de}{dt}\right]_{00} = \frac{m'n}{1+m} \cos \varphi \cdot \frac{1}{2\pi}\int_0^{2\pi} [\sin v \cdot R_0 + (\cos v + \cos E)S_0]dE.$$

In order to find the values of R_0, S_0, and W_0, it is first necessary to express R, S and W in terms of E'. For this purpose that part of the disturbing force arising from the action of the disturbing planet upon the sun need not be included, for it is known that this has no secular term.† Considering therefore only the action of m' upon m, it is evident from a figure that R, S and W will have the values,

$$R = \frac{m_0'}{\Delta^2}\left\{\frac{r' \cos \vartheta - r}{\Delta}\right\},$$

$$S = \frac{m_0'}{\Delta^2} \cdot \frac{r' \sin \vartheta \cos \gamma}{\Delta},$$

$$W = \frac{m_0'}{\Delta^2} \sin \gamma,$$

and also that

$$\Delta^2 = r^2 - 2rr' \cos \vartheta + r'^2,$$

in which ϑ is the angle included between the radii vectores, Δ is the distance between the two bodies, and γ is the inclination of the plane which includes r and r' to the plane of the orbit of the disturbed body.

If Π and Π' denote the angular distances respectively of the perihelia of the two orbits from the ascending node of the orbit of m' upon the orbit of m, and if I be their mutual inclination, we will have,

$$\cos \vartheta = \cos (v + \Pi) \cos (v' + \Pi') + \sin (v + \Pi) \sin (v' + \Pi') \cos I,$$

$$\sin \vartheta \cos \gamma = -\sin (v + \Pi) \cos (v' + \Pi') + \cos (v + \Pi) \sin (v' + \Pi') \cos I,$$

$$\sin \vartheta \sin \gamma = \sin I \sin (v' + \Pi')$$

The values of Π, Π', and I are obtained from the original elements by a direct solution of the spherical triangle whose sides are Π and Π', and in which the angle included between these sides is I. (See Article 7.)

† See Hill's "On Gauss's Method [8], . . ," page 321.

If we now eliminate v' from the above expressions by the equations,

$$r' \cos v' = a' (\cos E' - e'), \qquad r' \sin v' = a' \cos \varphi' \sin E', \qquad r' = a' (1 - e' \cos E'),$$

the resulting equations giving R, S, W, and Δ will be expressed wholly in terms of the variable E'. In order to simplify these results, we assume certain new auxiliaries defined by the equations,

$$k \cos (K - \Pi) = \cos \Pi', \qquad k \sin (K - \Pi) = - \cos I \sin \Pi', \qquad k' \cos (K' - \Pi) = \cos I \cos \Pi',$$

$$k' \sin (K' - \Pi) = - \sin \Pi',$$

$$A = r^2 + 2ka'e'r \cos (v + K) + a'^2,$$

$$B \cos \epsilon = ka'r \cos (v + K) + a'^2 e',$$

$$B \sin \epsilon = k'a' \cos \varphi' \cdot r \sin (v + K'),$$

$$A_c = ka' \cos (v + K), \qquad A_s = k'a' \cos \varphi' \sin (v + K')$$

$$B_c = - ka' \sin (v + K), \qquad B_s = k'a' \cos \varphi' \cos (v + K')$$

$$C_c = a' \sin \Pi' \sin I, \qquad C_s = a' \cos \varphi' \cos \Pi' \sin I.$$

$$C = a'^2 e'^2,$$

when the desired expressions become,

$$\frac{\Delta^3}{m_0} R = A_c (\cos E' - e') + A_s \sin E' - r$$

$$\frac{\Delta^3}{m_0} S = B_c (\cos E' - e') + B_s \sin E'$$

$$\frac{\Delta^3}{m_0} W = C_c (\cos E' - e') + C_s \sin E'$$

$$\Delta^2 = A - 2B \cos (E' - \epsilon) + C \cos^2 E'.$$

In order to effect the integrations, GAUSS here introduced a new variable, T, connected with E' by the relations,

$$N \sin E' = \alpha + \alpha' \sin T + \alpha'' \cos T$$

$$N \cos E' = \beta + \beta' \sin T + \beta'' \cos T$$

$$N = \gamma + \gamma' \sin T + \gamma'' \cos T,$$

the quantities α, α', α'', β, β' . . . being subject to the conditions that

$$(N \sin E')^2 + (N \cos E')^2 - N^2 \qquad \text{and} \qquad \sin^2 T + \cos^2 T - 1$$

shall be identically zero, and also being so chosen that the coefficients of $\sin T$, $\cos T$, and $\sin T \cos T$ shall vanish in the expression $N^2 \Delta^2$ which therefore must take the form,

$$G - G' \sin^2 T + G'' \cos^2 T.$$

From these conditions it is derived that the coefficients G, G' and $- G''$ in the trans-

formed expression for $N^2\Delta^2$ must severally satisfy the cubic equation,

$$x(x - A)(x + C) + B^2x + B^2C \sin^2 \epsilon = 0,$$

and hence that they must be the roots of this equation. By substituting for x the successive values, $-C$, 0, $a'^2 \cos^2 \varphi'$ and $+A$, the first member is seen to take in succession the corresponding values,

$$\begin{gathered} - B^2C \cos^2 \epsilon \\ + B^2C \sin^2 \epsilon \\ - a'^4 \cos^2 \varphi' \cdot r^2 \sin^2 I \sin^2 (v + \Pi) \\ + B^2(A + C \sin^2 \epsilon). \end{gathered}$$

Since, even when $\cos (v + K)$ has its maximum negative value, the value of A exceeds that of $(r - a')^2$, it is evident that A is always positive, and therefore that the above equation has one negative root which lies between $-C$ and 0, one positive root lying between 0 and $a'^2 \cos^2 \varphi'$, and that the third root lies between this value and $+A$. The roots are represented by $-G''$, G', and G, respectively, and thus G'', G' and G are always positive quantities, the last being the largest and the first the smallest except when φ' exceeds 45°, a case not met with in any of the planetary orbits.

Since α, β, γ, α', β' . . . must retain the same values whatever the values of E' and T, we may, by writing the equations arising from the three conditions above stated and equating the coefficients of the like terms in the two members, obtain a series of equations which are sufficient for the determination of these quantities in terms of G, G', G'' and the other known auxiliaries. Upon substituting the resulting expressions for $\sin E'$ and $\cos E'$ in the equations defining R_0, S_0, and W_0, and noticing that $N^2\Delta^2$ may be written,

$$G - G' \sin^2 T + G'' \cos^2 T = (G' + G'') \left\{ 1 - \frac{G' + G''}{G + G''} \sin^2 T \right\},$$

we obtain each of the components in the form,

$$R_0,\ S_0 \text{ or } W_0 = \frac{1}{2\pi} \int_0^{2\pi} \frac{m_s \sin^2 T + m_c \cos^2 T}{(G + G'')^{\frac{3}{2}} \left(1 - \frac{G' + G''}{G + G''} \sin^2 T \right)^{\frac{3}{2}}} dT.$$

If we now write,

$$\frac{G' + G''}{G + G''} = c^2 = \sin^2 \theta,$$

and consider that from LANDEN'S well-known transformation,

$$F\left(c, \frac{\pi}{2}\right) = \frac{\pi}{2} K = \frac{\pi}{2}(1 + c_0)(1 + c_{00})(1 + c_{000}) \cdots$$

$$E\left(c, \frac{\pi}{2}\right) = \frac{\pi}{2} KL = \frac{\pi}{2} K\left(1 - \frac{c^2}{2} - \frac{c^2 c_0}{4} - \frac{c^2 c_0 c_{00}}{8} \cdots\right),$$

and also notice that

$$\int_0^{\pi/2} \frac{dT}{(1 - c^2 \sin^2 T)^{\frac{3}{2}}} = \sec^2 \theta \cdot E\left(c, \frac{\pi}{2}\right)$$

$$\int_0^{\pi/2} \frac{\sin^2 T dT}{(1 - c^2 \sin^2 T)^{\frac{3}{2}}} = \frac{1}{c^2}\left[\sec^2 \theta \cdot E\left(c, \frac{\pi}{2}\right) - F\left(c, \frac{\pi}{2}\right)\right]$$

$$\int_0^{\pi/2} \frac{\cos^2 T dT}{(1 - c^2 \sin^2 T)^{\frac{3}{2}}} = \frac{1}{c^2}\left[F\left(c, \frac{\pi}{2}\right) - E\left(c, \frac{\pi}{2}\right)\right],$$

it is evident that each of the above three integrals becomes expressible wholly in terms of the rapidly convergent series of LANDEN.

For the purposes of the present computation HILL[8] has computed to ten places the logarithms of the quantities

$$K_0 = \sec^2 \theta \cdot KL, \qquad L'_0 = \frac{L - \cos^2 \theta}{c^2 L}, \qquad \text{and} \qquad N_0 = \sec^2 \theta \cdot (1 + L_0'),$$

and these correct to eight places are tabulated at intervals of one tenth of a degree for all values of θ from $\theta = 0°$ to $\theta = 50°$.

From a direct substitution it is now seen that the final resulting values of R_0, S_0 and W_0 are as follows, in which the symbols N, P, Q, etc., are written for abbreviation and have the meanings stated in Article 7:

$$R_0 = -N - QG' + VJ_1',$$
$$S_0 = PF_2 + VJ_2$$
$$W_0 = PF_3 + VJ_3$$

The integration with respect to E' having been thus entirely completed, that in regard to E is effected by mechanical quadratures. Since each variation is a function of E alone, it follows by the principles of quadratures that if any one of them be expanded into a periodic series involving the sines and cosines of E and its multiples, the secular term of the series, which is rigorously equal to $\frac{1}{2}\pi \int_0^{2\pi} f(E) dE$, may be also obtained by forming the values of $f(E)$ for $2j$ equidistant values of E, from $E = 0°$ to $E = 360°$, and dividing the sum by $2j$. The expression thus obtained,

$$\frac{1}{2j} \cdot \Sigma f(E),$$

will be subject only to the error involved in dropping those terms which contain a multiple of E not lower than $2j$. An inspection of the known forms of the series which express the variations renders it evident that the error thus committed is of

the order $2j$ in terms of the eccentricities and mutual inclinations of the orbits except in the one case of the variation of the Mean Longitude, in which, as this variation depends wholly upon the expansion of $-2(r/a)R_0$, it is of the order $2j+1$.

The resulting equations giving the values of all the secular variations are those stated in Article 7.

4. HILL'S SECOND MODIFICATION OF GAUSS'S METHOD. THE WORK OF CALLANDREAU AND INNES.

In HILL'S second modification of GAUSS'S method[8], the well-known expressions for the roots of a cubic equation when this is solved by the trigonometric method are introduced, and thus, throughout the integrals, the quantities p, q and θ' occur instead of the roots G, G' and G'', the equations connecting these quantities being,

$$G = 2q\sin\left(60^\circ - \frac{\theta'}{3}\right) + p, \qquad G' = 2q\sin\frac{\theta'}{3} + p,$$
$$G'' = 2q\sin\left(60^\circ + \frac{\theta'}{3}\right) - p.$$

It was shown in GAUSS'S original memoir[1] that

$$\int_0^{\pi/2} \frac{dT}{(m^2\cos^2 T + n^2\sin^2 T)^{\frac{1}{2}}} = \int_0^{\pi/2} \frac{dT}{(m'^2\cos^2 T + n'^2\sin^2 T)^{\frac{1}{2}}},$$

if $m' = \frac{1}{2}(m+n)$ and $n' = \sqrt{mn}$, and that by repeating this transformation by the employment of the equations,

$$m'' = \tfrac{1}{2}(m'+n'), \qquad n'' = \sqrt{m'n'},$$
$$m''' = \tfrac{1}{2}(m''+n''), \qquad n''' = \sqrt{m''n''},$$
$$\text{etc.} \qquad\qquad \text{etc.},$$

$m^{(k)}$ and $n^{(k)}$ very rapidly approach a single limit, μ, which GAUSS named the Arithmetico-geometrical Mean. It thus follows that our first integral is equal to $\pi/2\mu$, and that integrals of the form

$$\int_0^{\frac{\pi}{2}} \frac{(\sin^2 T - \cos^2 T)dT}{(m^2\cos^2 T + n^2\sin^2 T)^{\frac{1}{2}}}$$

become equal to $\pi/2 \cdot \omega/\mu$ in which ω is a very rapidly converging series involving m, n, m', n', etc., in its successive terms.

The integral expressions which actually enter into the equations for R_0, S_0, and W_0 are

$$\chi(\theta') = \frac{\sin\left(60^\circ - \frac{\theta'}{3}\right) - \frac{\sqrt{3}}{4}\cdot\frac{\omega}{(m^2-n^2)}}{24\sqrt[4]{12}\, m^4 n^4 \mu},$$

$$\psi(\theta') = \frac{\frac{1}{2} + m^2n^2 - \frac{\sqrt{3}}{4} \cdot \frac{\omega \sin \theta'}{(m^2 - n^2)}}{24\sqrt[4]{12}\, m^4n^4\mu},$$

in which

$$m^2 = \cos \frac{\theta'}{3}$$

and

$$n^2 = \cos\left(60° + \frac{\theta'}{3}\right),$$

the values of R_0, S_0, and W_0 being connected by comparatively simple relations with these quantities and with known auxiliaries.

HILL accordingly suggested that tables of these functions should be computed, and this was first done by MONS. O. CALLANDREAU[13] who however adopted as an argument the quantity α defined by the relation

$$\frac{1 - \alpha}{1 + \alpha} = \sqrt{\frac{\cos\left(60° + \frac{\theta'}{3}\right)}{\cos \frac{\theta'}{3}}}$$

and tabulated the logarithms of the functions $m^4n^4\chi(\theta')$ and $\psi(\theta') \div \chi(\theta')$ at intervals of 0.001 from $\alpha = 0.000$ to $\alpha = 0.400$; of 0.002 from $\alpha = 0.400$ to $\alpha = 0.600$ and of 0.005 from this point to the extreme value, $\alpha = 1.000$. This paper repeats the derivation of all formulas necessary when the second method alone is employed, essentially as this was given by HILL, and also contains a direct proof that R_0, S_0, and W_0 can be expressed wholly in terms of the complete elliptic integrals, F and E.

Similar tables were also computed by MR. R. T. A. INNES[22], the functions here tabulated being $(1 - \alpha)/(1 + \alpha^4) \cdot \psi(\theta')$ and $\psi(\theta') \div \chi(\theta')$ to the argument θ' at intervals of one degree, from $\theta' = -90°$ to $\theta' = +90°$.

Whether the first or second methods be employed, the values of the integrals involved may also, as was pointed out by HILL[38], be approximated to with great rapidity by the use of JACOBI'S Nome, q (American Journal of Mathematics, Vol. 23, page 321. In the Astronomical Journal, No. 511, a brief application is given to a case in the action of Venus on the Earth). This function is defined by the equation,

$$q = e^{-\pi \frac{K'}{K}},$$

in which K' is the complete elliptic integral of the first kind complementary to K, from which there may be derived,

$$K = \frac{\pi}{2}(1 + 2q + 2q^4 + 2q^9 + 2q^{16} \cdots)^2$$

$$KE = (1 + 3^2q^2 + 5^2q^6 \cdots) \div (1 + q^2 + q^6 \cdots).$$

The values of $\log [q/\tan^2 \frac{1}{2}\theta]$ computed to ten decimal places for each degree of θ from $\theta = 0°$ to $\theta = 45°$ are given by INNES[39]. When θ exceeds 45°, the values of K and E are readily obtained from their expressions in terms of the complementary complete integrals whose moduli are $\sin(\pi/2 - \theta)$, and to which the table is therefore directly applicable.

Lastly, in the second method, HILL recommends that the quadratures be performed upon the quantities $a/r \cdot R_0$, $a/r \cdot S_0$ and $a/r \cdot W_0$ directly, all constant and evanescent factors which appear in the expressions for the variations being removed from under the integral signs and reserved until the integration has been completed.

5. THE METHOD OF HALPHEN AND ITS MODIFICATIONS BY ARNDT AND INNES.

It was first pointed out by BRUNS[29] that the periods of the elliptic functions of the first and second integrals can be evaluated without a knowledge of the three roots, but it was HALPHEN[28] who first applied this remarkably elegant method of analysis to the present problem. It was shown by him that if ω and η are the two periods in question, then R_0, S_0, and W_0 may be obtained in the form $a\omega + b\eta$, in which a and b are rational functions of the coefficients of the cubic equation and ω and η are expressible in terms of certain hyper-geometric series in which the common variable is an absolute invariant of the elliptic functions.

The three integrals entering into the problem have the form,

$$\int_0^{\pi/2} \frac{l dT}{(G - G' \sin^2 T + G'' \cos^2 T)^{\frac{3}{2}}},$$

in which l has the values 1, $\sin^2 T$ and $\cos^2 T$, respectively, in the three cases; by introducing the new variable, s, defined by the relation,

$$\sin^2 T = \frac{G + G''}{G' + G''} \cdot \frac{s - G'}{s - G},$$

these become,

$$-2\frac{G' + G''}{n} \int_{G'}^{-G''} \frac{ds}{\sqrt{S}}(s - G), \qquad -2\frac{G + G''}{n} \int_{G'}^{-G''} \frac{ds}{\sqrt{S}}(s - G')$$

and

$$-2\frac{G' - G}{n} \int_{G'}^{-G''} \frac{ds}{\sqrt{S}}(s + G''),$$

respectively, in which

$$n = (G + G'')(G - G')(G' + G'')$$

and

$$S = -4(s - G)(s - G')(s + G'').$$

Introducing the WEIERSTRASSIAN $\wp$ function through the relation

$$u = \int_{\wp(u)} \frac{ds}{\sqrt{4(s-e_1)(s-e_2)(s-e_3)}},$$

u being the elliptic integral of the first kind and e_1, e_2, and e_3 the roots of the cubic equation increased by one third of the coefficient of x^2, and considering that from the theory of these functions,

$$s - G = \wp(u) - e_1, \qquad \wp(\omega) = e_1, \qquad \wp(\omega + \omega') = e_2, \quad \text{and} \qquad \wp(\omega') = e_3,$$

the first integral will become,

$$2\frac{G' + G''}{n}\int_{\omega}^{\omega+\omega'} [\wp(u) - e_1]du = 2\left[e_1\omega + \frac{\sigma'}{\sigma}(\omega + \omega') - \frac{\sigma'}{\sigma}\cdot\omega'\right],$$

σ and σ' being the second WEIERSTRASSIAN functions, which are connected with the periods, ω and η, by the equations,

$$\frac{\sigma'}{\sigma}(\omega + \omega') = \eta + \eta'; \qquad \frac{\sigma'}{\sigma}\omega' = \eta'.$$

The three integrals consequently take the final forms,

$$2\frac{G' + G''}{n}(e_1\omega + \eta);\ 2\frac{G + G''}{n}(e_2\omega + \eta), \text{ and } 2\frac{G' - G}{n}(e_3\omega + \eta).$$

A direct substitution of these expressions for the integrals in the equations which define R_0, S_0 and W_0 leads, after some reduction, to forms which are seen to contain only these integrals themselves, the coefficients of the cubic equation with other known auxiliaries, and the quantity n. But if, for brevity, we write the original cubic equation in the form,

$$x^3 - P_1x^2 + P_2x - P_3 = 0,$$

and let

$$\lambda = P_1^2 - 3P_2 \qquad \text{and} \qquad \rho = P_1P_2 - 9P_3,$$

then the invariants, g_2 and g_3, and the absolute invariant, g, will have the values,

$$g_2 = \tfrac{4}{3}\lambda; \qquad g_3 = \tfrac{4}{27}(2P_1\lambda - 3\rho), \qquad \text{and} \qquad g = g_2^3 \div 27g_3^2,$$

and n will be given by,

$$n^2 = \tfrac{1}{16}(g_2^3 - 27g_3^2),$$

in which the last factor is the discriminant. Thus, except for ω and η, our final expressions are obtained wholly in terms of the coefficients of the cubic equation, and a knowledge of the roots becomes unnecessary.

In the paper by BRUNS, before referred to, it is shown that ω and η are directly

expressible in terms of a hyper-geometric series whose variable is the absolute invariant, g. By a simple transformation the relations may be placed in the following forms, which are more convenient in practical application.

$$\omega = \frac{\pi}{\sqrt[4]{12}} g_2^{-\frac{1}{4}} F\left(\frac{1}{12}, \frac{5}{12}, 1, \frac{g-1}{g}\right),$$

$$\eta = \frac{\pi}{\sqrt[4]{1728}} g_2^{\frac{1}{4}} F\left(\frac{1}{12}, -\frac{1}{12}, 1, \frac{g-1}{g}\right).$$

Dr. Louis Arndt[30] has fully developed this method, deriving all the formulas necessary for its application and stating tables for $F(\omega)$ and $F(\eta)$ for values of $(g-1)/g$ from $(g-1)/g = 0.000$ to $(g-1)/g = 0.980$, the interval being 0.001.

In a recent paper by Innes[31] the complete formulas for this method are derived when the quadrature is applied directly to the expressions $(a/r)R_0$, $(a/r)S_0$ and $(a/r)W_0$, as suggested in the second method of Hill. The development is nearly identical with that of Arndt except that the forms of the hyper-geometric series are slightly changed, the variable,

$$\sin^2 \frac{i}{2} = \frac{\sqrt{g}-1}{2\sqrt{g}}$$

being preferred. The values of the logarithms of

$$F(\omega) = F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2 \frac{i}{2}\right) \quad \text{and} \quad F(\eta) = F\left(-\frac{1}{6}, \frac{5}{6}, 2, \sin^2 \frac{i}{2}\right).$$

were published by Mr. Frank Robbins[32] for all values of i, at intervals of one degree from $i = 1°$ to $i = 90°$, the computation having been made to ten places and published to seven, and these tables, computed with seven place logarithms, have been extended from $i = 90°$ to $i = 180°$ by Mr. C. J. Merfield[33].

Although the preceding methods are of great mathematical elegance, it is doubtful whether their formulas lead to so accurate results as those of Hill's first method when seven place logarithms are employed. (See the computations of Jupiter on Mars[24] and of Saturn on Mars[25], Article 10.) Moreover, when the method is applied which is explained in the computation of Jupiter on Mercury (Article 10), the roots of the cubic equation are so readily obtained that the avoidance of its solution becomes a matter of no practical importance. Accordingly Hill's first modification of Gauss's method has been employed throughout all of the following computation.

THE COMPUTATION.

6. THE ELEMENTS OF THE ORBITS AND THE ADOPTED MASSES.

The values adopted for the elements of the several orbits, to serve as the basis for this computation, were taken in each case from HILL's "*New Theory of Jupiter and Saturn.*"[16] Those of the four inner planets will be found on page 192; those of Jupiter and Saturn on page 558; of Uranus on page 109, and of Neptune on page 161. The epoch throughout is 1850.0 G. M. T.

The values of the masses finally selected by HILL, and here adopted, will be found on page 554 for Mercury, Venus and the Earth; on page 192 for Mars; on page 19 for Jupiter and Saturn, and on page 161 for Neptune. The mass of Uranus as stated in the "*New Theory*" is $1 \div 22640$, but at DR. HILL's suggestion this is here diminished to $1 \div 22800$, (*A. J.*, No. 316). The value assumed for the mass of Mercury when the first of these computations were made was $1 \div 5000000$, but all of the results are here changed to agree with the value $1 \div 7500000$ stated below. It seems not improbable that even this latter fraction is too large, but the true value of this element is still very uncertain.

	π	i	Ω	e	n
Mercury	75° 7′ 13.62″	7° 0′ 7.71″	46° 33′ 8.63″	0.20560476	5381016.260″
Venus	129 27 42.83	3 23 35.01	75 19 53.08	0.00684311	2106641.357
Earth	100 21 39.73	0 0 0.00	—	0.01677114	1295977.416
Mars	333 17 51.74	1 51 2.24	48 23 54.59	0.09326803	689050.784
Jupiter	11 54 31.67	1 18 42.10	98 56 19.79	0.04825511	109256.626
Saturn	90 6 41.37	2 29 40.19	112 20 49.05	0.05606025	43996.21506
Uranus	168 15 6.70	0 46 20.54	73 14 8.00	0.0469236	15425.752
Neptune	43 17 30.30	1 47 1.68	130 7 31.83	0.0084962	7864.935

	$\log a$	$1 \div m$
Mercury	9.5878217	7 500 000
Venus	9.8593378	408 134
Earth	0.0000000	327 000
Mars	0.1828971	3 093 500
Jupiter	0.7162374	1 047.879
Saturn	0.9794956	3 501.6
Uranus	1.2831044	22 800
Neptune	1.4781414	19 700

7. THE FORMULAS EMPLOYED IN THE COMPUTATION.

The following formulas are written in the order in which they were applied. When the right hand member appears in two different forms, one of these was used in the first computation and the other in the duplication, though sometimes other obvious modifications were made use of in the several cases differing from those which are here written.

The values of I, Π, and Π' were obtained from the general equations:

$$\begin{aligned}
\sin I \sin(\Pi - \omega) &= -\sin i' \sin(\Omega' - \Omega),\\
\sin I \cos(\Pi - \omega) &= -\sin i \cos i' + \cos i \sin i' \cos(\Omega' - \Omega)\\
&= \cos i \cos i' [-\tan i + \tan i' \cos(\Omega' - \Omega)],\\
\sin I \sin(\Pi' - \omega') &= -\sin i \sin(\Omega' - \Omega),\\
\sin I \cos(\Pi' - \omega') &= \cos i \sin i' - \sin i \cos i' \cos(\Omega' - \Omega),\\
&= \cos i \cos i' [\tan i_{\prime} - \tan i \cos(\Omega' - \Omega)].
\end{aligned}$$

When the Earth is the disturbing body, these become,

$$I = i; \qquad \Pi = 180^\circ + \omega; \qquad \Pi' = 180^\circ + \pi' - \Omega;$$

and when the Earth is the disturbed body,

$$I = i'; \qquad \Pi = \pi - \Omega'; \qquad \Pi' = \pi' - \Omega'.$$

As i, i' and I are always small, eight place logarithms were generally here used to insure the accuracy of Π and Π'.

The auxiliaries k, k', K, K' and C were then found from the relations:

$$k \sin(K - \Pi) = -\cos I \sin \Pi'; \qquad k \cos(K - \Pi) = \cos \Pi'; \qquad k' \sin(K' - \Pi) = -\sin \Pi';$$
$$k' \cos(K' - \Pi) = \cos I \cos \Pi'; \qquad C = a'^2 e'^2,$$

and their values were tested by the equations,

$$\tan \sigma = \frac{k}{k'}; \qquad \tan \tfrac{1}{2}(K - K' + 90^\circ) \cot \tfrac{1}{2}(K + K' - 90^\circ - 2\Pi) = \frac{\sin(\Pi' - \sigma)}{\sin(\Pi' + \sigma)},$$
$$\sin(K - K') = \sin I \tan I \sin(K' - \Pi) \sin(K - \Pi) \cot \Pi'.$$

The orbit of the disturbed planet being then divided into $2j$ parts in regard to the eccentric anomaly, the following equations were applied to each point of division, of which those marked with an asterisk are test equations upon the sums of the functions corresponding respectively to the odd and even points of division of the orbit. The sums corresponding to the odd points are designated by Σ_1, those to the even points by Σ_2, and

$$\Sigma = \Sigma_1 + \Sigma_2.$$

$$r \sin v = a \cos \varphi \sin E,$$
$$r \cos v = a (\cos E - e),$$
$$r^2 = a^2 (1 - 2 e \cos E + e^2 \cos^2 E),$$

(the last equation giving the value of r^2 for use in A, N, and J_3. Since

$$\tfrac{1}{2} \log r^2 = \log r,$$

this affords also an independent test of r).

$$*\Sigma_1 v + 180^\circ = \Sigma_2 v; \qquad *\Sigma_1 r = \Sigma_2 r = ja.$$

$$A = r^2 + 2ka'e'r \cos (v + K) + a'^2 = [r + ka'e' \cos (v + K)]^2 + a'^2[1 - k^2 e'^2 \cos^2 (v + K)],$$

(the second form used with ZECH's tables in the duplication).

$$*\Sigma_1 A = \Sigma_2 A = ja^2 + \tfrac{1}{2} ja^2 e^2 + j[a'^2 - 2kaa'ee' \cos K]$$
$$B \sin \epsilon = k'a' \cos \varphi' r \sin (v + K')$$
$$B \cos \epsilon = ka'r \cos (v + K) + a'^2 e'$$
$$*\Sigma_1 B \sin \epsilon = \Sigma_2 B \sin \epsilon = - jk'aa' \cos \varphi' \cdot e \sin K'$$
$$*\Sigma_1 B \cos \epsilon = \Sigma_2 B \cos \epsilon = j[a'^2 e' - kaa' e \cos K]$$
$$g = B^2 C \sin^2 \epsilon$$

To effect the solution of the cubic equation, h and l were found from the equations,

$$h = \tfrac{1}{2}[A - C + \sqrt{(A + C)^2 - 4B^2}]; \qquad l = \tfrac{1}{2}[A - C - \sqrt{(A + C)^2 - 4B^2}],$$

the very convenient test equation, $hl = B^2 - AC$, being applied to each pair of values. The first approximation to G was then obtained from

$$G = h - \frac{g}{h(h - l)},$$

and further approximations by successive applications of

$$G = h - \frac{g}{G(G - l)}.$$

(The number of trials required never exceeded three.) G' and G'' then follow from the equations,

$$G' = \frac{1}{2}\left[(A - C - G) + \sqrt{(A - C - G)^2 + \frac{4g}{G}}\right]; \qquad G'' = \frac{g}{GG'},$$

and we have for verification,

$$G + G' - G'' = A - C; \qquad G' = h + \frac{g}{G'(h - G')}; \qquad G'' = \frac{g}{(h + G'')(l + G'')};$$
$$*\Sigma G + \Sigma G' - \Sigma G'' = \Sigma A - 2jC.$$

(In some cases the first approximations to G were found by,

$$p = \tfrac{1}{3}(A - C); \qquad q^2 = p^2 - \tfrac{1}{3}(B^2 - AC); \qquad r = \tfrac{1}{2}p(p^2 - 3q^2) + \tfrac{1}{2}g;$$
$$\sin\theta' = \frac{r}{q^3}; \qquad G = 2q \sin(60^\circ - \tfrac{1}{3}\theta') + p,$$

the solution being then finished as before).

The modulus, ($c = \sin\theta$), of the elliptic integrals employed in the computation was separately found by the two equations,

$$\sin^2\theta = \frac{G' + G''}{G + G''}; \qquad \tan^2\theta = \frac{G' + G''}{G - G'},$$

and with θ as an argument the values of $\log K_0$, $\log L_0'$, and $\log N_0$ were taken from the tables of HILL's memoir[8], the interpolation being effected in both directions to second differences by the well-known formulas,

$$f(\theta_\kappa + nw) = f(\theta_\kappa) + n\left(D'_{\kappa+\frac{1}{2}} - \frac{1-n}{2}D''_\kappa\right),$$
$$f(\theta_{\kappa+1} - n'w) = f(\theta_{\kappa+1}) - n'\left(D'_{\kappa+\frac{1}{2}} + \frac{1-n'}{2}D''_{\kappa+1}\right),$$

in which $n + n' = 1$.

The logarithms of N, P, Q, and V were then obtained from,

$$N = \frac{ar^2K_0}{(G+G'')^{\frac{1}{2}}}; \qquad P = \frac{NL_0'}{(G+G'')^2}; \qquad Q = \frac{NN_0}{G+G''}; \qquad V = Q - PG'',$$

the first three being verified by similar operations performed upon the values of Σ_1 and Σ_2 formed from the respective logarithms, and the last by the use of ZECH's tables and also by the equation,

$$V = ar^2(G+G'')^{-\frac{1}{2}}[GN_0 + G''(N_0 - L_0')]K_0.$$

The following auxiliaries were next obtained:

$$\begin{aligned} J_1' &= a'^2\cos^2\varphi'[1 - \sin^2 I \sin^2(v+\Pi)] + G'' \\ &= [a'\cos\varphi' + a'\cos\varphi'\sin I\sin(v+\Pi)][a'\cos\varphi' - a'\cos\varphi'\sin I\sin(v+\Pi)] + G'', \\ J_2 &= ka'e'r\sin(v+K) - \tfrac{1}{2}a'^2\cos^2\varphi'\sin^2 I\sin 2(v+\Pi) \\ &= ka'e'r\sin(v+K) - a'^2\cos^2\varphi'\sin(v+\Pi)\cos(v+\Pi)\sin^2 I, \end{aligned}$$

the second form being employed with ZECH's tables in the duplication

$$J_3 = \frac{a'^2}{a}\cos^2\varphi'\sin I\cos I\cdot r\sin(v+\Pi) - \frac{a'}{a}e'\sin I\sin\Pi'\cdot r^2,$$
$$*\Sigma_1 J_3 = \Sigma_2 J_3 = -ja^2\cos^2\varphi'\sin I\cos I\cdot e\sin\Pi - \frac{a'e'}{a}\sin I\sin\Pi'\cdot\Sigma_1 r^2.$$
$$F_2 = -a'^2\sin\varphi'\cos\varphi'\cos I\cdot B\sin\epsilon,$$

$$*\Sigma_1 F_2 = \Sigma_2 F_2 = jk'aa'^3 ee' \cos^2 \varphi' \cos I \sin K',$$

$$F_3 = -\frac{a'^2}{a} \sin \varphi' \cos \varphi' \sin I \cdot r \cos (v + \Pi) \cdot B \sin \epsilon.$$

There were next obtained,

$$R_0 = -N - QG' + VJ_1'; \qquad S_0 = PF_2 + VJ_2; \qquad W_0 = PF_3 + VJ_3;$$

$$R^{(n)} = \frac{1}{r} R_0 \sin E; \qquad S^{(n)} = \frac{1}{r} S_0; \qquad \tfrac{1}{2} A_1{}^{(s)} = \frac{a}{2j} \Sigma R^{(n)}; \qquad B_0{}^{(c)} = \frac{a}{2j} \Sigma S^{(n)},$$

and the very accurate test equation,

$$\sin \varphi \cdot \tfrac{1}{2} A_1{}^{(s)} + \cos \varphi \cdot B_0{}^{(c)} = 0,$$

was applied.

These values were then substituted in the following series of equations, and the final values of the differential coefficients obtained:

$$\left[\frac{de}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \cos \varphi \cdot \frac{1}{2j} \Sigma [\sin v \cdot R_0 + (\cos v + \cos E) S_0],$$

$$\left[\frac{d\chi}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \frac{\cos \varphi}{e} \cdot \frac{1}{2j} \Sigma \left[-\cos v \cdot R_0 + \left(\frac{r}{a} \sec^2 \varphi + 1\right) \sin v \cdot S_0 \right],$$

$$\left[\frac{di}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \sec \varphi \cdot \frac{1}{2j} \Sigma [\cos u \cdot W_0],$$

$$\left[\frac{d\Omega}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \frac{\sec \varphi}{\sin i} \cdot \frac{1}{2j} \Sigma [\sin u \cdot W_0],$$

$$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} + 2 \sin^2 \frac{i}{2} \left[\frac{d\Omega}{dt}\right]_{00},$$

$$\left[\frac{dL}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \frac{1}{2j} \Sigma \left[-2 \frac{r}{a} R_0 \right] + 2 \sin^2 \frac{\varphi}{2} \left[\frac{d\chi}{dt}\right]_{00} + 2 \sin^2 \frac{i}{2} \left[\frac{d\Omega}{dt}\right]_{00}.$$

When the Earth is the disturbed body, the third and fourth equations are replaced by,

$$\left[\frac{dp}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \sec \varphi \cdot \frac{1}{2j} \Sigma [\sin (v + \pi) \cdot W_0]$$

$$\left[\frac{dq}{dt}\right]_{00} = \frac{m'n}{1+m} \cdot \sec \varphi \cdot \frac{1}{2j} \Sigma [\cos (v + \pi) \cdot W_0]$$

In this case

$$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00},$$

and the last term of the expression for $[dL/dt]_{00}$ disappears, but the first two equations remain unaltered.

8. THE VALUES OF THE PRELIMINARY CONSTANTS.

The values obtained for those constants which are direct functions of the elements of the orbits in the several cases are shown in the following tables. The last

columns of these tables contain the differences between the values of $K - K'$ formed directly and the same angles obtained from the test formula of the preceding article. The other test equations were also exactly satisfied.

An examination of the formulas of the preceding article renders it evident that with any two planets I will have the same value whether the inner or the outer planet is the disturbing one, while the value of Π in the first case will differ 180° from that of Π' in the second, and that of Π' in the first case will similarly differ 180° from that of Π in the second. These conditions will be seen to be here satisfied very exactly, the minute discrepancies which occur being due to the fact that in some places eight place logarithms were employed, in others seven, and in still others the attainment of a higher accuracy throughout the entire computation was sought by the use of the dash, (—), which was placed above the last figure of each logarithm for which the interpolation led to a value coinciding more nearly with the mean of the two adjacent figures than with either one of them. In combining such logarithms the effect of the dash was taken into consideration by methods which are obvious.

Mercury by—	I	Π	Π'	K
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Venus	4 20 42.982	230 39 31.39	284 54 1.27	305 43 2.40
Earth	7 0 7.710	208 34 4.99	233 48 31.10	334 57 50.59
Mars	5 9 10.165	209 13 54.31	107 24 19.31	101 45 36.16
Jupiter	6 17 15.310	218 5 54.72	154 49 24.01	63 8 32.52
Saturn	6 23 44.130	229 26 43.69	244 17 50.53	345 17 17.36
Uranus	6 19 17.399	211 43 10.39	304 49 47.06	266 43 32.95
Neptune	7 1 42.654	223 12 39.15	191 16 25.42	324 34 3.91

Mercury by—	K'	$\log k$	$\log k'$	$\log C$	resid.
	° ′ ″				″
Venus	305 47 57.49	9.9988328	9.9999176	5.3891826	0.007
Earth	334 33 18.85	9.9978879	9.9988719	6.4491252	0.002
Mars	101 53 33.05	9.9983990	9.9998432	8.3052599	0.004
Jupiter	63 24 30.76	9.9995281	9.9978563	8.7995614	0.002
Saturn	345 0 30.90	9.9978013	9.9994926	9.4563012	0.013
Uranus	267 3 12.48	9.9982188	9.9991396	9.9089914	0.000
Neptune	324 24 7.31	9.9968502	9.9998757	8.8147322	0.001

Venus by—	I	Π	Π'	K
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mercury	4 20 42.980	104 54 1.27	50 39 31.37	54 19 21.08
Earth	3 23 35.010	234 7 49.75	205 1 46.65	29 8 21.75
Mars	1 56 2.460	208 26 43.81	52 18 22.07	156 9 18.63
Jupiter	2 15 11.352	247 36 52.56	130 2 45.43	117 32 48.56
Saturn	2 3 12.046	281 7 33.71	241 43 52.16	39 24 36.81
Uranus	2 37 16.883	233 30 46.37	272 18 13.25	321 12 24.44
Neptune	2 46 38.369	265 47 34.23	179 34 46.34	86 12 46.11

Venus by—	K'	$\log k$	$\log k'$	$\log C$	resid.
	° ′ ″				″
Mercury	54 9 38.85	9.9992531	9.9994984	7.8017097	0.000
Earth	29 3 44.28	9.9998637	9.9993746	6.4491252	0.003
Mars	156 7 24.89	9.9998450	9.9999075	8.3052599	0.000
Jupiter	117 35 25.70	9.9998033	9.9998610	8.7995614	0.001
Saturn	39 22 46.29	9.9997836	9.9999375	9.4563012	0.003
Uranus	321 12 41.79	9.9995460	9.9999992	9.9089914	0.001
Neptune	86 12 49.67	9.9999999	9.9994896	8.8147322	0.001

Earth by—	I	II	II′	K
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mercury	7 0 7.710	53 48 31.10	28 34 4.99	25 25 13.33
Venus	3 23 35.010	25 1 46.65	54 7 49.75	330 56 48.79
Mars	1 51 2.240	51 57 45.14	284 53 57.15	127 3 21.25
Jupiter	1 18 42.100	1 25 19.94	272 58 11.88	88 27 5.264
Saturn	2 29 40.190	348 0 50.68	337 45 52.32	10 13 49.89
Uranus	0 46 20.540	27 7 31.73	95 0 58.70	292 6 31.40
Neptune	1 47 1.680	330 14 7.90	273 9 58.47	57 4 3.92

Earth by—	K'	$\log k$	$\log k'$	$\log C$	resid.
	° ′ ″				″
Mercury	25 3 36.28	9.9992608	9.9974965	7.8017097	0.002
Venus	330 51 5.12	9.9994999	9.9997387	5.3891826	0.008
Mars	127 4 14.72	$9.999788\bar{5}$	9.9999850	8.3052599	0.009
Jupiter	88 27 10.859	9.9998865	$9.999999\bar{8}$	8.7995614	0.010
Saturn	10 16 6.88	9.9999411	9.9996473	9.4563012	0.003
Uranus	292 6 34.66	9.9999609	9.9999997	9.9089914	0.005
Neptune	57 4 14.94	$9.999790\bar{2}$	$9.999999\bar{4}$	8.8147322	0.001

Mars by—	I	II	II′	K
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mercury	5 9 10.165	287 24 19.31	29 13 54.31	258 16 20.56
Venus	1 56 2.460	232 18 22.07	28 26 43.81	203 52 27.49
Earth	1 51 2.240	104 53 57.15	231 57 45.14	232 57 4.23
Jupiter	1 26 6.381	149 47 4.35	188 22 45.31	321 24 28.37
Saturn	2 21 52.110	176 17 59.42	293 4 38.76	243 12 17.28
Uranus	1 11 40.460	120 39 30.31	315 36 26.40	165 2 41.49
Neptune	2 22 41.388	152 49 56.12	222 47 52.02	290 3 32.71

Mars by—	K'	$\log k$	$\log k'$	$\log C$	resid.
	° ′ ″				″
Mercury	258 4 28.68	9.9995819	9.9986621	7.8017097	0.017
Venus	203 50 49.03	9.9999439	9.9998087	5.3891826	0.012
Earth	232 55 19.79	$9.999859\bar{6}$	$9.999914\bar{1}$	6.4491252	0.009
Jupiter	321 24 9.72	9.9999971	9.9998667	8.7995614	0.007
Saturn	243 14 23.99	9.9996870	9.9999432	9.4563012	0.007
Uranus	165 3 26.31	9.9999538	$9.999951\bar{9}$	9.9089914	0.003
Neptune	290 0 35.50	9.9998274	9.9997986	8.8147322	0.006

9. THE RADII VECTORES AND THE TRUE ANOMALIES.

The values of log r and v for the points of division employed in the four different cases are given in the following tables. In each case the equations,

$$\Sigma_1 r = \Sigma_2 r = ja, \quad \text{and} \quad \Sigma_1 v + 180° = \Sigma_2 v$$

were exactly satisfied, and the values of r were also obtained from the equation stated in Article 7 for obtaining the value of r^2.

MERCURY.

E	log r	v		
°		°	′	″
0	9.4878584	0	0	0.00
15	9.4916716	18	25	28.96
22.5	9.4963313	27	32	14.93
30	9.5026623	36	32	7.50
45	9.5195925	54	4	7.02
60	9.5407098	70	50	41.41
67.5	9.5522314	78	55	7.36
75	9.5640735	86	46	40.73
90	9.5878217	101	51	53.65
105	9.6103385	116	9	54.15
112.5	9.6207149	123	3	1.59
120	9.6303194	129	46	44.60
135	9.6467730	142	49	52.77
150	9.6589887	155	27	29.02
157.5	9.6633518	161	39	20.97
165	9.6664956	167	48	0.75
180	9.6690267	180	0	0.00
195	9.6664956	192	11	59.25
202.5	9.6633518	198	20	39.03
210	9.6589887	204	32	30.98
225	9.6467730	217	10	7.23
240	9.6303194	230	13	15.40
247.5	9.6207149	236	56	58.41
255	9.6103385	243	50	5.85
270	9.5878217	258	8	6.35
285	9.5640735	273	13	19.27
292.5	9.5522314	281	4	52.64
300	9.5407098	289	9	18.59
315	9.5195925	305	55	52.98
330	9.5026623	323	27	52.50
337.5	9.4963313	332	27	45.08
345	9.4916716	341	34	31.04

VENUS.

E	log r	v		
°		°	′	″
0	9.8563557	0	0	0.00
15	9.8564576	15	6	6.54
30	9.8567564	30	11	47.87
45	9.8572313	45	16	40.52
60	9.8578493	60	20	24.50
75	9.8585680	75	22	44.64
90	9.8593378	90	23	31.50
105	9.8601064	105	22	42.20
120	9.8608213	120	20	20.31
135	9.8614342	135	16	35.65
150	9.8619040	150	11	43.65
165	9.8621990	165	6	4.12
180	9.8622996	180	0	0.00
195	9.8621990	194	53	55.88
210	9.8619040	209	48	16.35
225	9.8614342	224	43	24.35
240	9.8608213	239	39	39.69
255	9.8601064	254	37	17.80
270	9.8593378	269	36	28.50
285	9.8585680	284	37	15.36
300	9.8578493	299	39	35.50
315	9.8572313	314	43	19.48
330	9.8567564	329	48	12.13
345	9.8564576	344	53	53.46

THE EARTH.

E	$\log r$	v		
°		°	′	″
0	9.9926546	0	0	0.00
22.5	9.9932181	22	52	14.25
30	9.9936460	30	29	2.39
45	9.9948189	45	41	0.84
60	9.9963428	60	50	8.59
67.5	9.9972036	68	23	26.41
90	0.0000000	90	57	39.46
112.5	0.0027784	113	23	5.92
120	0.0036266	120	49	43.50
135	0.0051200	135	40	31.82
150	0.0062624	150	28	37.29
157.5	0.006677$\bar{6}$	157	51	53.7$\bar{2}$
180	0.0072232	180	0	0.00
202.5	0.006677$\bar{6}$	202	8	6.2$\bar{9}$
210	0.0062624	209	31	22.71
225	0.0051200	224	19	28.18
240	0.0036266	239	10	16.50
247.5	0.0027784	246	36	54.08
270	0.0000000	269	2	20.54
292.5	9.9972036	291	36	33.59
300	9.9963428	299	9	51.41
315	9.9948189	314	18	59.16
330	9.9936460	329	30	57.61
337.5	9.9932181	337	7	45.75

MARS.

E	$\log r$	v		
°		°	′	″
0	0.1403760	0	0	0.00
30	0.1463201	32	47	24.62
45	0.1532670	48	54	53.41
60	0.1621567	64	44	46.64
90	0.1828971	95	21	5.913
120	0.2026920	124	31	47.15
135	0.2106341	138	39	52.35
150	0.2166313	152	34	23.40
180	0.2216237	180	0	0.00
210	0.2166313	207	25	36.60
225	0.2106341	221	20	7.65
240	0.2026920	235	28	12.85
270	0.1828971	264	38	54.087
300	0.1621567	295	15	13.36
315	0.1532670	311	5	6.59
330	0.1463201	327	12	35.38

10. THE SEPARATE RESULTS.

The values found for the intermediate auxiliary functions which depend upon E, as well as the final perturbations of the four inner planets in each case are now stated in the following tables. The results of the application of the more important test equations are also shown, but all of the test equations of Article 7 were also applied, and each computation (except the first), was, after its completion, duplicated from the beginning, the forms of the equations being changed in the duplication when this was possible.

MERCURY.

Action of Venus on Mercury.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$1000000\times g$	h
0°	0.619543952	+ 0.13308441	− 0.18036925	0.7970904	0.5235861$\bar{4}$
30	0.627434998	+ 0.22218381	− 0.06982371	0.1194506	0.5239083$\bar{6}$
60	0.647116316	+ 0.24372756	+ 0.07193966	0.1268000	0.52384406
90	0.675632886	+ 0.19194286	+ 0.20693555	0.0491867	0.52344851
120	0.706503003	+ 0.08070542	+ 0.29899200	2.1902889	0.52319742
150	0.730295757	− 0.06017874	+ 0.32344233	1.5631633	0.52358280
180	0.738317327	− 0.19295989	+ 0.27373528	1.8358797	0.52446104
210	0.727259050	− 0.28205939	+ 0.16318979	0.6524819	0.5250077$\bar{8}$
240	0.701243272	− 0.30360314	+ 0.02142638	0.0112481	0.5247075$\bar{5}$
270	0.669559472	− 0.25181838	− 0.11356958	0.3160138	0.5239107$\bar{5}$
300	0.641856586	− 0.14058090	− 0.20562585	1.0359483	0.52329644
330	0.624398293	+ 0.00030325	− 0.23007624	1.2969588	0.52323374
Σ_1	4.054580456*	− 0.17962654†	+ 0.28009822‡	5.9972554	3.14309264
Σ_2	4.054580456	− 0.17962659	+ 0.28009814	5.9972551	3.1430919$\bar{3}$

E	l	G	G'	G''	θ		
°					°	′	″
0	0.0959333$\bar{2}$	0.5235825$\bar{8}$	0.09595274	0.000015866	25	20	53.90
30	0.1035021$\bar{5}$	0.5239078$\bar{2}$	0.10350489	0.000002203	26	23	25.33
60	0.12324776	0.5238434$\bar{6}$	0.12325032	0.000001964	29	0	59.15
90	0.15215988	0.52344311	0.15217844	0.000013171	32	37	46.70
120	0.1832810$\bar{9}$	0.52318510	0.18331625	0.000022837	36	17	45.71
150	0.20668846	0.52356735	0.20672760	0.000023681	38	55	52.70
180	0.21383179	0.52444977	0.21385942	0.000016369	39	41	12.31
210	0.2022267$\bar{8}$	0.5250039$\bar{3}$	0.20223677	0.000006145	38	21	51.31
240	0.1765112$\bar{3}$	0.5247074$\bar{9}$	0.17651140	0.000000121	35	27	1.91
270	0.1456242$\bar{3}$	0.52390915	0.14562996	0.000004142	31	49	7.06
300	0.11853565	0.52329155	0.11855723	0.000016698	28	25	30.42
330	0.10114005	0.52322787	0.10117042	0.000024501	26	5	20.70
Σ_1	0.9113408$\bar{3}$	3.14305994	0.91144736	0.000073855	194	13	23.40
Σ_2	0.9113415$\bar{4}$	3.14305922	0.91144808	0.000073843	194	13	23.80

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = + 4.054580460.$
† $6[a'^2e' - kaa'e\cos K] = - 0.17962650.$
‡ $- 6k'aa'\cos\varphi'\cdot e\sin K' = + 0.28009816.$

ACTION OF VENUS ON MERCURY.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.06678154	0.36107029	0.27485672	9.0518226	9.9748963	9.6076810
30	0.07267844	0.36875602	0.28344481	9.0869397	0.0171823	9.6511278
60	0.08883727	0.38974368	0.30686976	9.1792738	0.1306112	9.7669404
90	0.11429390	0.42259487	0.34345542	9.2994384	0.2842725	9.9240134
120	0.14429575	0.46098687	0.38608356	9.4147448	0.4383831	$0.082154\bar{1}$
150	0.16872258	0.49199359	0.42040790	$9.496033\bar{5}$	$0.550042\bar{8}$	0.1974492
180	0.17620114	0.50144271	0.43084933	9.5225000	0.5845077	0.2336318
210	0.16325170	0.48506821	0.41274966	$9.488799\bar{4}$	$0.533532\bar{3}$	$0.181381\bar{4}$
240	0.13698082	0.45165804	0.37573831	9.4055654	$0.417388\bar{9}$	0.0613864
270	0.10823963	0.41480523	0.33478930	9.2928156	0.2691020	9.9083455
300	0.08503270	0.38481172	0.30136861	$9.176137\bar{7}$	0.1234343	9.7587487
330	0.07094409	0.36649704	0.28092115	9.0860236	$0.015098\bar{3}$	$9.648233\bar{6}$
Σ_1	0.69812922	2.54971331	2.07576629	$5.750044\bar{3}$	$1.669221\bar{5}$	$9.510542\bar{4}$
Σ_2	0.69813034	2.54971496	2.07576824	5.7500501	$1.669230\bar{1}$	9.5105508

E	$\log V$	J_1'	$1000 \times J_2$	J_3	$1000 \times F_2$
0°	$9.607665\bar{0}$	0.521404654	− 2.7049984	− 0.024195167	+ 0.6439191
30	9.6511256	0.520191194	− 0.6256166	− 0.032354328	+ 0.2492710
60	9.7669384	0.521003862	+ 1.8268277	− 0.030122813	− 0.2568249
90	9.9240003	0.522559008	+ 2.6401451	− 0.018096342	− 0.7387609
120	$0.082131\bar{6}$	0.523207843	+ 2.0172777	+ 0.000503667	− 1.0674025
150	0.1974260	0.522626872	+ 1.0213969	+ 0.020692271	− 1.1546906
180	$0.233615\bar{9}$	0.521405157	+ 0.3980040	+ 0.037057786	− 0.9772364
210	$0.181375\bar{4}$	0.520383901	+ 0.3750436	+ 0.045213990	− 0.5825883
240	0.0613863	0.520288911	+ 0.7059590	+ 0.042976535	− 0.0764923
270	9.9083414	0.521364846	+ 0.6938449	+ 0.030947096	+ 0.4054437
300	9.7587319	0.522844254	− 0.4295092	+ 0.012350057	+ 0.7340853
330	9.6482088	0.523030858	− 2.2812182	− 0.007832614	+ 0.8213734
Σ_1	$9.510469\bar{0}$	3.130154681*	+ 1.8135608	+ 0.038570065	− 0.9999517
Σ_2	$9.510477\bar{5}$	3.130156679	+ 1.8235957	+ 0.038570073	− 0.9999517

* $\Sigma_1(J_1' - G'') = + 3.130080826.$
$\Sigma_2(J_1' - G'') = + 3.130082836.$

ACTION OF VENUS ON MERCURY.

E	$10000 \times F_3$	R_0	$1000 \times S_0$	W_0	$R^{(a)}$	$1000 \times S^{(a)}$
0°	− 0.24640136	0.05971623	− 0.4883004	− 0.009827036	0.00000000	− 1.5879204
30	− 0.00762098	0.06444673	− 0.0208471	− 0.014490447	+ 0.10127648	− 0.0655216
60	− 0.09149188	0.08146579	+ 0.7212240	− 0.017625381	+ 0.20314033	+ 2.0766362
90	− 0.49802851	0.11164771	+ 0.7947010	− 0.015287000	+ 0.28842180	+ 2.0529668
120	− 0.89441990	0.15077545	− 0.4917115	+ 0.000363093	+ 0.30587345	− 1.1518366
150	− 0.92808606	0.18433784	− 2.4881611	+ 0.032271915	+ 0.20211372	− 5.4561938
180	− 0.56751805	0.19359808	− 3.0725539	+ 0.063241297	0.00000000	− 6.5837414
210	− 0.13320998	0.17488253	− 1.4207354	+ 0.068605698	− 0.19174661	− 3.1154764
240	+ 0.01209827	0.14154301	+ 0.6131517	+ 0.049504580	− 0.28714394	+ 1.4363109
270	− 0.19301611	0.10799082	+ 1.3152406	+ 0.025022994	− 0.27897487	+ 3.3976883
300	− 0.46971306	0.08194590	+ 0.7289596	+ 0.007023584	− 0.20433749	+ 2.0989094
330	− 0.49748432	0.06575439	− 0.1643583	− 0.003535808	− 0.10333144	− 0.5165700
Σ_1	− 2.25744598	0.70904446	− 1.9892305	+ 0.092680137	+ 0.01753235	− 3.7116419
Σ_2	− 2.25744596	0.70906002	− 1.9841603	+ 0.092587352	+ 0.01775908	− 3.7031067

E	$R_0 \sin v + (\cos v + \cos E)S_0$	$-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v \cdot S_0$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	− 0.00097660	− 0.05971623	− 0.008630594	− 0.004699311	− 0.09487655
30	+ 0.03833160	− 0.05180530	− 0.006100201	− 0.013143840	− 0.10594280
60	+ 0.07755258	− 0.02541165	+ 0.002882594	− 0.017388064	− 0.14618182
90	+ 0.10909895	+ 0.02454507	+ 0.009914505	− 0.011635936	− 0.22329541
120	+ 0.11643405	+ 0.09565744	− 0.000337472	+ 0.000133975	− 0.33255092
150	+ 0.08098459	+ 0.16537954	− 0.032192272	− 0.002265848	− 0.43432170
180	+ 0.00614511	+ 0.19359808	− 0.055541660	− 0.030242129	− 0.46680548
210	− 0.07011629	+ 0.16039915	− 0.041182679	− 0.054870101	− 0.41204381
240	− 0.10947718	+ 0.08954949	− 0.009624818	− 0.048559933	− 0.31218788
270	− 0.10595403	+ 0.01957233	+ 0.007191939	− 0.023967193	− 0.21598164
300	− 0.07680517	− 0.02822239	+ 0.005196776	− 0.004724857	− 0.14704334
330	− 0.03941900	− 0.05265111	− 0.003501678	+ 0.000490095	− 0.10809245
Σ_1	+ 0.01287279	+ 0.26545474	− 0.066055174	− 0.105480319	− 1.49964599
Σ_2	+ 0.01292582	+ 0.26543968	− 0.065870386	− 0.105392823	− 1.49967781

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = -0.0000000083.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 11321.398	m'	p 4.0539001
$[d\chi/dt]_{00}$	$= +1133127.6$	m'	p 6.0542788
$[di/dt]_{00}$	$= -$ 60449.278	m'	n 4.7813911
$[d\Omega/dt]_{00}$	$= -$ 792605.00	m'	n 5.8990568
$[d\pi/dt]_{00}$	$= +1127216.0$	m'	p 6.0520072
$[dL/dt]_{00}$	$= -1326653.0$	m'	n 6.1227573

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= +0.027739414$
$[d\chi/dt]_{00}$	$= +2.7763615$
$[di/dt]_{00}$	$= -0.14811133$
$[d\Omega/dt]_{00}$	$= -1.9420214$
$[d\pi/dt]_{00}$	$= +2.7618772$
$[dL/dt]_{00}$	$= -3.2505323$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Hill.	Method of Gauss.
$[de/dt]_{00}$	$+0.02780$	$+0.02774$	$+0.0277391$	$+0.0277394$
$e[d\pi/dt]_{00}$	$+0.56811$	$+0.57086$	$+0.567852$	$+0.567855$
$[di/dt]_{00}$	-0.14812	-0.14806	-0.1481112	-0.1481113
$\sin i\,[d\Omega/dt]_{00}$	-0.23648	-0.23665	-0.2367447	-0.2367449
$[d\chi/dt]_{00}$			$+2.776347$	$+2.776361$
$[dL/dt]_{00}$	-3.2769		-3.250522	-3.250532

NOTES.

This is the only one of the twenty eight computations that was not duplicated, but the values of θ were computed by two different formulas and all known test equations were applied. As an illustration of his first modification of GAUSS's method HILL published this complete computation from exactly the same elements as here employed, and DR. LOUIS ARNDT states that he has verified the results and found them correct. (*Bulletin de la Societe des Sciences Naturelles de Neuchatel*, Vol. XXIV). INNES states however that the test arising from the constancy of the major axis is not satisfied, the residual being -0.00075 (*M. N.*, Vol. LII, page 87), but this statement is an error, for the residual obtained from HILL'S figures is -0.0000000088, a practically exact agreement with that here obtained.

Upon comparing the present computation with that of HILL, the following slight discrepancies may be noticed:

Π', K and K' differ by less than $0''.1$ from HILL's values, a difference doubtless due to the fact that the preliminary computation was here effected with eight place logarithms while HILL employed but seven. The value of l for 330° should be 0.10114009 instead of 0.11014009, and G'' for 180° should be 0.00001637 instead of 0.00001617. These are misprints merely. The values of the logarithms of K_0, L_0', and N_0 in HILL seem to be slightly in error throughout, a double interpolation to second differences from HILL's values of θ giving with the three functions most in error,

			Hill's Values.
For $E =$ 60°,	$\log N_0$	0.30686978	0.3068691
For $E =$ 150°,	$\log L_0'$	0.49199342	0.4919942
For $E =$ 180°,	$\log L_0'$	0.50144261	0.5014421

The effect of these differences upon the final coefficients is, however, almost inappreciable.

It is evident from an inspection of the final sums that a division into twelve parts is necessary in this case, the terms from the sixth to the eleventh orders, inclusive, amounting to 1/600th of the whole for $[di/dt]_{00}$ and to 1/1200th of the whole for $[d\Omega/dt]_{00}$.

ACTION OF THE EARTH ON MERCURY.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$10000\times g$	h
0°	1.10386215	+ 0.29403604	− 0.13175730	0.04882864	1.00008277
30	1.11164085	+ 0.32704393	+ 0.06103685	0.01047875	1.00111148
60	1.12870093	+ 0.25768623	+ 0.24661358	0.17106421	1.00173668
90	1.15278960	+ 0.10454706	+ 0.37524793	0.39606073	1.00128597
120	1.17861164	− 0.09134000	+ 0.41247210	0.47853578	1.00022588
150	1.19808875	− 0.27748723	+ 0.34831240	0.34124242	0.99978030
180	1.20368356	− 0.40401673	+ 0.19995995	0.11246339	1.00066133
210	1.19273750	− 0.43702459	+ 0.00716576	0.00014443	1.00216106
240	1.16934301	− 0.36766690	− 0.17841099	0.08952996	1.00285589
270	1.14208711	− 0.21452774	− 0.30704518	0.26517354	1.00212493
300	1.11943230	− 0.01864062	− 0.34426952	0.33336685	1.00067710
330	1.10628960	+ 0.16750659	− 0.28010968	0.22068934	0.99977580
Σ_1	6.90363359*	− 0.32994198†	+ 0.20460782‡	1.23378883	6.00623965
Σ_2	6.90363341	− 0.32994198	+ 0.20460808	1.23378921	6.00623954

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = +6.90363352$.

† $6[a'^2e' - kaa'e\cos K] = -0.32994198$.

‡ $-6k'aa'\cos\varphi'\cdot e\sin K' = +0.20460788$.

ACTION OF THE EARTH ON MERCURY.

E	l	G	G'	G''	θ		
					°	′	″
0°	0.10349811	1.00007732	0.10355071	0.00004715	18	46	28.61
30	0.11024810	1.00111031	0.11025877	0.00000949	19	22	58.72
60	0.12668299	1.00171716	0.12683714	0.00013464	20	51	17.40
90	0.15122236	1.00123943	0.15152995	0.00026105	22	54	42.04
120	0.17810449	1.00016768	0.17843084	0.00026815	25	0	4.02
150	0.19802718	0.99973772	0.19824193	0.00017219	26	27	9.31
180	0.20274096	1.00064732	0.20281046	0.00005541	26	45	34.58
210	0.19029517	1.00216104	0.19029526	0.00000008	25	50	0.71
240	0.16620585	1.00284522	0.16627021	0.00005369	24	1	53.32
270	0.13968091	1.00209425	0.13990074	0.00018915	21	57	13.77
300	0.11847394	1.00063933	0.11879216	0.00028045	20	10	34.78
330	0.10623253	0.99975109	0.10646458	0.00020734	19	3	48.08
Σ_1	0.89570634	6.00609403	0.89669152	0.00083949	135	35	52.71
Σ_2	0.89570625	6.00609384	0.89669123	0.00083930	135	35	52.63

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.03586144	0.32053269	0.22947519	8.5993188	8.9197434	8.8287400
30	0.03828768	0.32372814	0.23305754	8.6307049	8.9534609	8.8632763
60	0.04451494	0.33191845	0.24223558	8.7125510	9.0428624	8.9539831
90	0.05408514	0.34447345	0.25629369	8.8165734	9.1597444	9.0722159
120	0.06487989	0.35858872	0.27208270	8.9130565	9.2712667	9.1849500
150	0.07303960	0.36922627	0.28397010	8.9788974	9.3482020	9.2629066
180	0.07483693	0.37156571	0.28658308	8.0002544	9.3712099	9.2865324
210	0.06949181	0.36460451	0.27880654	8.9738846	9.3366140	9.2517535
240	0.05973313	0.35186484	0.26456369	8.9063079	9.2556584	9.1696145
270	0.04950056	0.33846383	0.24956627	8.8114799	9.1479627	9.0600557
300	0.04158092	0.32806159	0.23791429	8.7102233	9.0374863	8.9477383
330	0.03700237	0.32203558	0.23116016	8.6301758	8.9522476	8.8613541
Σ_1	0.32140725	2.06253200	1.53285453	2.8417119	4.8982271	4.3715583
Σ_2	0.32140716	2.06253178	1.53285430	2.8417160	4.8982316	4.3715621

ACTION OF THE EARTH ON MERCURY.

E	$\log V$	J_1'	J_2	J_3	$1000 \times F_2$
0°	8.8287147	0.9963685	− 0.0084115617	− 0.04554861	+ 2.1929308
30	8.8632712	0.9875038	− 0.0046146363	− 0.08975347	− 1.0158793
60	8.9539113	0.9853935	+ 0.0065532595	− 0.10655243	− 4.1045657
90	9.0720772	0.9913722	+ 0.0136249408	− 0.09143424	− 6.2455186
120	9.1848079	0.9979641	+ 0.0119853300	− 0.04844492	− 6.8650672
150	9.2628156	0.9998176	+ 0.0047535770	+ 0.01089164	− 5.7972107
180	9.2865032	0.9963768	− 0.0029434883	+ 0.07066640	− 3.3280756
210	9.2517534	0.9902153	− 0.0070671940	+ 0.11485779	− 0.1192649
240	9.1695861	0.9854773	− 0.0058653024	+ 0.13162975	+ 2.9694212
270	9.0599551	0.9862780	− 0.0010764519	+ 0.11649805	+ 5.1103718
300	8.9475885	0.9932756	+ 0.0016290397	+ 0.07352222	+ 5.7299237
330	8.8612431	0.9996406	− 0.0026304325	+ 0.01421265	+ 4.6620645
Σ_1	4.3711117	5.9548558*	+ 0.0029472768	+ 0.08527241	− 3.4054328
Σ_2	4.3711156	5.9548275	+ 0.0029898031	+ 0.08527242	− 3.4054372

	$1000 \times F_3$	R_0	$1000 \times S_0$	$1000 \times W_0$	$R^{(n)}$	$1000 \times S^{(n)}$
0°	− 0.18791333	0.020434768	− 0.3847186	− 3.085993	0.000000000	− 1.2510793
30	+ 0.04317409	0.021303927	− 0.4280932	− 6.547345	+ 0.033478638	− 1.3454762
60	− 0.07397385	0.025621038	+ 0.1363110	− 9.590571	+ 0.063887706	+ 0.3925293
90	− 0.49750230	0.033590134	+ 0.7062450	− 10.865960	+ 0.086774000	+ 1.8244563
120	− 0.86425140	0.043556201	+ 0.5521780	− 7.575463	+ 0.088361000	+ 1.2934800
150	− 0.83674077	0.051546760	− 0.4218487	+ 1.808294	+ 0.056517506	− 0.9250562
180	− 0.43280690	0.053431000	− 1.3516860	+ 13.566615	0.000000000	− 2.8963373
210	− 0.01035886	0.048659649	− 1.2877184	+ 20.505307	− 0.053351926	− 2.8237889
240	− 0.07819780	0.040457621	− 0.3317444	+ 19.465045	− 0.082075038	− 0.7771130
270	+ 0.18040025	0.032377496	+ 0.5949004	+ 13.349022	− 0.083641596	+ 1.5368184
300	− 0.46718624	0.026190705	+ 0.7690320	+ 6.465459	− 0.065308303	+ 2.2142904
330	− 0.46610559	0.022213151	+ 0.2265587	+ 0.990810	− 0.034907444	+ 0.7120630
Σ_1	− 1.94793392	0.209691333	− 0.6106280	+ 19.245092	+ 0.004865365	− 1.0242299
Σ_2	− 1.94793368	0.209691117	− 0.6099562	+ 19.240128	+ 0.004869178	− 1.0209836

* $\Sigma_1(J_1' - G'') = 5.9540163.$
$\Sigma_2(J_1' - G'') = 5.9539882.$

Action of the Earth on Mercury.

E	$\sin v \cdot R_0 + (\cos v + \cos E)S_0$	$-\cos v \cdot R_0 + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$100 \times W_0 \cos u$	$100 \times W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	− 0.000769437	− 0.020434774	− 0.2710273	− 0.1475729	− 0.03246657
30	+ 0.011967951	− 0.017591057	− 0.2756307	− 0.5938896	− 0.03502118
60	+ 0.024315363	− 0.008157573	− 0.1568518	− 0.9461439	− 0.04597425
90	+ 0.032727249	+ 0.008319102	+ 0.7047203	− 0.8270793	− 0.06718020
120	+ 0.032844271	+ 0.028781472	+ 0.7040911	− 0.2795215	− 0.09606764
150	+ 0.022159505	+ 0.046499174	− 0.1803831	− 0.0126962	− 0.12145034
180	+ 0.002703372	+ 0.053431012	− 1.1914874	− 0.6487582	− 0.12883338
210	− 0.017924683	+ 0.045456375	− 1.2308941	− 1.6399928	− 0.11464781
240	− 0.030714227	+ 0.026434445	− 0.3784440	− 1.9093573	− 0.08923343
270	− 0.031808132	+ 0.005466897	+ 0.3836684	− 1.2785779	− 0.06475510
300	− 0.024103783	− 0.010000896	+ 0.4783818	− 0.4349396	− 0.04699652
330	− 0.012845686	− 0.018098650	+ 0.0981246	− 0.0137335	− 0.03651581
Σ_1	+ 0.004275559	+ 0.070053686	− 0.5016340	− 4.3662934	− 0.43957179
Σ_2	+ 0.004276204	+ 0.070051841	− 0.5003946	− 4.3659693	− 0.43957044

$\sin \varphi \cdot \frac{1}{2}A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = -0.00000000016.$

Differential Coefficients.

			log coeff.
$[de/dt]_{00}$	$= +3752.8345''$	m'	p 3.5743594
$[d\chi/dt]_{00}$	$= +299037.72$	m'	p 5.4757260
$[di/dt]_{00}$	$= -4591.3713$	m'	n 3.6619424
$[d\Omega/dt]_{00}$	$= -328217.95$	m'	n 5.5161623
$[d\pi/dt]_{00}$	$= +296589.74$	m'	p 5.4721561
$[dL/dt]_{00}$	$= -390282.17$	m'	n 5.5913787

Final Values Corresponding to the Above Value of m'.

$$[de/dt]_{00} = +0.011476557''$$
$$[d\chi/dt]_{00} = +0.91448833$$
$$[di/dt]_{00} = -0.014040890$$
$$[d\Omega/dt]_{00} = -1.0037245$$
$$[d\pi/dt]_{00} = +0.90700208$$
$$[dL/dt]_{00} = -1.1935233$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.01153	+0.01147	+0.0114766
$e[d\pi/dt]_{00}$	+0.18658	+0.18799	+0.186484
$[di/dt]_{00}$	−0.01414	−0.01404	−0.0140409
$\sin i\,[d\Omega/dt]_{00}$	−0.12219	−0.12233	−0.122360
$[dL/dt]_{00}$	−1.1942		−1.19352

NOTES.

As a' and e' are both small in this case, the sums, up to and including R_0, are in very exact agreement. But as I and e are unusually large, the final sums differ considerably, the greatest discrepancy being in $W_0 \cos u$, which shows that a neglect of the terms from the 6th to the 11th orders would produce an error in $[di/dt]_{00}$ of slightly more than 1/1000th of the whole value of this coefficient.

ACTION OF MARS ON MERCURY.

E	A	B cos ϵ	B sin ϵ	g	h
0°	2.3984504	+ 0.12138918	+ 0.45632916	0.0042055	2.3024514
15	2.3737047	− 0.02025060	+ 0.40611421	0.0033309	2.3025214
30	2.3556032	− 0.14407564	+ 0.32017434	0.0020703	2.3025321
45	2.3456111	− 0.24164776	+ 0.20436670	0.0008435	2.3024889
60	2.3447258	− 0.30631712	+ 0.06658267	0.0000896	2.3024184
75	2.3533241	− 0.33367707	− 0.08378742	0.0001418	2.3023625
90	2.3710514	− 0.32186246	− 0.23649650	0.0011296	2.3023658
105	2.3967846	− 0.27167903	− 0.38113729	0.0029337	2.3024640
120	2.4286852	− 0.18654648	− 0.50785318	0.0052088	2.3026710
135	2.4643476	− 0.07226635	− 0.60800831	0.0074658	2.3029751
150	2.5010251	+ 0.06337314	− 0.67477750	0.0091956	2.3033343
165	2.5359018	+ 0.21112855	− 0.70361065	0.0099983	2.3036852
180	2.5663693	+ 0.36093056	− 0.69254258	0.0096862	2.3039570
195	2.5902662	+ 0.50257031	− 0.64232765	0.0083325	2.3040870
210	2.6060491	+ 0.62639549	− 0.55638795	0.0062519	2.3040409
225	2.6128739	+ 0.72396747	− 0.44058010	0.0039202	2.3038234
240	2.6105920	+ 0.78863682	− 0.30279611	0.0018517	2.3034773
255	2.5996753	+ 0.81599664	− 0.15242602	0.0004692	2.3030742
270	2.5810993	+ 0.80418226	+ 0.00028298	0.0000000	2.3026229
285	2.5562149	+ 0.76399902	+ 0.14492387	0.0004242	2.3023962
300	2.5266327	+ 0.66886612	+ 0.27163969	0.0014902	2.3022243
315	2.4941374	+ 0.55458616	+ 0.37179487	0.0027917	2.3021798
330	2.4606272	+ 0.41894655	+ 0.43856414	0.0038844	2.3022350
345	2.4280691	+ 0.27119119	+ 0.46739720	0.0044120	2.3023425
Σ_1	29.7509107*	+ 2.89391842†	− 1.41728084‡	0.0450638	27.6343997
Σ_2	29.7509107	+ 2.89391853	− 1.41728059	0.0450638	27.6344002

* $12a^2 + 6a^2e^2 + 12[a'^2 - 2kaa'ee' \cos K] = 29.7509106.$

† $12[a'^2e' - kaa'e \cos K] = + 2.89391844.$

‡ $- 12k'aa' \cos \varphi' \cdot e \sin K' = - 1.41728062.$

ACTION OF MARS ON MERCURY.

E	l	G	G'	G''	θ		
°					°	′	″
0	0.0758032	2.3016305	0.0957141	0.0190900	12	51	3.65
15	0.0509875	2.3018785	0.0717874	0.0201570	11	28	40.15
30	0.0328754	2.3021359	0.0509293	0.0176577	9	54	3.88
45	0.0229265	2.3023282	0.0338958	0.0108086	7	59	27.93
60	0.0221116	2.3024014	0.0237650	0.0016363	6	1	37.49
75	0.0307659	2.3023354	0.0326775	0.0018845	7	2	5.28
90	0.0484898	2.3021481	0.0572743	0.0085668	9	43	5.24
105	0.0741248	2.3018919	0.0890147	0.0143178	12	11	37.36
120	0.1058185	2.3016404	0.1249596	0.0181104	14	22	46.33
135	0.1411768	2.3014735	0.1626257	0.0199473	16	17	11.47
150	0.1774951	2.3014531	0.1994129	0.0200367	17	54	21.70
165	0.2120208	2.3016063	0.2327627	0.0186630	19	13	8.83
180	0.2422166	2.3019140	0.2604177	0.0161582	20	12	26.11
195	0.2659835	2.3023097	0.2806562	0.0128954	20	51	34.72
210	0.2818125	2.3026974	0.2924401	0.0092842	21	10	38.08
225	0.2888548	2.3029783	0.2954612	0.0057613	21	10	27.64
240	0.2869190	2.3030785	0.2900893	0.0027715	20	52	41.30
255	0.2764054	2.3029737	0.2772408	0.0007349	20	19	35.05
270	0.2582114	2.3026922	0.2582114	0.0000000	19	33	51.59
285	0.2336230	2.3023071	0.2344977	0.0007857	18	38	24.76
300	0.2042127	2.3019157	0.2076391	0.0031178	17	36	2.08
315	0.1717618	2.3016103	0.1791035	0.0067722	16	29	5.95
330	0.1381965	2.3014548	0.1502129	0.0112362	15	19	13.32
345	0.1055309	2.3014695	0.1221038	0.0156999	14	6	53.42
Σ_1	1.8741623	27.6251620	2.0110657	0.1276658	185	31	50.77
Σ_2	1.8741617	27.6251624	2.0118270	0.1284276	185	48	12.56

ACTION OF MARS ON MERCURY.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.01657774	0.29504379	0.20087027	8.0316820	7.5954802	7.8669295
15	0.01319292	0.29055307	0.19582511	8.0355546	7.5943701	7.8655109
30	0.00979383	0.28603832	0.19075139	8.0547661	7.6099058	7.8800682
45	0.00636436	0.28147805	0.18562486	8.0870691	7.6401445	7.9084927
60	0.00361341	0.27781623	0.18150720	8.1291203	7.6819573	7.9481379
75	0.00492729	0.27956556	0.18347444	8.1771100	7.7316276	7.9980604
90	0.00943284	0.28555854	0.19021211	8.2272783	7.7853440	8.0537440
105	0.01490785	0.29282894	0.19838217	8.2762397	7.8395128	8.1098440
120	0.02081742	0.30066160	0.20717937	8.3211159	7.8908949	8.1628540
135	0.02681707	0.30859802	0.21608807	8.3595540	7.9366442	8.2098882
150	0.03254501	0.31616047	0.22457224	8.3896939	7.9743208	8.2484993
165	0.03762579	0.32285663	0.23208058	8.4101312	8.0019112	8.2766735
180	0.04171228	0.32823434	0.23810787	8.4198969	8.0178772	8.2928778
195	0.04453613	0.33194630	0.24226678	8.4184648	8.0212321	8.2961421
210	0.04594748	0.33380024	0.24434354	8.4057702	8.0116018	8.2861294
225	0.04593448	0.33378318	0.24432441	8.3822399	7.9892733	8.2631894
240	0.04461766	0.33205341	0.24238678	8.3488317	7.9552231	8.2283875
255	0.04222050	0.32890264	0.23885673	8.3070780	7.9111256	8.1835072
270	0.03902753	0.32470208	0.23414923	8.2591387	7.8593690	8.1310520
285	0.03533705	0.31984157	0.22870029	8.2078386	7.8030574	8.0742275
300	0.03141955	0.31467566	0.22290683	8.1566450	7.7459663	8.0168746
315	0.02748746	0.30948385	0.21708212	8.1095325	7.6924008	7.9633068
330	0.02367787	0.30444739	0.21142959	8.0706478	7.6468600	7.9179598
345	0.02004728	0.29964170	0.20603415	8.0437755	7.6135016	7.8848519
Σ_1	0.31918262	3.69919207	2.58841642	8.8145868	3.7748004	7.0335140
Σ_2	0.31939818	3.69947951	2.58873971	8.8145879	3.7748012	7.0338946

ACTION OF MARS ON MERCURY.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	7.8624692	2.3161036	+ 0.034714816	− 0.08283714	− 0.09798270
15	7.8607965	2.3114562	+ 0.028725995	− 0.12496689	− 0.08720058
30	7.8759318	2.3036617	+ 0.023022893	− 0.15748255	− 0.06874762
45	7.9059548	2.2939379	+ 0.017028648	− 0.17817560	− 0.04388148
60	7.9477527	2.2850812	+ 0.009527241	− 0.18564570	− 0.01429659
75	7.9976170	2.2883310	− 0.000387926	− 0.17939386	+ 0.01799079
90	8.0517340	2.2994643	− 0.012768514	− 0.15985316	+ 0.05078038
105	8.1064943	2.3097716	− 0.026802369	− 0.12835823	+ 0.08183755
120	8.1586286	2.3171693	− 0.041120811	− 0.08705254	+ 0.10904591
135	8.2052458	2.3210340	− 0.054165965	− 0.03874380	+ 0.13055114
150	8.2438461	2.3213532	− 0.064480743	+ 0.01328579	+ 0.14488783
165	8.2723467	2.3185110	− 0.070890916	+ 0.06550049	+ 0.15107882
180	8.2891363	2.3131717	− 0.072601961	+ 0.11434922	+ 0.14870229
195	8.2931581	2.3062073	− 0.069243148	+ 0.15650567	+ 0.13792019
210	8.2839812	2.2986444	− 0.060886015	+ 0.18909426	+ 0.11946725
225	8.2618557	2.2916136	− 0.048054879	+ 0.20988694	+ 0.09460109
240	8.2277453	2.2862701	− 0.031734816	+ 0.21745674	+ 0.06501619
255	8.1833367	2.2836646	− 0.013368882	+ 0.21127772	+ 0.03272882
270	8.1310520	2.2845306	+ 0.005188260	+ 0.19176389	− 0.00006076
285	8.0740447	2.2890080	+ 0.021821056	+ 0.16024221	− 0.03111796
300	8.0161484	2.2963713	+ 0.034481515	+ 0.11886354	− 0.05832630
315	7.9617277	2.3049376	+ 0.041716329	+ 0.07045517	− 0.07983156
330	7.9153379	2.3123762	+ 0.043244020	+ 0.01832594	− 0.09416821
345	7.8811861	2.3164879	+ 0.040207969	− 0.03396170	− 0.10035921
Σ_1	7.0037635	27.6341976*	− 0.133414115	+ 0.19026829	+ 0.30431767
Σ_2	7.0037641	27.6349607	− 0.133414088	+ 0.19026812	+ 0.30431761

* $\Sigma_1(J_1' - G'') = 27.5065318.$
$\Sigma_2(J_1' - G'') = 27.5065331.$

ACTION OF MARS ON MERCURY.

E	F_3	R_0	$1000 \times S_0$	$100 \times W_0$	$R^{(n)}$
0°	+ 0.006125234	0.005413043	− 0.1331172	− 0.05793911	0.000000000
15	+ 0.004244733	0.005395946	− 0.1341971	− 0.08902862	+ 0.004501863
30	+ 0.002091460	0.005581735	− 0.1069853	− 0.11749705	+ 0.008771561
45	+ 0.000394535	0.005978460	− 0.0544826	− 0.14331159	+ 0.012778556
60	− 0.000202379	0.006587462	+ 0.0157366	− 0.16470025	+ 0.016426268
75	+ 0.000673582	0.007397547	+ 0.0931210	− 0.17804909	+ 0.019496561
90	+ 0.003010065	0.008379102	+ 0.1659322	− 0.17823953	+ 0.021645905
105	+ 0.006397644	0.009479745	+ 0.2230373	− 0.15960687	+ 0.022459598
120	+ 0.010124788	0.010622853	+ 0.2557091	− 0.11755697	+ 0.021550254
135	+ 0.013355105	0.011710967	+ 0.2593958	− 0.05060872	+ 0.018676901
150	+ 0.015340435	0.012635664	+ 0.2351775	+ 0.03775311	+ 0.013854131
165	+ 0.015613320	0.013293529	+ 0.1902485	+ 0.13831063	+ 0.007415508
180	+ 0.014107795	0.013605711	+ 0.1366988	+ 0.23722085	0.000000000
195	+ 0.011176655	0.013535474	+ 0.0883172	+ 0.31912559	− 0.007550472
210	+ 0.007500348	0.013096767	+ 0.0561741	+ 0.37133333	− 0.014359702
225	+ 0.003911780	0.012350565	+ 0.0447317	+ 0.38738325	− 0.019696946
240	+ 0.001183526	0.011390146	+ 0.0503246	+ 0.36844987	− 0.023106835
255	− 0.000166297	0.010320547	+ 0.0628133	+ 0.32211263	− 0.024451640
270	+ 0.000001635	0.009239635	+ 0.0697179	+ 0.25931088	− 0.023868989
285	+ 0.001425688	0.008225494	+ 0.0610457	+ 0.19093564	− 0.021678655
300	+ 0.003528129	0.007331827	+ 0.0329172	+ 0.12533212	− 0.018282391
315	+ 0.005583303	0.006590510	− 0.0111915	+ 0.06726176	− 0.014086776
330	+ 0.006923128	0.006018069	− 0.0617561	+ 0.01815027	− 0.009457250
345	+ 0.007124120	0.005623237	− 0.1063107	− 0.02290733	− 0.004691492
Σ_1	+ 0.069734164	0.109902014	+ 0.7165294	+ 0.78161752	− 0.006827048
Σ_2	+ 0.069734168	0.109902021	+ 0.7165286	+ 0.78161728	− 0.006826894

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = + 0.000000000104.$

ACTION OF MARS ON MERCURY.

E	$1000 \times S^{(n}$	$1000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$	$1000 \times \left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1 \right) \sin v S_0 \right]$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$-2 \frac{r}{a} R_0$
0°	−0.43288822	−0.2662344	− 5.4130432	−0.5088501	− 0.2770661	−0.008600192
15	−0.43258505	+1.4484906	− 5.1972533	−0.6072560	− 0.6510373	−0.008648634
30	−0.33624961	+3.1443027	− 4.6032128	−0.4946402	− 1.0657799	−0.009175713
45	−0.16468928	+4.7703837	− 3.5917373	−0.1836694	− 1.4212974	−0.010218569
60	+0.04531053	+6.2357682	− 2.1327368	+0.2693639	− 1.6248266	−0.011820509
75	+0.25408211	+7.4151862	− 0.2308954	+0.7622000	− 1.6090993	−0.014007787
90	+0.42865608	+8.1659686	+ 2.0547246	+1.1559864	− 1.3566975	−0.016758205
105	+0.54706506	+8.3522568	+ 4.6004959	+1.3031443	− 0.9215481	−0.019968410
120	+0.59900000	+7.8723845	+ 7.2196098	+1.0926174	− 0.4337649	−0.023429816
135	+0.58504581	+6.6852166	+ 9.6761487	+0.5003962	− 0.0756834	−0.026827124
150	+0.51571179	+4.8307371	+11.7119607	−0.3765995	− 0.0265069	−0.029771109
165	+0.41003953	+2.4394871	+13.0838349	−1.3270492	− 0.3897732	−0.031867221
180	+0.29291257	−0.2733976	+13.6057094	−2.0833919	− 1.1343956	−0.032806220
195	+0.19034846	−3.0319641	+13.1877783	−2.4169344	− 2.0838761	−0.032447216
210	+0.12318210	−5.5396099	+11.8615372	−2.2290420	− 2.9698842	−0.030857525
225	+0.10088867	−7.5290371	+ 9.7823314	−1.5918740	− 3.5316447	−0.028292299
240	+0.11788553	−8.8108894	+ 7.2045350	−0.7163503	− 3.6141908	−0.025122156
255	+0.15406833	−9.3069299	+ 4.4325573	+0.1350559	− 3.2182941	−0.021739497
270	+0.18010328	−9.0565656	+ 1.7602448	+0.7452938	− 2.4836977	−0.018479268
285	+0.16656410	−8.1932578	− 0.5835177	+1.0058653	− 1.6229225	−0.015575563
300	+0.09477911	−6.8986293	− 2.4659985	+0.9273370	− 0.8431253	−0.013156194
315	−0.03382955	−5.3509513	− 3.8502686	+0.6070918	− 0.2895753	−0.011264707
330	−0.19409650	−3.6857779	− 4.7671354	+0.1797507	− 0.0251579	−0.009892996
345	−0.34269706	−1.9808199	− 5.2732726	−0.2254930	− 0.0403424	−0.009012936
Σ_1	+1.43430766	−4.2819430	+36.0361948	−2.0385248	−15.8550934	−0.229869903
Σ_2	+1.43430113	−4.2819391	+36.0362016	−2.0385225	−15.8550938	−0.229869963

DIFFERENTIAL COEFFICIENTS.

			log coeff.
		"	
$[de/dt]_{00}$	=	− 1879.077	m' n 3.2739445
$[d\chi/dt]_{00}$	=	+76914.75	m' p 4.8860096
$[di/dt]_{00}$	=	− 934.0667	m' n 2.9703779
$[d\Omega/dt]_{00}$	=	−59594.26	m' n 4.7752044
$[d\pi/dt]_{00}$	=	+76470.27	m' p 4.8834926
$[dL/dt]_{00}$	=	−101879.0	m' n 5.0080846

FINAL VALUES CORRESPONDING TO THE ABOVE VALUES OF m'.

$$
\begin{aligned}
[de/dt]_{00} &= -0.00060742746 \\
[d\chi/dt]_{00} &= +0.024863343 \\
[di/dt]_{00} &= -0.00030194497 \\
[d\Omega/dt]_{00} &= -0.019264347 \\
[d\pi/dt]_{00} &= +0.024719659 \\
[dL/dt]_{00} &= -0.032933242
\end{aligned}
$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	−0.00060	−0.00061	−0.000607
$e[d\pi/dt]_{00}$	+0.00508	+0.00511	+0.005082
$[di/dt]_{00}$	−0.00030	−0.00030	−0.000302
$\sin i\,[d\Omega/dt]_{00}$	−0.00234	−0.00235	−0.002348
$[dL/dt]_{00}$	−0.0331		−0.032933

NOTES.

On account of the very large values of the eccentricities of both orbits and their high mutual inclination, the approximate test is here wholly inapplicable if but twelve points of division are employed. Thus the two sums differ by 1° 38′ 46″.90 for θ and by 40′ 42″.47 for ϵ, while the sums of the functions immediately dependent upon these quantities differ by proportionate amounts. When the number of points of division is increased to twenty-four, the final sums are in almost exact agreement, showing that the combined effect of all terms from the 11th to the 23rd orders is wholly inappreciable. The greatest difference which arises in the variations from the employment of twenty-four points of division, instead of twelve, occurs in the case of $[di/dt]_{00}$ and here produces a decrease of but three units in the seventh decimal of the logarithm of the coefficient.

ACTION OF JUPITER ON MERCURY.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	p	q^2
0°	27.23340536	+2.0282403	+1.4219711	9.05679111	80.552426
30	27.14356714	+1.0282450	+1.6206004	9.02684503	80.826371
60	27.06879996	+0.0526625	+1.2863778	9.00192264	81.050843
90	27.03145602	−0.6371012	+0.5088565	8.98947466	81.156986
120	27.04270097	−0.8562240	−0.5036265	8.99322298	81.117340
150	27.09836241	−0.5459922	−1.4797775	9.01177679	80.952183
180	27.18120744	+0.2104677	−2.1580381	9.03939180	80.714558
210	27.26787830	+1.2104630	−2.3566674	9.06828209	80.466952
240	27.33631108	+2.1860457	−2.0224444	9.09109301	80.265977
270	27.37048776	+2.8758091	−1.2449232	9.10248524	80.156946
300	27.36240999	+3.0949315	−0.2324396	9.09979265	80.170254
330	27.31308297	+2.7846997	+0.7437107	9.08335031	80.311910
Σ_1	163.22483480*	+6.7161237†	−2.2081997‡	54.28221419	483.871398
Σ_2	163.22483460	+6.7161234	−2.2082005	54.28221412	483.871348

E	$-\theta'$	g	h	l	G
	° ′ ″				
0°	88 49 5	0.12745094	27.006742	+0.163630	27.006566
30	89 2 45	0.16554389	27.007491	+0.073044	27.007263
60	89 19 0	0.10430339	27.007569	−0.001801	27.007426
90	89 39 30	0.01632120	27.006899	−0.038475	27.006877
120	89 41 40	0.01598743	27.006246	−0.026577	27.006224
150	89 11 43	0.13801114	27.006455	+0.028875	27.006265
180	88 41 46	0.29354831	27.007536	+0.110638	27.007132
210	88 20 11	0.35007250	27.008599	+0.196247	27.008116
240	87 48 4	0.25781881	27.008698	+0.264581	27.008341
270	88 19 32	0.09768920	27.007728	+0.299728	27.007593
300	88 22 22	0.00340551	27.006562	+0.292816	27.006557
330	88 35 32	0.03486337	27.006176	+0.243874	27.006128
Σ_1	532° 41′ 57″	0.80250439	162.043353	+0.803287	162.042246
Σ_2	533 9 13	0.80250130	162.043348	+0.803293	162.042242

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = +163.22483477.$

† $6[a'^2 e' - kaa'e \cos K] = +6.7161238.$

‡ $-6k'aa' \cos \varphi' \cdot e \sin K' = -2.2082004.$

ACTION OF JUPITER ON MERCURY.

E	G'	G''	θ	$\log K_0$	$\log L_0'$
°			° ′ ″		
0	0.188801	0.024995	5 6 8.07	0.0025877	0.2764500
30	0.123075	0.049804	4 35 5.15	0.0020887	0.2757852
60	0.061321	0.062980	3 53 7.75	0.0014995	0.2750001
90	0.011983	0.050435	2 45 10.71	0.0007524	0.2740043
120	0.014441	0.040995	2 35 41.32	0.0006684	0.2738923
150	0.087484	0.058419	4 12 38.17	0.0017613	0.2753489
180	0.173639	0.062597	5 21 37.00	0.0028566	0.2768082
210	0.248823	0.052092	6 3 11.63	0.0036449	0.2778582
240	0.297071	0.032133	6 20 5.30	0.0039929	0.2783216
270	0.311476	0.011613	6 16 40.56	0.0039213	0.2782263
300	0.293250	0.000430	5 59 8.38	0.0035638	0.2777502
330	0.249105	0.005182	5 34 4.63	0.0030827	0.2771094
			° ′ ″		
Σ_1	1.028523	0.224130	29 15 47.82	0.0151689	1.6582224
Σ_2	1.031946	0.227545	29 26 50.85	0.0152513	1.6583323

E	$\log N_0$	$\log N$	$\log P$	$\log Q$	$\log V$
0°	0.1799707	6.4183196	3.8310274	5.1664192	5.1659174
30	0.1792229	6.4468140	3.8580378	5.1937562	5.1927566
60	0.1783398	6.5219986	3.9320091	5.2678436	5.2665795
90	0.1772196	6.6157906	4.0252257	5.3607256	5.3597128
120	0.1770937	6.7009450	4.1105921	5.4459161	5.4450927
150	0.1787322	6.7589559	4.1694988	5.5052851	5.5041125
180	0.1803735	6.7800058	4.1918462	5.5278954	5.5266396
210	0.1815544	6.7609474	4.1741434	5.5101707	5.5091259
240	0.1820755	6.7044320	4.1187250	5.4544932	5.4538485
270	0.1819683	6.6198775	4.0347585	5.3701732	5.3699402
300	0.1814329	6.5255910	3.9403890	5.2755478	5.2755392
330	0.1807123	6.4489104	3.8629284	5.1980770	5.1979730
Σ_1	1.0792861	9.6512920	4.1245888	2.1381153	2.1336169
Σ_2	1.0794097	9.6512958	4.1245926	2.1381878	2.1336210

ACTION OF JUPITER ON MERCURY.

E	J_1'	J_2	J_3	F_2	F_3
0°	26.907806	−0.08848287	−1.4438584	−1.8440911	+0.12702123
30	26.754757	−0.00410757	−2.3330959	−2.1016844	+0.05043578
60	26.779325	+0.16212886	−2.4984534	−1.6682462	−0.05353664
90	26.922525	+0.18465069	−1.8956909	−0.6599133	−0.05566955
120	27.032841	+0.04256694	−0.6863554	+0.6531309	+0.07759141
150	27.046752	−0.14516517	+0.8055488	+1.9190575	+0.24215469
180	26.945408	−0.26169947	+2.1803373	+2.7986639	+0.29255865
210	26.802715	−0.24648650	+3.0696706	+3.0562500	+0.18231424
240	26.714620	−0.10777763	+3.2352193	+2.6228182	+0.00934620
270	26.757114	+0.06770763	+2.6325527	+1.6144859	−0.07863175
300	26.911783	+0.13569580	+1.4231210	+0.3014406	−0.02506303
330	27.011072	+0.02690450	−0.0689744	−0.9644853	+0.08731406
Σ_1	161.291783*	−0.11756837	+2.2100104	+2.8637163	+0.42791782
Σ_2	161.294935	−0.11649642	+2.2100109	+2.8637104	+0.42791747

E	$1000 \times R_0$	$1000{,}000 \times S_0$	$100{,}000 \times W_0$	$1000 \times R^{(n)}$	$100{,}000 \times S^{(n)}$
0°	0.12949124	−0.25462237	− 2.1070330	0.00000000	−0.82801500
30	0.13531974	−0.15796961	− 3.6329106	+0.21265173	−0.49649079
60	0.16094783	+0.15688051	− 4.6204172	+0.40133398	+0.45170948
90	0.20322837	+0.35279259	− 4.3458069	+0.52500398	+0.91137646
120	0.25064630	+0.20287634	− 1.9026678	+0.50847837	+0.47523891
150	0.28657259	−0.17989808	+ 2.6073819	+0.31420703	−0.39449173
180	0.29756923	−0.44461029	+ 7.3765067	0.00000000	−0.95269261
210	0.28082350	−0.33962404	+ 9.9405132	−0.30790355	−0.74474862
240	0.24483424	+0.03827880	+ 9.2004732	−0.49668759	+0.08966829
270	0.20310516	+0.33360150	+ 6.1619379	−0.52468566	+0.86179960
300	0.16660105	+0.28219872	+ 2.6818065	−0.41543058	+0.81254093
330	0.14104055	−0.02790130	− 0.1024400	−0.22164184	−0.08769243
Σ_1	1.25008989	−0.01899829	+10.6286684	−0.00230582	+0.04845000
Σ_2	1.25008991	−0.01899894	+10.6286755	−0.00236831	+0.04975249

$\sin\varphi \cdot \frac{1}{2} A_1^{(6)} + \cos\varphi \cdot B_0^{(6)} = +0.00000000000073.$

* $\Sigma_1(J_1' - G'') = 161.067653.$
$\Sigma_2(J_1' - G'') = 161.067390.$

ACTION OF JUPITER ON MERCURY.

E	$1000 \times [R_0 \sin v + (\cos v + \cos E)S_0]$	$1000\times\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$1000\times W_0 \cos u$	$1000\times W_0 \sin u$	$1000\times\left(-2\frac{r}{a}R_0\right)$
0°	−0.00509245	−0.12949124	−0.018505013	−0.010075877	−0.20573445
30	+0.07792118	−0.11047546	−0.015293859	−0.032953015	−0.22244969
60	+0.15333585	−0.04994118	+0.007556597	−0.045582053	−0.28880400
90	+0.19816106	+0.04884221	+0.028185071	−0.033078773	−0.40645673
120	+0.19031365	+0.16372520	+0.017684089	−0.007020516	−0.55282671
150	+0.12222497	+0.25901646	−0.026009473	−0.001830673	−0.67519875
180	+0.00889221	+0.29756923	−0.064784149	−0.035274618	−0.71750180
210	−0.11061196	+0.25859897	−0.059670973	−0.079503145	−0.66165318
240	−0.18859570	+0.15601907	−0.017887815	−0.090249100	−0.54000750
270	−0.19945173	+0.03508579	+0.017710220	−0.059019459	−0.40621028
300	−0.15503995	−0.05982940	+0.019842790	−0.018040859	−0.29894808
330	−0.08443003	−0.11301587	−0.001014512	+0.000141991	−0.23185401
Σ_1	+0.00381361	+0.37805168	−0.056093501	−0.206243023	−2.60382254
Σ_2	+0.00381349	+0.37805210	−0.056093526	−0.206243074	−2.60382263

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 3.3470577	m'	p 0.5246632
$[d\chi/dt]_{00}$	$= +1613.8089$	m'	p 3.2078521
$[di/dt]_{00}$	$= -$ 51.404941	m'	n 1.7110049
$[d\Omega/dt]_{00}$	$= -1550.4039$	m'	n 3.1904449
$[d\pi/dt]_{00}$	$= +1602.2454$	m'	p 3.2047290
$[dL/dt]_{00}$	$= -2312.2863$	m'	n 3.3640416

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$
\begin{aligned}
[de/dt]_{00} &= +0.00319413 \\
[d\chi/dt]_{00} &= +1.540072 \\
[di/dt]_{00} &= -0.049056191 \\
[d\Omega/dt]_{00} &= -1.4795642 \\
[d\pi/dt]_{00} &= +1.5290366 \\
[dL/dt]_{00} &= -2.2066350
\end{aligned}
$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.00320	+0.00320	+0.003194
$e[d\pi/dt]_{00}$	+0.31437	+0.31664	+0.314377
$[di/dt]_{00}$	−0.04907	−0.04905	−0.049056
$\sin i\,[d\Omega/dt]_{00}$	−0.18042	−0.18037	−0.180368
$[dL/dt]_{00}$	−2.2078		−2.20663

NOTES.

The above results were published in 1896 in *A. J.*, No. 386. In 1911, upon applying to the various computations all of the test equations devised or learned of by that time, a slight error was detected in the value of F_3 for 240°. This rendered the values of W_0, $W_0 \cos u$, $W_0 \sin u$, $[di/dt]_{00}$ and $[d\Omega/dt]_{00}$ incorrect.

In this computation the device was for the first time applied of finding the root G by approximations and then depressing the cubic equation and solving the resulting quadratic equation directly. When a' and hence g is large, some such device becomes necessary as the solution by HILL'S formulas involves a great amount of labor. Thus, while but three approximations were necessary with the Earth on Mercury, no less than eleven were required in some cases with Mars on Mercury, and in the latter, as well as in the present case, if the formulas of HILL'S second method are employed the angle θ' will be found so nearly equal to 90° as to render the values of the roots obtained from it but little better than first approximations. Accordingly all the remaining computations have been effected by the method here outlined, a method which, since the approximation to the value of G is always very rapid, leads so quickly to the values of the roots that special devices for avoiding the solution of the cubic seem unnecessary.

The final sums are here practically in exact agreement, showing that the effect of all terms from the sixth to the eleventh orders inclusive is inappreciable.

ACTION OF SATURN ON MERCURY.

E	A	$B \cos \epsilon$	$B \sin \epsilon$	g	h
0°	91.40055452	7.9236831	−0.7566872	0.1637322	90.704247
22.5	91.41316673	8.0026121	+0.6479546	0.1200578	90.704730
45	91.37113281	7.5285556	+1.9837667	1.1253368	90.704973
67.5	91.28182268	6.5736833	+3.0473832	2.6555561	90.704844
90	91.15980423	5.2833659	+3.6768805	3.8659866	90.704445
112.5	91.02405557	3.8540430	+3.7764200	4.0781377	90.704075
135	90.89484135	2.5033169	+3.3308508	3.1725737	90.704038
157.5	90.79086206	1.4368221	+2.4080044	1.6581198	90.704442
180	90.72697664	0.8169216	+1.1483775	0.3771123	90.705145
202.5	90.71250894	0.7379931	−0.2562644	0.0187792	90.705802
225	90.75006384	1.2120495	−1.5920765	0.7248182	90.706059
247.5	90.83489465	2.1669211	−2.6556933	2.0167735	90.705781
270	90.95505790	3.4572383	−3.2851902	3.0861872	90.705088
292.5	91.09266180	4.8865615	−3.3847297	3.2760409	90.704321
315	91.22635509	6.2372881	−2.9391612	2.4702909	90.703855
337.5	91.33481374	7.3037834	−2.0163140	1.1625662	90.703855
Σ_1	728.48478638*	34.9624190‡	+1.5667604‡	14.9860379	725.637848
Σ_2	728.48478617	34.9624206	+1.5667608	14.9860312	725.637850

* $8a^2 + 4a^2e^2 + 8[a'^2 - 2kaa'ee' \cos K] = +728.48478640$.

† $8(a'^2e' - kaa'e \cos K] = +34.9624198$.

‡ $-8k'aa' \cos \varphi' \cdot e \sin K' = +1.5667610$.

ACTION OF SATURN ON MERCURY.

E	l	G	G'	G''	θ
					° ′ ″
0°	+0.41035$\bar{1}$	90.704227	0.4147226	0.0043526	3 53 50.78
22.5	+0.422480	90.704715	0.4256040	0.0031100	3 56 31.53
45	+0.380202	90.704836	0.4105578	0.0302188	3 59 47.96
67.5	+0.291021	90.704520	0.3703889	0.0790439	4 2 4.89
90	+0.16940$\bar{3}$	90.703974	0.3081768	0.1383038	4 1 12.34
112.5	+0.034023	90.703579	0.2300012	0.1954823	3 55 22.98
135	−0.09515$\bar{4}$	90.703652	0.1455476	0.2403156	3 44 5.05
157.5	−0.199537	90.704241	0.0683029	0.2676388	3 29 2.13
180	−0.264126	90.705099	0.0149026	0.2789825	3 15 29.12
202.5	−0.279250	90.705800	0.0007394	0.2799874	3 11 3.15
225	−0.24195$\bar{3}$	90.705971	0.0294521	0.2713169	3 17 46.29
247.5	−0.156844	90.705536	0.0901199	0.2467190	3 29 20.24
270	−0.03598$\bar{7}$	90.704713	0.1675089	0.2031209	3 39 39.24
292.5	+0.102383	90.703922	0.2482641	0.1454820	3 46 28.97
315	+0.236543	90.703554	0.3215440	0.0847000	3 50 7.92
337.5	+0.345001	90.703713	0.3789646	0.0338216	3 52 2.72
					° ′ ″
Σ_1	+0.559279	725.636026	1.8124124	1.2513111	29 41 58.70
Σ_2	+0.55927$\bar{7}$	725.636026	1.8123850	1.2512850	29 41 56.61

ACTION OF SATURN ON MERCURY.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.00150875	0.27501245	0.17835367	5.6285747	1.9882904	3.8492800
22.5	0.00154356	0.27505883	0.17840585	5.6455608	2.0053300	3.8663219
45	0.00158664	0.27511623	0.17847042	5.6919307	2.0514967	3.9126260
67.5	0.00161702	0.27515671	0.17851595	5.7568907	2.1160330	3.9773995
90	0.00160533	0.27514113	0.17849842	5.8276384	2.1862035	4.0478488
112.5	0.00152867	0.27503898	0.17838353	5.8929411	2.2508613	4.1127652
135	0.00138529	0.27484791	0.17816860	5.9445922	2.3018923	4.1639869
157.5	0.00120533	0.27460806	0.17789879	5.9773700	2.3341639	4.1963617
180	0.00105402	0.27440638	0.17767193	5.9884810	2.3449566	4.2071875
202.5	0.00100673	0.27434335	0.17760102	5.9770716	2.3334678	4.1956990
225	0.00107884	0.27443945	0.17770914	5.9440470	2.3006205	4.1628231
247.5	0.00120882	0.27461270	0.17790402	5.8922401	2.2492258	4.1113306
270	0.00133101	0.27477556	0.17808722	5.8268942	2.1844672	4.0463801
292.5	0.00141515	0.27488768	0.17821333	5.7562165	2.1144600	3.9761077
315	0.00146116	0.27494901	0.17828232	5.6914235	2.0503133	3.9116762
337.5	0.00148559	0.27498156	0.17831893	5.6452895	2.0046971	3.8658214
Σ_1	0.01101104	1.19868812	1.42524172	6.5435816	7.4082403	2.3018085
Σ_2	0.01101087	1.19868787	1.42524142	6.5435802	7.4082388	2.3018068

ACTION OF SATURN ON MERCURY.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	3.8492539	90.058195	−0.59752575	− 6.047237	+ 3.829727
22.5	3.8663033	89.638288	−0.20995430	− 7.910876	− 3.279413
45	3.9124452	89.669758	+0.36734212	− 8.328272	−10.040194
67.5	3.9769269	90.090673	+0.71855783	− 7.235732	−15.423345
90	4.0470220	90.582478	+0.67967125	− 4.799462	−18.609342
112.5	4.1115969	90.879839	+0.35660262	− 1.390299	−19.113128
135	4.1625507	90.892982	−0.04825504	+ 2.472681	−16.858027
157.5	4.1947623	90.670885	−0.36411899	+ 6.201244	−12.187337
180	4.2055204	90.332824	−0.49291773	+ 9.227617	− 5.812141
202.5	4.1940258	90.018885	−0.40939877	+11.090997	+ 1.296998
225	4.1612018	89.853321	−0.15646464	+11.507770	+ 8.057780
247.5	4.109856$\bar{2}$	89.914463	+0.15541932	+10.414614	+13.440932
270	4.0451660	90.199819	+0.35977661	+ 7.978084	+16.626927
292.5	3.9752380	90.576579	+0.29271946	+ 4.569179	+17.130715
315	3.9111697	90.780941	−0.07365934	+ 0.706820	+14.875613
337.5	3.8656191	90.580690	−0.50178023	− 3.021123	+10.204923
Σ_1	2.2943297	722.370318*	+0.03796748	+12.718001	− 7.929657
Σ_2	2.294328$\bar{4}$	722.370302	+0.03804694	+12.718004	− 7.929655

ACTION OF SATURN ON MERCURY.

E	F_3	$10000 \times R_0$	$1000000 \times S_0$	$1000000 \times W_0$	$1000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	−0.2217173	0.20835581	−0.38501122	− 4.2759258	0.0000000	−1.2520309
22.5	+0.0670688	0.21359630	−0.18752118	− 5.8140290	+2.6067657	−0.5980245
45	−0.2247552	0.23766012	+0.18723103	− 6.8102258	+5.0798256	+0.5659595
67.5	−0.9885705	0.27944181	+0.4799050$\bar{7}$	− 6.8742610	+7.2389467	+1.3456245
90	−1.8298399	0.33354940	+0.47167629	− 5.3763783	+8.6166490	+1.2184913
112.5	−2.2911663	0.39056810	+0.12052582	− 1.8384740	+8.6416600	−0.2886458
135	−2.1148434	0.43919690	−0.40799123	+ 3.5527836	+7.0044048	−0.9201906
157.5	−1.3919042	0.46950930	−0.83324243	+ 9.6804438	+3.9006063	−1.8089208
180	−0.5106652	0.47592790	−0.91983080	+14.8005740	0.0000000	−1.9709759
202.5	+0.0653940	0.45862525	−0.61203702	+17.3393050	−3.8101033	−1.3286969
225	+0.0610980	0.42282850	−0.06578321	+16.6811060	−6.7433594	−0.1483686
247.5	−0.4587079	0.37651300	+0.43874423	+13.4040390	−8.3306788	+1.0507430
270	−1.1366346	0.32772690	+0.65346841	+ 8.8350920	−8.4662365	+1.6881183
292.5	−1.5401525	0.28276850	+0.49946088	+ 4.2959022	−7.3251220	+1.4004581
315	−1.4203565	0.24587472	+0.10699355	+ 0.5601261	−5.2554073	+0.3234187
337.5	−0.8596732	0.22009701	−0.26508243	− 2.2258014	−2.6861019	−0.8453756
Σ_1	−7.3977141	2.69112025	−0.35924718	+27.9671518	+0.2358762	−0.4955782
Σ_2	−7.3977118	2.69111927	−0.3592470$\bar{7}$	+27.9671246	+0.2365727	−0.4955464

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = + 0.000000000034.$

* $\Sigma_1(J_1' - G'') = 721.119007.$ $\Sigma_2(J_1' - G'') = 721.119017.$

ACTION OF SATURN ON MERCURY.

E	$1000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000\times\left[-R_0\cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v\, S_0\right]$	$1000000\times W_0 \cos u$	$1000000\times W_0 \sin u$	$1000\times -2\frac{r}{a}R_0$
0°	−0.000770023	−0.0208355810	− 3.7553310	− 2.0447571	−0.033103367
22.5	+0.009535644	−0.0190997850	− 3.2422806	− 4.8260289	−0.034604548
45	+0.019486084	−0.0136593860	− 0.8728046	− 6.7540641	−0.040621607
67.5	+0.027699067	−0.0044469025	+ 2.0656152	− 6.5565758	−0.051490988
90	+0.032545329	+0.0078015052	+ 3.4868920	− 4.0923123	−0.066709877
112.5	+0.032625198	+0.0215155020	+ 1.6174933	− 0.8739008	−0.084259712
135	+0.027148276	+0.0344565700	− 3.5128323	+ 0.5313050	−0.100609845
157.5	+0.016337313	+0.0439769600	− 9.5267356	− 1.7182294	−0.111738865
180	+0.001839662	+0.0475927900	−12.9986018	− 7.0776661	−0.114756176
202.5	−0.013287669	+0.0439638930	−11.8447875	−12.6630324	−0.109148595
225	−0.025446823	+0.0337808020	− 6.8547683	−15.2076154	−0.096860250
247.5	−0.031966152	+0.0197522140	− 1.0475614	−13.3630400	−0.081227519
270	−0.032206863	+0.0054309792	+ 2.5393222	− 8.4623118	−0.065545379
292.5	−0.027462505	−0.0063965152	+ 2.7411608	− 3.3076901	−0.052103952
315	−0.019770536	−0.0145922640	+ 0.5055592	− 0.2411456	−0.042025673
337.5	−0.010655672	−0.0192899750	− 2.2254410	− 0.0400332	−0.035657730
Σ_1	+0.002825107	+0.0799753913	−21.4625646	−43.3484220	−0.560232174
Σ_2	+0.002825224	+0.0799754154	−21.4625365	−43.3485702	−0.560231909

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 1.8596825	m'	p 0.2694389
$[d\chi/dt]_{00}$	$= +256.04618$	m'	p 2.4083183
$[di/dt]_{00}$	$= -$ 14.751452	m'	n 1.1688348
$[d\Omega/dt]_{00}$	$= -244.39983$	m'	n 2.3881009
$[d\pi/dt]_{00}$	$= +254.22335$	m'	p 2.4052154
$[dL/dt]_{00}$	$= -373.17967$	m'	n 2.5719180

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$[de/dt]_{00} = +0.00053109524$$
$$[d\chi/dt]_{00} = +0.073122627$$
$$[di/dt]_{00} = -0.0042127757$$
$$[d\Omega/dt]_{00} = -0.069796619$$
$$[d\pi/dt]_{00} = +0.072602050$$
$$[dL/dt]_{00} = -0.10657405$$

Comparison with Other Results.

		Leverrier.	Newcomb.	Method of Gauss.
		"	"	"
	$[de/dt]_{00}$	+0.00053	+0.00053	+0.0005311
	$e[d\pi/dt]_{00}$	+0.01494	+0.01503	+0.0149273
	$[di/dt]_{00}$	−0.00421	−0.00421	−0.0042128
$\sin i$	$[d\Omega/dt]_{00}$	−0.00853	−0.00850	−0.0085087
	$[dL/dt]_{00}$	−0.1070		−0.106574

Notes.

The considerable disagreement of the first sums is caused by the rather large value of e'. The very exact agreement toward the close of the computation shows, however, that all terms above the 15th order are wholly inappreciable, the total effect of all terms from the 8th to the 15th orders inclusive occurring with $[de/dt]_{00}$ and amounting to but 1/30000th of the value of this coefficient.

Action of Uranus on Mercury.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	368.36907643	16.94656315	−5.8755608	27.995613	367.49553
45	368.87526360	22.1817430	−3.9726882	12.798540	367.49652
90	369.14240318	24.5977597	+1.1481399	1.069008	367.49606
135	369.01848670	22.7793354	+6.4872119	34.127748	367.49535
180	368.57162431	17.7916836	+8.9169755	64.480235	367.49625
225	368.05910248	12.5565017	+7.0141016	39.896574	367.49742
270	367.78562896	10.1404905	+1.8932751	2.906824	367.49671
315	367.91587927	11.9589113	−3.4457976	9.628773	367.49535
Σ_1	1473.86873228*	69.4764969†	+6.0828297‡	96.451680	1469.98455
Σ_2	1473.86873205	69.4764914	+6.0828277	96.451635	1469.98464

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee' \cos K] = +1473.86873246.$

† $4[a'^2e' - kaa'e \cos K] = +69.4764933.$

‡ $-4k'aa' \cos \varphi' \cdot e \sin K' = +6.0828300.$

ACTION OF URANUS ON MERCURY.

E	l	G	G'	G''	θ
					° ′ ″
0°	+0.06261	367.49532	0.30919	0.24638	2 13 39.23
45	+0.56779	367.49643	0.6237$\bar{3}$	0.0558$\bar{4}$	2 27 51.84
90	+0.83539	367.49605	0.8388$\bar{8}$	0.00347	2 44 38.85
135	+0.71220	367.49510	0.8250$\bar{1}$	0.1125$\bar{7}$	2 53 41.24
180	+0.26443	367.49577	0.5717$\bar{8}$	0.3068$\bar{7}$	2 48 5.47
225	−0.24927	367.49712	0.22774	0.47670	2 30 27.70
270	−0.52202	367.49669	0.0147$\bar{4}$	0.53674	2 13 6.46
315	−0.39041	367.49528	0.05839	0.44873	2 7 39.31
Σ_1	+0.64041	1469.98383	1.7345$\bar{8}$	1.0934$\bar{6}$	9 59 30.01
Σ_2	+0.64031	1469.98393	1.73486	1.09383	9 59 40.09

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.00049251	0.27365789	0.17682994	4.7157168	9.8582891	2.3270039
45	0.00060285	0.27380499	0.17699541	4.7796309	9.9227977	2.3913072
90	0.00074754	0.27399787	0.17721240	4.9163$\bar{2}7\bar{6}$	0.0598$\bar{1}2\bar{1}$	2.5282$\bar{8}3\bar{3}$
135	0.00083193	0.27411036	0.17733893	5.0341227	0.1774639	2.6460770
180	0.00077916	0.27404002	0.17725980	5.0782321	0.2210425	2.6898771
225	0.00062422	0.27383348	0.17702747	5.0332$\bar{6}6\bar{9}$	0.1754$\bar{6}7\bar{0}$	2.6444$\bar{7}7\bar{7}$
270	0.00048849	0.27365254	0.17682391	4.9151$\bar{2}2\bar{7}$	0.0570$\bar{0}0\bar{6}$	2.5260$\bar{5}9\bar{3}$
315	0.00044928	0.27360025	0.17676511	4.7787$\bar{8}3\bar{5}$	9.9208203	2.3897$\bar{6}6\bar{9}$
Σ_1	0.00250770	1.09534833	0.70812605	9.6253991	0.1961442	0.0712235
Σ_2	0.00250828	1.09534908	0.70812692	9.6258039	0.1965$\bar{4}8\bar{9}$	0.0716217

E	log V	J_1'	J_2	J_3	F_2
0°	2.3266400	366.509888	−2.2679092	−1.6777615	+100.81429
45	2.3912247	363.119586	−0.5138777	−3.4254958	+ 68.16435
90	2.5282$\bar{7}8\bar{2}$	365.160838	+2.2768225	−2.9100648	− 19.70006
135	2.6459108	367.567473	+0.7238013	−0.4333073	−111.30915
180	2.6894241	366.570373	−1.5747120	+2.5538323	−152.99965
225	2.6437740	364.094438	−1.1669936	+4.3014333	−120.34967
270	2.5252669	364.090394	+1.5139175	+3.7858697	− 32.48527
315	2.3891044	367.299619	+1.4067899	+1.3092445	+ 59.12384
Σ_1	0.0696$\bar{0}9\bar{2}$	1462.331493*	−0.0518812	+1.7518757	−104.37069
Σ_2	0.0700139	1462.082117	−0.4497199	+1.7518747	−104.37063

* $\Sigma_1(J_1' - G'') = 1461.238038.$
$\Sigma_2(J_1' - G'') = 1460.987287.$

ACTION OF URANUS ON MERCURY.

E	F_3	$1000000 \times R_0$	$1000000000 \times S_0$	$1000000 \times W_0$	$100000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	− 7.546840	2.572319	− 40.838728	−0.3564792	0.00000000	−0.13280483
45	− 0.473969	2.902866	− 6.943520	−0.8432739	+0.62046843	−0.02098878
90	− 1.504590	4.048391	+ 74.582644	−0.9823294	+1.04582909	+0.19267091
135	−14.059822	5.410853	+ 15.278801	−0.1938530	+0.86293520	+0.03446006
180	−17.382080	5.928252	−102.476485	+1.2462639	0.00000000	−0.21958237
225	− 5.500356	5.225873	− 69.412255	+1.8932075	−0.83343288	−0.15655363
270	+ 1.222261	3.978026	+ 47.038051	+1.2690546	−1.02765154	+0.12151438
315	− 5.177088	2.987389	+ 39.388463	+0.3202880	−0.63853471	+0.11906293
Σ_1	−25.211249	16.526988	− 21.694518	+1.1765099	+0.01817755	−0.03820191
Σ_2	−25.211235	16.526981	− 21.688511	+1.1763686	+0.01143604	−0.02401942

E	$1000000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$	$1000000 \times \left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1 \right) \sin v S_0 \right]$	$1000000 \times W_0 \cos u$	$1000000 \times W_0 \sin u$	$1000000 \times -2 \frac{r}{a} R_0$
0°	−0.0816775	−2.5723195	−0.31307783	−0.17046914	− 4.0868774
45	+2.3415248	−1.7140875	−0.10807474	−0.83631981	− 4.9616693
90	+3.9465628	+0.9815683	+0.63709735	−0.74771500	− 8.0967815
135	+3.2460626	+4.3319665	+0.19167313	−0.02898997	−12.3950199
180	+0.2049530	+5.9282527	−1.09453081	−0.59596603	−14.2942599
225	−3.0528880	+4.2563802	−0.77797582	−1.72597460	−11.9712672
270	−3.9027069	+0.7238029	+0.36474303	−1.21550896	− 7.9560519
315	−2.3679844	−1.8134015	+0.28908589	−0.13789041	− 5.1061400
Σ_1	+0.1671314	+5.0613044	−0.40576826	−2.72965913	−34.4339707
Σ_2	+0.1667150	+5.0608577	−0.40529154	−2.72917479	−34.4340964

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = -0.00000000000024.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 0.21975650	m'	p 9.3419417
$[d\chi/dt]_{00}$	$= +32.406731$	m'	p 1.5106352
$[di/dt]_{00}$	$= -$ 0.55745051	m'	n 9.7462063
$[d\Omega/dt]_{00}$	$= -30.777028$	m'	n 1.4882267
$[d\pi/dt]_{00}$	$= +32.177180$	m'	p 1.5075480
$[dL/dt]_{00}$	$= -45.859693$	m'	n 1.6614312

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$
\begin{aligned}
[de/dt]_{00} &= +0.0000096384435'' \\
[d\chi/dt]_{00} &= +0.0014213479 \\
[di/dt]_{00} &= -0.000024449584 \\
[d\Omega/dt]_{00} &= -0.0013498699 \\
[d\pi/dt]_{00} &= +0.0014112801 \\
[dL/dt]_{00} &= -0.0020113907
\end{aligned}
$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.00000	+0.00001	+0.0000096
$e[d\pi/dt]_{00}$	+0.00029	+0.00029	+0.0002902
$[di/dt]_{00}$	−0.00001	−0.00002	−0.0000244
$\sin i\ [d\Omega/dt]_{00}$	−0.00016	−0.00016	−0.0001646

NOTES.

In the results of this computation, published in *A. J.*, No. 398, the residual from the test equation which arises from the constancy of the major axis was stated very much too large. Its true value is as here given.

A comparison of the above figures with the corresponding tabulation for Saturn on Mercury and Uranus on Venus shows that a division into eight parts is fully sufficient. The effect of all terms of the 4th and higher orders may, however, in some cases amount to 1/1000th of the whole.

ACTION OF NEPTUNE ON MERCURY.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	904.45979027	+15.1625927	− 5.380877	1.889897	904.17356
45	904.50658592	+17.0408109	+ 3.150684	0.647955	904.17446
90	904.46648308	+12.3030460	+10.652425	7.406776	904.17363
135	904.36745259	+ 3.7246157	+12.729927	10.577523	904.17288
180	904.26302599	− 3.6693551	+ 8.166225	4.352861	904.17435
225	904.20989585	− 5.5475736	− 0.365336	0.008712	904.17570
270	904.24366422	− 0.8098086	− 7.867076	4.039717	904.17446
315	904.34902898	+ 7.7686225	− 9.944584	6.455136	904.17292
Σ_1	3617.43296356*	+22.9864685†	+ 5.570698‡	17.689251	3616.69599
Σ_2	3617.43296334	+22.9864755	+ 5.570691	17.689326	3616.69596

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee'\cos K] = +3617.4329635$,

† $4[a'^2e' - kaa'e\cos K] = +22.986469$.

‡ $-4k'aa'\cos\varphi' \cdot e\sin K' = +5.570695$,

ACTION OF NEPTUNE ON MERCURY.

E	l	G	G'	G''	θ °	′	″
0°	0.22096	904.17356	0.2300460	0.0090860	0	55	54.56
45	0.26686	904.17446	0.2695190	0.0026589	0	59	38.88
90	0.2275$\bar{9}$	904.17362	0.2591945	0.0316047	1	1	39.23
135	0.1293$\bar{1}$	904.17286	0.1906740	0.0613537	0	57	23.73
180	0.02341	904.17434	0.0820755	0.0586556	0	42	53.31
225	0.03107	904.17570	0.0003070	0.0313770	0	20	20.99
270	0.0039$\bar{4}$	904.17445	0.0688415	0.0649018	0	41	48.59
315	0.11084	904.17291	0.1564755	0.0456255	0	51	23.82
Σ_1	0.57589	3616.69597	0.6401575	0.1642481	3	22	15.69
Σ_2	0.5759$\bar{4}$	3616.69593	0.6169755	0.1410151	3	8	47.42

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.00008616	0.27311614	0.17622049	4.1292404	8.4898441	1.3492047
45	0.00009807	0.27313202	0.17623836	4.192724$\bar{6}$	8.553349$\bar{6}$	1.412709$\bar{5}$
90	0.00010478	0.27314097	0.17624841	4.3291694	8.6897764	1.5491508
135	0.00009080	0.27312233	0.17622746	4.447037$\bar{2}$	8.807597$\bar{7}$	1.666983$\bar{8}$
180	0.00005070	0.27306887	0.17616731	4.491505$\bar{4}$	8.852013$\bar{7}$	1.711392$\bar{4}$
225	0.00001141	0.27301649	0.17610837	4.446977$\bar{3}$	8.807458$\bar{0}$	1.666817$\bar{8}$
270	0.00004818	0.27306552	0.17616353	4.3290882	8.6895869	1.5489683
315	0.00007281	0.27309835	0.17620048	4.1926694	8.553221$\bar{0}$	1.4125965
Σ_1	0.00028982	1.09239150	0.70479974	7.279003$\bar{4}$	4.721221$\bar{1}$	6.158716$\bar{2}$
Σ_2	0.00027309	1.09236919	0.70477467	7.279408$\bar{4}$	4.7216261	6.159107$\bar{5}$

E	log V	J_1'	J_2	J_3	F_2
0°	1.3491992	897.83542	−6.8010114	− 59.717292	+41.026915
45	1.4127079	890.85490	+1.7283795	− 93.077013	−24.022644
90	1.5491318	899.76750	+6.4446186	− 62.851349	−81.220264
135	1.6669469	904.08417	−1.3097917	+ 13.254268	−97.060333
180	1.7113571	897.88500	−6.6871696	+ 90.657875	−62.264029
225	1.6667989	891.04449	−2.2340656	+124.017084	+ 2.785531
270	1.5489293	894.36354	+5.9480179	+ 93.790933	+59.983153
315	1.4125691	903.73833	+2.4206062	+ 17.685807	+75.823263
Σ_1	6.1586174	3589.85146*	−1.0955445	+ 61.880167	−42.474225
Σ_2	6.1590228	3589.72189	+0.6051284	+ 61.880146	−42.474183

* $\Sigma_1(J_1' - G'') = 3589.68721.$
$\Sigma_2(J_1' - G'') = 3589.58087.$

ACTION OF NEPTUNE ON MERCURY.

E	F_3	$1000000 \times R_0$	$1000000000 \times S_0$	$1000000000 \times W_0$	$1000000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	− 2.928627	0.6591808	−15.070783	−133.45316	0.000000	−0.04900917
45	− 0.320722	0.7449111	+ 4.384511	−240.74232	+1.592198	+0.01325344
90	− 8.201274	1.0513250	+22.423107	−222.59983	+2.715911	+0.05792607
135	−13.630154	1.3989891	− 6.706655	+ 61.47306	+2.231138	−0.01512625
180	− 6.745208	1.5178690	−34.846090	+466.35639	0.000000	−0.07466679
225	+ 0.065738	1.3382857	−10.354891	+575.81202	−2.134326	−0.02335460
270	− 3.847073	1.0317860	+21.345904	+331.94457	−2.665434	+0.05514330
315	− 7.846125	0.7779772	+ 6.529864	+ 45.70113	−1.662874	+0.01973839
Σ_1	−21.722182	4.2601608	− 6.147862	+442.24897	+0.050477	−0.01060659
Σ_2	−21.731263	4.2601631	− 6.147171	+442.24389	+0.026136	−0.00548902

E	$1000000000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$	$1000000000 \times \left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1 \right) \sin v S_0 \right]$	$1000000000 \times W_0 \cos u$	$1000000000 \times W_0 \sin u$	$1000000000 \times -2 \frac{r}{a} R_0$
0°	− 30.1416	− 659.1808	−117.20523	− 63.81760	−1047.3000
45	+ 608.8428	− 430.4075	− 30.85375	−238.75703	−1273.2255
90	+1024.2533	+ 261.0141	+144.36884	−169.43527	−2102.6500
135	+ 855.3050	+1105.9013	− 60.78178	+ 9.19306	−3204.7615
180	+ 69.6922	+1517.8690	−409.57745	−223.01267	−3659.9008
225	− 792.9698	+1080.1647	−236.61842	−524.94878	−3065.7028
270	−1014.1308	+ 169.4383	+ 95.40526	−317.93876	−2063.5719
315	− 621.4949	− 466.5348	+ 41.24897	− 19.67525	−1329.7429
Σ_1	+ 49.6731	+1289.1406	−287.00858	−774.20430	−8873.4227
Σ_2	+ 49.6831	+1289.1237	−287.00498	−774.18800	−8873.4327

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = +0.000000000000013.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 0.065401848	m'	p 8.8155900
$[d\chi/dt]_{00}$	$= +$ 8.2544736	m'	p 0.9166894
$[di/dt]_{00}$	$= -$ 0.39452600	m'	n 9.5960756
$[d\Omega/dt]_{00}$	$= -$ 8.7298700	m'	n 0.9410078
$[d\pi/dt]_{00}$	$= +$ 8.1893623	m'	p 0.9132501
$[dL/dt]_{00}$	$= -11.826130$	m'	n 1.0728427

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$
\begin{aligned}
[de/dt]_{00} &= +0.''00000331989 \\
[d\chi/dt]_{00} &= +0.00041900885 \\
[di/dt]_{00} &= -0.00002002670 \\
[d\Omega/dt]_{00} &= -0.00044314061 \\
[d\pi/dt]_{00} &= +0.00041570371 \\
[dL/dt]_{00} &= -0.00060031125
\end{aligned}
$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.00000	+0.00000	+0.0000033
$e[d\pi/dt]_{00}$	+0.00009	+0.00009	+0.0000855
$[di/dt]_{00}$	−0.00001	−0.00002	−0.0000200
$\sin i\ [d\Omega/dt]_{00}$	−0.00005	−0.00005	−0.0000508

NOTES.

In the final results of this computation, published in *A. J.*, No. 398, the value of the residual arising from the equation $[da/dt]_{00} = 0$ is greatly overstated. Its true value is that given above.

The very large disagreement in G', G'', θ, etc., arises from the large values of e' and I but the gradual lessening of the discrepancies as the end of the computation is approached shows that terms of the 8th and higher orders are wholly inappreciable. The greatest effect produced by all terms of the 4th and higher orders here occurs with $[de/dt]_{00}$ and amounts to but 1/10000th of the value of this coefficient.

VENUS.

Action of Mercury on Venus.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$1000\times g$	h
0°	0.73249627	+0.19271542	+0.22036223	0.30759837	0.58840054
15	0.70628935	+0.12839518	+0.25427819	0.40956972	0.56402284
30	0.67778527	+0.05734848	+0.27076206	0.46439247	0.53673295
45	0.64892716	−0.01558282	+0.26869056	0.45731389	0.50812387
60	0.62168257	−0.08542863	+0.24820463	0.39023784	0.48000449
75	0.59791027	−0.14742905	+0.21070058	0.28121662	0.45436878
90	0.57923027	−0.19735886	+0.15873412	0.15960636	0.43333042
105	0.56691636	−0.23181556	+0.09584678	0.05819218	0.41895978
120	0.56180731	−0.24845074	+0.02632410	0.00438952	0.41292353
135	0.56425026	−0.24613080	−0.04509594	0.01288203	0.41598961
150	0.57407787	−0.22501408	−0.11354628	0.08166866	0.42773114
165	0.59061890	−0.18653936	−0.17436209	0.19258116	$0.4467037\bar{3}$
180	0.61274521	−0.13332873	−0.22339892	0.31613445	0.47092914
195	0.63894853	−0.06900848	−0.25731488	0.41941058	0.49830819
210	0.66744400	+0.00203821	−0.27379880	0.47486769	0.52683118
225	0.69628968	+0.07496950	−0.27172719	0.46770903	0.55465379
240	0.72352217	+0.14481528	−0.25124139	0.39984532	0.58013161
255	0.74728586	+0.20681571	−0.21373734	0.28938127	$0.6018400\bar{4}$
270	0.76596211	+0.25674559	−0.16177082	0.16577156	0.61860745
285	0.77827946	+0.29120222	−0.09888350	0.06193800	$0.6295459\bar{9}$
300	0.78339732	+0.30783739	−0.02936080	0.00546066	0.63407808
315	0.78096661	+0.30551754	+0.04205926	0.01120554	0.63196045
330	0.77115125	+0.28440072	+0.11050957	0.07735875	0.62329194
345	0.75461912	+0.24592608	+0.17132542	0.18593159	0.60851491
Σ_1	8.07130162*	0.35632005†	−0.01822030‡	2.84733165	6.33299247
Σ_2	8.07130156	0.35632016	−0.01822015	2.84733161	$6.3329919\bar{7}$

* $12a^2 + 6a^2e^2 + 12[a'^2 - 2kaa'ee'\cos K] = 8.07130158.$

† $12[a'^2e' - kaa'e\cos K] = +0.35632010.$

‡ $-12k'aa'\cos\varphi'\cdot e\sin K' = -0.01822024.$

ACTION OF MERCURY ON VENUS.

E	l	G	G'	G''	ζ		
°					°	′	″
0	0.13776127	0.58723527	0.14259981	0.00367327	29	50	12.97
15	0.13593205	0.56231468	0.14274284	0.00510263	30	41	37.56
30	0.13471786	0.53456025	0.14296704	0.00607649	31	40	18.70
45	0.13446883	0.50568767	0.14321940	0.00631437	32	42	44.77
60	0.13534362	0.47761736	0.14342737	0.00569662	33	44	34.65
75	0.13720703	0.45239646	0.14351084	0.00433149	34	40	36.18
90	0.13956539	0.43206750	0.14340426	0.00257595	35	25	4.49
105	0.14162212	0.41845750	0.14309623	0.00097183	35	52	44.73
120	0.14254931	0.41288415	0.14266320	0.00007451	36	0	33.62
135	0.14192619	0.41587654	0.14225701	0.00021775	35	48	50.91
150	0.14001227	0.42706500	0.14202490	0.00134648	35	20	41.47
165	$0.1375807\overline{2}$	0.44529837	0.14203102	0.00304495	34	40	10.49
180	0.13548161	0.46890720	0.14224327	0.00473972	33	51	11.1^{3}
195	0.13430588	0.49597000	0.14257523	0.00593116	32	57	11.97
210	0.13427836	0.52451120	0.14293246	0.00633413	32	1	25.39
225	0.13530143	0.55262582	0.14323803	0.00590863	31	6	52.56
240	0.13705610	0.57856627	0.14343948	0.00481804	30	16	21.83
255	$0.1391113\overline{7}$	0.60079677	0.14351100	0.00335627	29	32	27.98
270	0.14102020	0.61804514	0.14345226	0.00186975	28	57	29.65
285	$0.1423990\overline{2}$	0.62934391	0.14328793	0.00068684	28	33	26.40
300	0.14298478	0.63406059	0.14306246	0.00006020	28	21	52.79
315	0.14267170	0.63192420	0.14283210	0.00012415	28	23	50.37
330	0.14152485	0.62303406	0.14265313	0.00087040	28	39	39.46
345	0.13976975	0.60786141	$0.1425687\overline{3}$	0.00214548	29	8	52.62
Σ_1	1.66229562	6.31855399	1.71486964	0.03813556	384	9	26.15
Σ_2	1.66229608	6.31855323	$1.7148703\overline{6}$	0.03813555	384	9	26.54

ACTION OF MERCURY ON VENUS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.09428087	0.39679023	0.31472571	0.0090498	0.8627996	0.5522553
15	0.10017341	0.40440450	0.32320948	0.0415726	0.9381720	$0.610879\bar{6}$
30	0.10716555	0.41342180	0.33324961	0.0806578	1.0282684	0.6810018
45	0.11492344	0.42340404	0.34435531	$0.124816\bar{3}$	$1.129676\bar{9}$	$0.759899\bar{9}$
60	0.12293987	0.43369427	0.35579389	$0.171632\bar{4}$	1.2368681	$0.843197\bar{0}$
75	0.13049856	0.44337414	0.36654485	0.2174860	1.3415449	0.9243732
90	0.13670353	0.45130400	0.37534554	0.2575171	1.4325547	0.9947294
105	0.14065788	0.45634998	0.38094249	0.2862203	1.4972527	1.0445040
120	0.14178785	0.45779080	0.38254019	0.2989083	1.5248859	1.0655419
135	0.14009658	0.45563407	0.38014861	0.2935151	1.5107656	1.0544719
150	0.13608370	0.45051253	0.37446742	$0.271437\bar{9}$	$1.458228\bar{2}$	$1.014044\bar{2}$
165	0.13043971	0.44329885	0.36646127	$0.236759\bar{5}$	1.3768369	0.9516100
180	0.12381666	0.43481823	0.35704266	$0.194571\bar{5}$	1.2784801	0.8761594
195	0.11676724	0.42577304	0.34698954	0.1495758	1.1741125	$0.795947\bar{2}$
210	0.10975153	0.41675185	0.33695547	$0.105445\bar{2}$	$1.072260\bar{8}$	0.7174326
225	0.10314673	0.40824135	0.32748247	$0.064777\bar{9}$	0.9789193	0.6452104
240	0.09725105	0.40063000	0.31900464	$0.029299\bar{4}$	$0.898020\bar{0}$	$0.582349\bar{3}$
255	0.09229597	0.39422222	0.31186325	$0.000126\bar{3}$	0.8320547	$0.530842\bar{6}$
270	0.08845847	0.38925289	0.30632243	$9.977973\bar{8}$	$0.782562\bar{5}$	$0.491964\bar{1}$
285	0.08587486	0.38590395	0.30258708	$9.963306\bar{2}$	0.7504867	0.4665316
300	0.08464926	0.38431433	0.30081370	9.9564277	0.7363980	$0.455069\bar{4}$
315	0.08485629	0.38458290	0.30111334	9.9575313	0.7406137	$0.457894\bar{4}$
330	0.08653835	0.38676424	0.30354673	$9.966711\bar{9}$	0.7632399	$0.475140\bar{5}$
345	0.08969689	0.39085723	0.30811150	9.9839477	0.8041353	0.5067244
Σ_1	1.32942669	5.01604517	4.05980799	1.3196325	$13.074566\bar{1}$	$8.748884\bar{7}$
Σ_2	1.32942756	5.01604627	4.05980919	1.3196348	13.0745711	$8.748889\bar{0}$

ACTION OF MERCURY ON VENUS.

E	log V	J_1'	J_2	J_3	$1000 \times F_2$
0°	0.5489818	0.14641488	+0.046569366	+0.007078444	−6.6249758
15	0.6061455	0.14799570	+0.053805593	+0.005998321	−7.6446250
30	0.6750909	0.14917691	+0.057280924	+0.004274648	−8.1401963
45	0.7534266	0.14962155	+0.056746961	+0.002024896	−8.0779189
60	0.8370288	0.14915409	+0.052252409	−0.000597633	−7.4620305
75	0.9194294	0.14784239	+0.044134510	−0.003414224	−6.3345072
90	0.9916527	0.14602955	+0.032988273	−0.006232935	−4.7721868
105	1.0433053	0.14427332	+0.019614339	−0.008861684	−2.8815397
120	1.0654487	0.14317010	+0.004953440	−0.011121317	−0.7914084
135	1.0542014	0.14310862	−0.009985226	−0.012857840	+1.3557653
150	1.0124149	0.14408811	−0.024195183	−0.013952904	+3.4136578
165	0.9480753	0.14573205	−0.036737898	−0.014331875	+5.2420265
180	0.8709298	0.14748132	−0.046798849	−0.013968920	+6.7162708
195	0.7897501	0.14882171	−0.053732494	−0.012888777	+7.7359211
210	0.7111602	0.14942892	−0.057095855	−0.011165046	+8.2314924
225	0.6396416	0.14920890	−0.056671026	−0.008915214	+8.1692132
240	0.5779992	0.14827058	−0.052477157	−0.006292610	+7.5533276
255	0.5279163	0.14686716	−0.044770588	−0.003475962	+6.4258044
270	$0.490375\bar{7}$	0.14532896	−0.034036602	−0.000657227	+4.8634820
285	0.4659576	0.14399775	−0.020965237	+0.001971497	+2.9728356
300	0.4550194	0.14316554	−0.006416142	+0.004231074	+0.8827039
315	0.4577910	0.14302191	+0.008631431	+0.005967519	−1.2644706
330	0.4744060	0.14361483	+0.023141851	+0.007062502	−3.3223626
345	0.5048724	0.14483260	+0.036096092	+0.007441419	−5.1507310
Σ_1	$8.710508\bar{1}$	1.75532379*	−0.003833505	−0.041341924	+0.5477741
Σ_2	8.7105125	1.75532376	−0.003833543	−0.041341924	+0.5477737

* $\Sigma_1(J_1' - G'') = 1.71718823.$
$\Sigma_2(J_2' - G'') = 1.71718811.$

ACTION OF MERCURY ON VENUS.

E	$1000 \times F_3$	R_0	S_0	W_0	$R^{(n)}$	$S^{(n)}$
0°	+0.12855762	−1.0113711	+0.11654330	+0.02599380	0.0000000	+0.16223015
15	+0.28853773	−1.0855630	+0.15095431	+0.02672256	−0.3910151	+0.21008155
30	+0.43551240	−1.1839703	+0.18420453	+0.02487781	−0.8232930	+0.25617923
45	+0.52994293	−1.3088599	+0.21275230	+0.01862044	−1.2857216	+0.29555816
60	+0.54641763	−1.4594255	+0.23029016	+0.00532100	−1.7533282	+0.31946706
75	+0.48046967	−1.6276742	+0.22753508	−0.01781201	−2.1774255	+0.31512321
90	+0.34977540	−1.7935752	+0.19439994	−0.05167293	−2.4796011	+0.26875615
105	+0.18941983	−1.9243133	+0.12616235	−0.09195653	−2.5651527	+0.17410988
120	+0.04248891	−1.9847155	+0.03108865	−0.12787906	−2.3681413	+0.04283321
135	−0.05148113	−1.9570464	−0.06917608	−0.14733858	−1.9039342	−0.09517474
150	−0.06711060	−1.8525243	−0.15091802	−0.14550282	−1.2730044	−0.20741354
165	+0.00001044	−1.7023416	−0.20114407	−0.12716787	−0.6051233	−0.27625376
180	+0.13212520	−1.5390947	−0.22013939	−0.10126618	0.0000000	−0.30227222
195	+0.29405223	−1.3852923	−0.21561000	−0.07503510	+0.4924233	−0.29612150
210	+0.44259806	−1.2520945	−0.19638955	−0.05218717	+0.8604054	−0.26990714
225	+0.53811675	−1.1428753	−0.16935246	−0.03375802	+1.1118587	−0.23300075
240	+0.55512295	−1.0569737	−0.13887026	−0.01942443	+1.2611694	−0.19133211
255	+0.48911303	−0.9922445	−0.10732663	−0.00839923	+1.3226839	−0.14811570
270	+0.35776744	−0.9463685	−0.07579537	+0.00013574	+1.3083458	−0.10478640
285	+0.19621617	−0.9174587	−0.04456334	+0.00686904	+1.2273328	−0.06171770
300	+0.04762640	−0.9042927	−0.01348260	+0.01232297	+1.0864013	−0.01870356
315	−0.04835263	−0.9063942	+0.01780841	+0.01685710	+0.8903706	+0.02473967
330	−0.06620421	−0.9240677	+0.04973149	+0.02067163	+0.6425655	+0.06916321
345	−0.00136711	−0.9584216	+0.08262385	+0.02378861	+0.3452194	+0.11498675
Σ_1	+2.90467772	−5.9084737	+0.01066288	−0.40860954	−3.5384806	+0.02421404
Σ_2	+2.90467759	−5.9084840	+0.01066372	−0.40860959	−3.5384837	+0.02421607

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = +0.000000033.$

ACTION OF MERCURY ON VENUS.

E	$[R_0 \sin v + (\cos v + \cos E) S_0]$	$\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	+0.2330866	+1.0113711	+0.015230839	+0.021064164	2.0089005
15	+0.0087244	+1.1264720	+0.009475283	+0.024986282	2.1567748
30	−0.2767659	+1.2080644	+0.002459142	+0.024755972	2.3539076
45	−0.6298334	+1.2226121	−0.003043897	+0.018369962	2.6050539
60	−1.0391050	+1.1217558	−0.002204101	+0.004843036	2.9088638
75	−1.4586399	+0.8508163	+0.011332120	−0.013742297	3.2495836
90	−1.7948636	+0.3765263	+0.042079553	−0.029990049	3.5871504
105	−1.9215248	−0.2668018	+0.086138100	−0.032190511	3.8554443
120	−1.7441606	−0.9487539	+0.127283754	−0.012324508	3.9830138
135	−1.2790794	−1.4880990	+0.145357191	+0.024082111	3.9330324
150	−0.6591294	−1.7579567	+0.132584634	+0.059936528	3.7270060
165	−0.0490234	−1.7488888	+0.098503636	+0.080428204	3.4271881
180	+0.4402788	−1.5390947	+0.059336027	+0.082061396	3.0992536
195	+0.7728021	−1.2274791	+0.026854296	+0.070065081	2.7888981
210	+0.9628351	−0.8906639	+0.005513895	+0.051895060	2.5190296
225	+1.0443017	−0.5731024	−0.005195888	+0.033355748	2.2968106
240	+1.0518039	−0.2937729	−0.007836004	+0.017773731	2.1211810
255	+1.0129579	−0.0559799	−0.005257515	−0.006550236	1.9880041
270	+0.9468650	+0.1451147	+0.000109448	−0.000080285	1.8927371
285	+0.8649660	+0.3177537	+0.006402057	−0.002489441	1.8316679
300	+0.7723975	+0.4708818	+0.012250672	−0.001332937	1.8023971
315	+0.6691423	+0.6125550	+0.016656295	+0.002594179	1.8040170
330	+0.5508294	+0.7487960	+0.018894170	+0.008386104	1.8371831
345	+0.4092814	+0.8824106	+0.018479728	+0.014979907	1.9041728
Σ_1	−0.5559282	−0.3477320	+0.405702029	+0.226988212	31.8406236
Σ_2	−0.5559251	−0.3477313	+0.405701406	+0.226989461	31.8406376

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= -$	$97592.111''\ m'$	n 4.9894147
$[d\chi/dt]_{00}$	$= -8920493.9$	m'	n 6.9503889
$[di/dt]_{00}$	$= +$	$71223.820\ m'$	p 4.8526253
$[d\Omega/dt]_{00}$	$= +$	$673299.06\ m'$	p 5.8282080
$[d\pi/dt]_{00}$	$= -8919313.6$	m'	n 6.9503315
$[dL/dt]_{00}$	$= +5590689.3$	m'	p 6.7474654

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$
\begin{aligned}
[de/dt]_{00} &= -0''.013012279 \\
[d\chi/dt]_{00} &= -1.1893992 \\
[di/dt]_{00} &= +0.0094965089 \\
[d\Omega/dt]_{00} &= +0.089773204 \\
[d\pi/dt]_{00} &= -1.1892420 \\
[dL/dt]_{00} &= +0.74542525
\end{aligned}
$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$-0''.01304$	$-0''.01301$	$-0''.013012$
$e[d\pi/dt]_{00}$	-0.00810	-0.00814	-0.008138
$[di/dt]_{00}$	$+0.00950$	$+0.00949$	$+0.009497$
$\sin i\ [d\Omega/dt]_{00}$	$+0.00529$	$+0.00531$	$+0.005301$
$[dL/dt]_{00}$	$+0.747$		$+0.745425$

NOTES.

This computation was originally made with but twelve points of division, but it was found that, notwithstanding the small eccentricity of the orbit of Venus, the values of e' and I are here so large that the tests which arise by comparing the sums of the functions were, toward the close of the computation, entirely inapplicable. The sums for $[de/dt]_{00}$ agreed to but a single significant figure, while those for $[d\chi/dt]_{00}$, $[di/dt]_{00}$, and $[d\Omega/dt]_{00}$ agreed to but two. It will be noticed that the increase of the number of points of division almost wholly removes the discrepancy.

Notwithstanding the entire disagreement of the test equations when but twelve points of division were employed, it is evident that this number would have been sufficient. The greatest error would have occurred with $[dL/dt]_{00}$, its amount being $0''.00000016$, showing that with this coefficient the sum of all terms of an order higher than the 12th amounts to but 1/4000000th of the remaining terms.

ACTION OF THE EARTH ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$1000\times g$	h
0°	1.53711333	+0.64403672	+0.34841048	0.03414346	1.00234463
30	1.52932317	+0.38337105	+0.61703357	0.10708849	$1.0037255\bar{4}$
60	1.51985488	+0.02331740	+0.71967967	0.14568124	$1.0031407\bar{0}$
90	1.51125435	−0.33964844	+0.62884478	0.11122751	1.00120600
120	1.50583081	−0.60827014	+0.36886855	0.03827088	$0.999831\bar{3}1$
150	1.50503256	−0.71057033	+0.00941083	0.00002491	$1.000355\bar{3}1$
180	1.50906482	−0.61913831	−0.35321171	0.03509098	$1.0022640\bar{2}$
210	1.51684286	−0.35847281	−0.62183471	0.10876150	$1.003709\bar{3}5$
240	1.52628659	+0.00158081	−0.72448100	0.14763156	$1.0032674\bar{7}$
270	1.53487476	+0.36454672	−0.63364618	0.11293247	$1.0013352\bar{7}$
300	1.54031063	+0.63316838	−0.37366983	0.03927365	0.99985008
330	1.54113325	+0.73546881	−0.01421211	0.00005681	1.00036791
Σ_1	9.13846106*	+0.07469486†	−0.01440384‡	0.44009177	6.01069819
Σ_2	9.13846095	+0.07469500	−0.01440382	0.44009169	6.01069936

E	l	G	G'	G''	θ		
					°	′	″
0°	0.53448743	1.00227181	0.53462398	0.00006372	46	55	3.02
30	$0.5253163\bar{7}$	1.00350237	0.52574251	0.00020298	46	22	33.07
60	$0.5164329\bar{2}$	1.00284204	0.51701255	0.00028098	45	53	55.40
90	0.50976708	1.00097979	0.51021108	0.00021779	45	33	45.30
120	$0.5057182\bar{4}$	0.99975382	0.50587139	0.00007567	45	20	44.51
150	$0.5043959\bar{9}$	1.00035525	0.50439608	0.00000005	45	14	29.83
180	$0.5065195\bar{4}$	1.00219338	0.50665928	0.00006911	45	19	11.90
210	$0.5128522\bar{5}$	1.00348844	0.51328431	0.00021116	45	39	53.44
240	$0.5227378\bar{6}$	1.00296095	0.52332564	0.00028127	46	15	21.69
270	$0.5332582\bar{3}$	1.00109414	0.53371072	0.00021137	46	54	18.41
300	0.54017928	0.99976461	0.54033745	0.00007270	47	19	22.49
330	0.54048407	1.00036779	0.54048430	0.00000011	47	18	38.53
Σ_1	3.12607525	6.00978661	3.12783029	0.00084345	277	3	39.01
Σ_2	3.12607397	6.00978778	3.12782900	0.00084346	277	3	38.58

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 9.13846101.$

† $6[a'^2e' - kaa'e \cos K] = +0.07469471.$

‡ $-6k'aa' \cos\varphi' \cdot e \sin K' = -0.04440383.$

ACTION OF THE EARTH ON VENUS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.26000336	0.60598257	0.54577154	9.8305330	0.4344894	0.3752914
30	0.25288171	0.59719127	0.53614961	9.8233230	0.4173018	0.3578664
60	0.24673133	0.58958546	0.52781896	9.8197364	0.4066135	0.3462012
90	0.24246622	0.58430371	0.52203035	9.8196999	0.4029640	0.3412105
120	0.23974361	0.58092896	0.51833025	9.8208350	0.4019120	0.3392393
150	0.23844519	0.57931865	0.51656434	9.8213596	0.4003696	0.3377696
180	0.23942219	0.58053038	0.51789317	9.8218870	0.4004544	0.3387987
210	0.24375790	0.58590391	0.52378440	9.8244982	0.4071947	0.3466789
240	0.25132635	0.59526901	0.53404476	9.8301980	0.4226554	0.3628370
270	0.25983876	0.60577958	0.54554439	9.8370023	0.4416487	0.3819851
300	0.26543283	0.61267386	0.55308969	9.8405753	0.4533906	0.3937357
330	0.26526804	0.61247091	0.55286781	9.8378789	0.4500302	0.3905869
Σ_1	1.50265967	3.56497024	3.19694837	8.9637646	2.5195152	2.1561032
Σ_2	1.50265782	3.56496803	3.19694594	8.9637618	2.5195089	2.1560973

E	log V	J_1'	J_2	J_3	F_2
0°	0.3752596	0.9974825	+0.004202020	−0.047236424	−0.005832177
30	0.3577653	0.9964541	+0.010025286	−0.058127293	−0.010328762
60	0.3460609	0.9970986	+0.013405599	−0.053273840	−0.012046994
90	0.3411015	0.9987570	+0.012206904	−0.033976547	−0.010526476
120	0.3392013	0.9997616	+0.006516023	−0.005406098	−0.006174632
150	0.3377695	0.9991244	−0.001172845	+0.024782063	−0.000157532
180	0.3387641	0.9974877	−0.007608410	+0.048499056	+0.005912547
210	0.3465734	0.9964670	−0.010818504	+0.059389945	+0.010409132
240	0.3626968	0.9970681	−0.010873469	+0.054536468	+0.012127366
270	0.3818798	0.9987050	−0.008962465	+0.035239146	+0.010606846
300	0.3936994	0.9997505	−0.005884342	+0.006668715	+0.006255003
330	0.3905868	0.9991427	−0.001520954	−0.023519435	+0.000237902
Σ_1	2.1556821	5.9886490*	−0.000242579	+0.003787877	+0.000241113
Σ_2	2.1556763	5.9886502	−0.000242578	+0.003787879	+0.000241110

* $\Sigma_1(J_1' - G'') = 5.9878055$.
$\Sigma_2(J_1' - G'') = 5.9878067$.

ACTION OF THE EARTH ON VENUS.

E	$1000 \times F_3$	R_0	$1000 \times S_0$	W_0	$R^{(n)}$	$S^{(n)}$
0°	+0.20122394	0.42126107	−5.890097	−0.11153496	0.0000000	−0.008199111
30	+0.06017487	0.40675651	−4.150418	−0.13232119	+0.2828447	−0.005772122
60	−0.29485277	0.40441131	−0.984602	−0.11894044	+0.4858527	−0.001365876
90	−0.50823894	0.41101802	+0.151227	−0.07580674	+0.5682286	+0.000209069
120	−0.36563277	0.41646740	−1.349097	−0.01272800	+0.4969244	−0.001858754
150	−0.00856116	0.41403810	−2.948800	+0.05391799	+0.2845157	−0.004052670
180	+0.20680805	0.40710579	−1.730893	+0.10632289	0.0000000	−0.002376680
210	+0.06559223	0.40536093	+2.554045	+0.13208012	−0.2785530	+0.003510140
240	−0.29105383	0.41526071	+7.029019	+0.12494379	−0.4954847	+0.009684416
270	−0.50707628	0.43289840	+7.732079	+0.08349760	−0.5984781	+0.010689527
300	−0.36741788	0.44451245	+3.199250	+0.01546615	−0.5340294	+0.004438118
330	−0.01281582	0.43893384	−3.068000	−0.05784756	−0.3052198	−0.004266768
Σ_1	−0.91092526	2.50901873	+0.273580	+0.00352943	−0.0467370	+0.000322113
Σ_2	−0.91092510	2.50900580	+0.270133	+0.00352022	−0.0466619	+0.000317176

E	$[R_0 \sin v + (\cos v + \cos E)S_0]$	$\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	−0.01178019	−0.42126107	−0.065352939	−0.09038273	−0.83675673
30	+0.19740436	−0.35572418	−0.013079792	−0.13167315	−0.80869188
60	+0.35044516	−0.20183130	+0.049268295	−0.10825646	−0.80605518
90	+0.41100745	+0.00311509	+0.061732775	−0.04399687	−0.82203604
120	+0.36078900	+0.20803111	+0.012668751	−0.00122668	−0.83578481
150	+0.21090709	+0.35633148	−0.049130989	−0.02221027	−0.83298365
180	+0.00346179	+0.40710579	−0.062298971	−0.08615914	−0.81978340
210	−0.20590981	+0.34919573	−0.013955080	−0.13134085	−0.81552660
240	−0.36545661	+0.19760050	+0.050403535	−0.11432600	−0.83336328
270	−0.43294230	−0.01250179	+0.067326200	−0.04938656	−0.86579680
300	−0.38308894	−0.22551772	+0.015375410	−0.00167292	−0.88598306
330	−0.22607885	−0.37629530	−0.052873488	−0.02346769	−0.87266531
Σ_1	−0.04562979	−0.03587269	+0.000064081	−0.40202393	−5.01772646
Σ_2	−0.04561206	−0.03587897	+0.000019626	−0.40207539	−5.01770028

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = +0.00000000814.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= -$	$16017''.410\ m'$	n 4.2045923
$[d\chi/dt]_{00}$	$= -1840673.3$	m'	n 6.2649767
$[di/dt]_{00}$	$= +$	$14.695\ m'$	p 1.1671802
$[d\Omega/dt]_{00}$	$= -2385136.3$	m'	n 6.3775132
$[d\pi/dt]_{00}$	$= -1844854.1$	m'	n 6.2659621
$[dL/dt]_{00}$	$= -1765973.3$	m'	n 6.2469841

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= -0''.04898290$
$[d\chi/dt]_{00}$	$= -5.6289701$
$[di/dt]_{00}$	$= +0.000044940$
$[d\Omega/dt]_{00}$	$= -7.293993$
$[d\pi/dt]_{00}$	$= -5.6417558$
$[dL/dt]_{00}$	$= -5.4005288$

COMPARISON WITH OTHER RESULTS.

		Leverrier.	Newcomb.	Method of Gauss.
	$[de/dt]_{00}$	$-0''.04875$	$-0''.04896$	$-0''.048982$
	$e[d\pi/dt]_{00}$	-0.03873	-0.03852	-0.038607
	$[di/dt]_{00}$	$+0.00006$	$+0.00004$	$+0.0000449$
$\sin i$	$[d\Omega/dt]_{00}$	-0.43154	-0.43169	-0.431698
	$[dL/dt]_{00}$	-5.397		-5.4005

NOTES.

The close agreement of the sums of the functions toward the beginning of the computation is here caused by the smallness of the term a'^2e'; the ratio of the major axes is, however, so large that the expansion of the perturbing function is not very rapidly convergent. The greatest error arising from a division of the orbit into but six parts would here occur with the coefficient $[d\Omega/dt]_{00}$, its amount being $0''.0004$, which is 1/16000th of the whole.

ACTION OF MARS ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	2.6510232	−0.78427108	+0.4410282	0.003928189	2.30246275
30	2.6356206	−0.87195528	−0.1200582	0.000291101	2.30200670
60	2.6766222	−0.66613155	−0.6497891	0.008527164	2.30404590
90	2.7630522	−0.22195070	−1.0062238	0.020447915	2.30677230
120	2.8717556	+0.34156979	−1.0938564	0.024164644	2.30762720
150	2.9736020	+0.87343460	−0.8892048	0.015968474	2.30563865
180	3.0412895	+1.23113141	−0.4471057	0.004037199	2.30274975
210	3.0566810	+1.31881567	+0.1139805	0.000262374	2.30215960
240	3.0156549	+1.11299237	+0.6437115	0.008368400	2.30456350
270	2.9292114	+0.66881112	+1.0001462	0.020201647	2.30723845
300	2.8205208	+0.10529076	+1.0877783	0.023896846	2.30739440
330	2.7187000	−0.42657423	+0.8831271	0.015750931	2.30502860
Σ_1	17.0768662*	+1.34058170†	−0.0182332‡	0.072922442	13.82884350
Σ_2	17.0768672	+1.34058118	−0.0182329	0.072922442	13.82884430

E	l	G	G'	G''	θ
					° ′ ″
0°	0.32836475	2.30159781	0.33433455	0.00510484	22 33 26.00
30	0.31341820	2.30194311	0.31388465	0.00040288	21 40 58.40
60	0.35238060	2.30214618	0.36444375	0.01016345	23 44 4.25
90	0.43608420	2.30201186	0.46014840	0.01930383	27 1 51.11
120	0.54393270	2.30165423	0.56837725	0.01847156	30 11 38.49
150	0.64776765	2.30144287	0.66243750	0.01047414	32 38 59.04
180	0.71834405	2.30164190	0.72188175	0.00242983	34 6 10.10
210	0.73432570	2.30208690	0.73455360	0.00015516	34 23 46.69
240	0.69089570	2.30230785	0.69835610	0.00520478	33 30 59.53
270	0.60177725	2.30207736	0.62106785	0.01412953	31 34 45.83
300	0.49293070	2.30165418	0.51868770	0.02001680	28 47 46.93
330	0.39347570	2.30144156	0.41360955	0.01654686	25 31 1.93
Σ_1	3.12684850	13.81100215	3.20608110	0.06139126	172 54 5.30
Σ_2	3.12684870	13.81100366	3.20570155	0.06101240	172 51 23.00

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 17.0768669.$

† $6[a'^2e' - kaa'e\cos K] = +1.34058156.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = -0.0182328.$

ACTION OF MARS ON VENUS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.05236235	0.34221622	0.25376720	9.0799241	8.6961571	8.9706997
30	0.04824440	0.36381569	0.24772073	9.0778391	8.6903136	8.9633892
60	0.05820469	0.34986592	0.26232763	9.0871721	8.7089460	8.9854537
90	0.07644597	0.37365893	0.28892064	9.1058581	8.7480486	9.0290445
120	0.09671054	0.39993150	0.31822634	$9.129423\bar{7}$	8.7983322	$9.082138\bar{5}$
150	0.11444649	0.42279102	0.34867356	9.1516340	9.8464806	9.1313354
180	0.12581914	0.43738414	0.35989311	9.1660121	8.8784042	9.1634092
210	0.12819869	0.44043118	0.36327720	$9.168118\bar{2}$	$8.884247\bar{6}$	9.1692445
240	0.12114959	0.43139835	0.35324262	$9.157413\bar{9}$	8.8625240	9.1475124
270	0.10649224	0.41255431	0.33228403	9.1373395	8.8203390	9.1048461
300	0.08740987	0.38789402	0.30480686	$9.113745\bar{3}$	8.7700379	$9.052751\bar{5}$
330	0.06771795	0.36229172	0.27622192	9.0929018	8.7249711	9.0040125
Σ_1	0.54165618	2.34869015	1.85226376	$4.733691\bar{0}$	2.7144014	4.4019649
Σ_2	0.54154574	2.34854285	1.85209808	$4.733690\bar{7}$	$2.714400\bar{5}$	4.4018722

E	log V	J_1'	J_2	J_3	F_2
0°	8.9695199	2.3059508	+0.040158678	−0.039429682	−0.09502735
30	8.9632959	2.2999322	−0.012465787	−0.068607310	+0.02586866
60	8.9831123	2.3089841	−0.060962061	−0.080069400	+0.14000859
90	9.0246325	2.3187331	−0.093160848	−0.070744754	+0.21680880
120	9.0779450	2.3192082	−0.101285499	−0.043131936	+0.23569076
150	9.1289682	2.3119133	−0.083189540	−0.004629729	+0.19159498
180	9.1628614	2.3032758	−0.042922865	+0.034445271	+0.09633687
210	9.1692095	2.2997005	+0.009559512	+0.063622976	−0.02455913
240	9.1463381	2.3040271	+0.060251438	+0.075085208	−0.13869908
270	9.1016472	2.3135287	+0.094787774	+0.065760616	−0.21549926
300	9.0481938	2.3207258	+0.103054621	+0.038147729	−0.23438118
330	9.0002162	2.3179852	+0.082763214	−0.000354612	−0.19028541
Σ_1	4.3879705	13.8621718*	−0.001705675	−0.014952810	+0.00392861
Σ_2	4.3879695	13.8617930	−0.001705688	−0.014952813	+0.00392864

* $\Sigma_1(J_1' - G'') = 13.800780\bar{6}$.
$\Sigma_2(J_1' - G'') = 13.8007806$.

ACTION OF MARS ON VENUS.

E	$1000 \times F_3$	R_0	$1000 \times S_0$	$1000 \times W_0$	$R^{(n)}$	$1000 \times S^{(n)}$
0°	+ 2.8021516	0.06350844	−0.9770071	− 3.5365233	0.00000000	−1.3600097
30	− 0.4518739	0.06287361	+0.1223535	− 6.3268753	+0.04372018	+0.1701610
60	− 0.0998524	0.06461988	+1.2993923	− 7.7066718	+0.07763320	+1.8025651
90	+ 3.5312016	0.06860530	+2.2776450	− 7.2896438	+0.09484620	+3.1488232
120	+ 6.8298095	0.07412522	+2.6943730	− 4.7318401	+0.08844541	+3.7122436
150	+ 6.5062493	0.07970667	+2.2590820	− 0.1661617	+0.05477227	+3.1047600
180	+ 2.8799133	0.08340064	+1.0358414	+ 5.2294311	0.00000000	+1.4223079
210	− 0.4389788	0.08380114	−0.4699266	+ 9.3597858	−0.05758587	−0.6458416
240	− 0.1552801	0.08095204	−1.6671420	+10.5057004	−0.09659109	−2.2969487
270	+ 3.4223024	0.07610223	−2.2705920	+ 8.5365166	−0.10521063	−3.1390719
300	+ 6.6966141	0.07080065	−2.2876620	+ 4.6568417	−0.08505867	−3.1735299
330	+ 6.3844559	0.06631737	−1.8208410	+ 0.3034394	−0.04611486	−2.5323023
Σ_1	+18.9533560	0.43740687	−0.0977956	+ 4.4169380	−0.01557115	+0.1066283
Σ_2	+18.9533565	0.43740632	−0.0977209	+ 4.4170612	−0.01557271	+0.1065284

E	$[R_0 \sin v + (\cos v + \cos E) S_0]$	$\left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1\right) \sin v S_0\right]$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$-2\frac{r}{a} R_0$
0°	−0.001954014	−0.06350844	− 2.072195	− 2.865834	−0.12614768
30	+0.031835191	−0.05421922	− 0.625404	− 6.295890	−0.12500201
60	+0.057445987	−0.02972268	+ 3.192309	− 7.014411	−0.12879747
90	+0.068588125	+0.00502476	+ 5.936279	− 4.230780	−0.13721060
120	+0.061265775	+0.04210058	+ 4.709813	− 0.456037	−0.14875771
150	+0.035700934	+0.07141598	+ 0.151409	+ 0.068446	−0.16035811
180	−0.002071682	+0.08340064	− 3.064140	− 4.237688	−0.16794274
210	−0.040838000	+0.07318497	− 0.988919	− 9.307396	−0.16859554
240	−0.068190156	+0.04377273	+ 4.238101	− 9.612920	−0.16245809
270	−0.076084930	+0.00506196	+ 6.883205	− 5.049116	−0.15220446
300	−0.063800118	−0.03106656	+ 4.629519	− 0.503716	−0.14111682
330	−0.036506252	−0.05549212	+ 0.277348	+ 0.123100	−0.13184872
Σ_1	−0.017304208	+0.04497627	+11.633407	−24.690606	−0.87522051
Σ_2	−0.017304932	+0.04497633	+11.633918	−24.691636	−0.87521944

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = + 0.0000000018.$

Differential Coefficients.

			log coeff.
$[de/dt]_{00}$	$= -$ 6075.5972	m'	n 3.7835890
$[d\chi/dt]_{00}$	$= +2307588.8$	m'	p 6.3631584
$[di/dt]_{00}$	$= +$ 4084.7434	m'	p 3.6111648
$[d\Omega/dt]_{00}$	$= -$ 146478.61	m'	n 5.1657742
$[d\pi/dt]_{00}$	$= +2307332.0$	m'	p 6.3631101
$[dL/dt]_{00}$	$= -$ 307497.75	m'	n 5.4878419

Final Values Corresponding to the Above Value of m'.

$$[de/dt]_{00} = -0.0019639882''$$
$$[d\chi/dt]_{00} = +0.74594759$$
$$[di/dt]_{00} = +0.0013204280$$
$$[d\Omega/dt]_{00} = -0.047350446$$
$$[d\pi/dt]_{00} = +0.74586465$$
$$[dL/dt]_{00} = -0.099401232$$

Comparison with Other Results.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$-0.00195''$	$-0.00196''$	$-0.001964''$
$e[d\pi/dt]_{00}$	$+0.00510$	$+0.00510$	$+0.005104$
$[di/dt]_{00}$	$+0.00131$	$+0.00132$	$+0.001320$
$\sin i\,[d\Omega/dt]_{00}$	-0.00280	-0.00281	-0.002802
$[dL/dt]_{00}$	-0.099		-0.099401

Notes.

The close agreement of the final sums shows that, notwithstanding the high eccentricity of the orbit of Mars, the expansion of the perturbing function is quite rapidly convergent for this case. The greatest error arising from a division into but six parts would here occur with the coefficient $[d\Omega/dt]_{00}$, and would amount to 1/50000th of the whole value of this coefficient.

ACTION OF JUPITER ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	27.41848845	−0.4215334	+3.3076466	0.68960365	27.007779̄
30	27.28099847	−1.8560261	+1.9912702	0.24993195	27.007266̄
60	27.22724715	−2.4400062	+0.1352278	0.00115264	27.006632
90	27.27164617	−2.0169963	−1.7631551	0.19594869	27.006516̄
120	27.40230326	−0.7003432	−3.1952096	0.64351702	27.007039
150	27.58420446	+1.1571588	−3.7772161	0.89930104	27.007701̄
180	27.76860046	+3.0577930	−3.3532271	0.70874081	27.007851̄
210	27.90607834	+4.4922856	−2.0368516	0.26150512	27.007330
240	27.95980514	+5.0762663	−0.1808092	0.00206064	27.006665
270	27.91539376	+4.6532564	+1.7175743	0.18594838	27.006520
300	27.78474891	+3.3366031	+3.1496275	0.62528757	27.007025
330	27.60287233	+1.4791014	+3.7316353	0.87772776	27.007658
Σ_1	165.56119337*	+7.9087796†	−0.1367440‡	2.67036233	162.042990
Σ_2	165.56119353	+7.9087799	−0.1367430	2.67036294	162.042990̄

E	l	G	G'	G''	θ
	°				° ′ ″
0	0.347678̄	27.0068207	0.4107944	0.06215867	7 35 44.57
30	0.21070̄1	27.0069201	0.2483150	0.03726867	5 53 53.53
60	0.157583	27.0066304	0.1578551	0.00027037	4 23 18.41
90	0.20209̄9	27.0062448	0.2334496	0.03108029	5 40 35.79
120	0.332232	27.0061457	0.393656̄8	0.06053126	7 26 34.62
150	0.51347̄2	27.0064436	0.572857̄6	0.05812876	8 46 58.16
180	0.69771̄8	27.0068530	0.7344471	0.03573164	9 42 56.80
210	0.835716	27.0069600	0.847511̄4	0.01142507	10 16 14.68
240	0.890108	27.0066621	0.8901968	0.00008571	10 27 38.97
270	0.845842	27.0062568	0.854165̄9	0.00806094	10 17 29.01
300	0.714692	27.0061444	0.7465850	0.03101258	9 45 50.06
330	0.532182	27.0064305	0.5886244	0.05521465	8 52 22.88
Σ_1	3.140010	162.0392563	3.3335352	0.18979023	49 22 3.43
Σ_2	3.14001̄1	162.0392558	3.344923̄8	0.20117838	49 47 34.05

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 165.5611934.$

† $6[a'^2e' - kaa'e\cos K] = +7.9087800.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = -0.1367433.$

ACTION OF JUPITER ON VENUS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.00574768	0.28065745	0.18470221	7.429089$\bar{1}$	4.8448027	6.1813194
30	0.00346017	0.27761215	0.18127767	7.4282000	4.8416643	6.1774038
60	0.00191340	0.27555166	0.17896019	7.4297378	4.842340$\bar{1}$	6.1772233
90	0.00320448	0.27727161	0.18089469	7.433272$\bar{4}$	4.846616$\bar{6}$	6.1822034
120	0.00551791	0.28035168	0.18435840	7.4378460	4.8533279	6.1897695
150	0.00769513	0.28324824	0.18761506	7.4422392	4.8606851	6.197453$\bar{2}$
180	0.00942826	0.28555246	0.19020526	7.4452930	4.8667491	6.2034501
210	0.01054435	0.28703562	0.19187230	7.446201$\bar{1}$	4.869917$\bar{9}$	6.206414$\bar{0}$
240	0.01094130	0.28756298	0.19246500	7.4447133	4.8693318	6.205706$\bar{1}$
270	0.01058711	0.28709242	0.19193614	7.4412095	4.8651139	6.2015516
300	0.00952253	0.28567775	0.19034610	7.4366174	4.858373$\bar{2}$	6.1950025
330	0.00785489	0.28346069	0.18785389	7.4321742	4.8509265	6.1876739
Σ_1	0.04307108	1.69535398	1.12103716	4.623296$\bar{5}$	9.134924$\bar{7}$	7.1524707
Σ_2	0.04334613	1.69572073	1.12144975	4.623296$\bar{3}$	9.1349242	7.152699$\bar{8}$

E	log V	J_1'	J_2	J_3	F_2
0°	6.1800737	27.032601604	+0.14514043	−0.9796507	−4.3121624
30	6.1766560	27.002428557	+0.10192507	−1.0502482	−2.5960090
60	6.1772178	26.980449138	+0.02691302	−0.8390983	−0.1762960
90	6.1815797	27.031356993	−0.07055224	−0.4027785	+2.2986164
120	6.1885563	27.065863046	−0.15999110	+0.1417995	+4.1655781
150	6.196289$\bar{1}$	27.048572013	−0.20267009	+0.6487166	+4.9243381
180	6.2027348	27.006174576	−0.17674090	+0.9821448	+4.3715859
210	6.206185$\bar{3}$	26.976509919	−0.09316228	+1.0527424	+2.6554331
240	6.2057043	26.979786076	+0.01124440	+0.8415929	+0.2357201
270	6.2013902	27.007935305	+0.09774464	+0.4052732	−2.2391928
300	6.1943816	27.036474318	+0.14682513	−0.1393049	−4.1061538
330	6.1865682	27.045934111	+0.16010585	−0.6462222	−4.8649146
Σ_1	7.148668$\bar{5}$	162.101348758*	−0.00660902	+0.0074833	+0.1782719
Σ_2	7.1486684	162.112736898	−0.00660905	+0.0074833	+0.1782712

* $\Sigma_1(J_1' - G'') = 161.911558528.$
$\Sigma_2(J_1' - G'') = 161.911558518.$

ACTION OF JUPITER ON VENUS.

E	F_3	$1000 \times R_0$	$100000 \times S_0$	$1000 \times W_0$	$1000 \times R^{(a)}$	$100000 \times S^{(a)}$
0°	+0.06417143	1.3439836	−0.81929943	−0.14785238	0.0000000	−1.1404782
30	−0.01379964	1.3378723	−0.27202988	−0.15783804	+0.9303113	−0.3783208
60	−0.00425156	1.3439282	+0.28211779	−0.12622122	+1.6145721	+0.3913643
90	+0.08385809	1.3588647	+0.54292475	−0.06059607	+1.8786178	+0.7505883
120	+0.16287446	1.3765538	+0.50194299	+0.02305118	+1.6424891	+0.6915652
150	+0.15397825	1.3917093	+0.38822446	+0.10305711	+0.9563448	+0.5335546
180	+0.06595223	1.4018961	+0.39768404	+0.15712801	0.0000000	+0.5460578
210	−0.01357095	1.4066592	+0.47042587	+0.16914113	−0.9666180	+0.6465278
240	−0.00563629	1.4053074	+0.35504098	+0.13510528	−1.6767973	+0.4891671
270	+0.08123098	1.3964225	−0.08725072	+0.06503287	−1.9305409	−0.1206233
300	+0.15970886	1.3800824	−0.66641223	−0.02064190	−1.6580071	−0.9244718
330	+0.15112247	1.3602249	−0.99122256	−0.09822805	−0.9458544	−1.3785254
Σ_1	+0.44281913	8.2517515	+0.05107414	+0.02056897	−0.0777432	+0.0532044
Σ_2	+0.44281920	8.2517529	+0.05107192	+0.02056895	−0.0777394	+0.0532012

E	$1000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000\times\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v\cdot S_0\right]$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	−0.0163860	−1.3439836	−0.086632804	−0.11981268	−0.0026695736
30	+0.6682014	−1.1590573	−0.015602101	−0.15706504	−0.0026598877
60	+1.1706512	−0.6601482	+0.052284193	−0.11488325	−0.0026786592
90	+1.3587959	+0.0201574	+0.049346057	−0.03516889	−0.0027177294
120	+1.1829926	+0.7039968	−0.022943878	+0.00222159	−0.0027625280
150	+0.6850081	+1.2114930	−0.093907380	−0.04245200	−0.0027999142
180	−0.0079537	+1.4018961	−0.092067787	−0.12732924	−0.0028229786
210	−0.7073256	+1.2159042	−0.017870798	−0.16819438	−0.0028299915
240	−1.2164221	+0.7037025	+0.054502775	−0.12362395	−0.0028202318
270	−1.3963839	+0.0113009	+0.052437627	−0.03846518	−0.0027928449
300	−1.2058917	−0.6713715	−0.020520787	+0.00223277	−0.0027507212
330	−0.7013026	−1.1657067	−0.089781837	−0.03984932	−0.0027043278
Σ_1	−0.0930097	+0.1340921	−0.115378288	−0.48119476	−0.0165046924
Σ_2	−0.0930067	+0.1340915	−0.115378432	−0.48119481	−0.0165046855

$\sin\varphi\cdot\frac{1}{2}A_1^{(c)} + \cos\varphi\cdot B_0^{(c)} = +0.0000000000028.$

Differential Coefficients.

			log coeff.
$[de/dt]_{00}$	$= -$ 32.654970	m'	n 1.5139493
$[d\chi/dt]_{00}$	$= +6879.8159$	m'	p 3.8375768
$[di/dt]_{00}$	$= -$ 40.510972	m'	n 1.6075727
$[d\Omega/dt]_{00}$	$= -2854.6599$	m'	n 3.4555544
$[d\pi/dt]_{00}$	$= +6874.8117$	m'	p 3.8372608
$[dL/dt]_{00}$	$= -5799.7390$	m'	n 3.7634084

Final Values Corresponding to the Above Value of m'.

$[de/dt]_{00}$	$= -0.031162921$
$[d\chi/dt]_{00}$	$= +6.5654682$
$[di/dt]_{00}$	$= -0.038659982$
$[d\Omega/dt]_{00}$	$= -2.7242270$
$[d\pi/dt]_{00}$	$= +6.5606924$
$[dL/dt]_{00}$	$= -5.5347410$

Comparison with Other Results.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	-0.03117	-0.03117	-0.0311629
$e[d\pi/dt]_{00}$	$+0.04482$	$+0.04491$	$+0.0448955$
$[di/dt]_{00}$	-0.03865	-0.03865	-0.0386600
$\sin i\,[d\Omega/dt]_{00}$	-0.16114	-0.16122	-0.1612345
$[dL/dt]_{00}$	-5.535		-5.5347410

Notes.

The term a^2e' is here so large that the sums of the functions B, ϵ, G', G'', and θ, as well as those of the functions immediately dependent upon these quantities are in great disagreement; but, as the expansion of the perturbing function is here rapidly convergent, the final sums agree almost exactly. The greatest effect of all terms from the 6th to the 11th orders is here produced with the coefficient $[di/dt]_{00}$ and amounts to $0''.00000002$.

ACTION OF SATURN ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	92.09886822	+10.392665	+4.3401600	5.386574	90.704833
30	91.77434432	+ 7.489741	+6.4166597	11.773870	90.705340
60	91.37861335	+ 3.936959	+6.7658078	13.090030	90.705178
90	91.01772022	+ 0.686282	+5.2940524	8.014522	90.704527
120	90.78837022	− 1.391274	+2.3957451	1.641279	90.704015
150	90.75201312	− 1.739027	−1.1525126	0.379833	90.704174
180	90.91838168	− 0.263800	−4.3999687	5.536055	90.704848
210	91.24289345	+ 2.639122	−6.4764687	11.994380	90.705369
240	91.63859982	+ 6.191904	−6.8256188	13.322485	90.705222
270	91.99948069	+ 9.442580	−5.3538604	8.196630	90.704548
300	92.22884291	+11.520135	−2.4555545	1.724250	90.704026
330	92.26522467	+11.867891	+1.0927030	0.341433	90.704165
Σ_1	549.05167620*	+30.386589†	−0.1794291‡	40.700673	544.228122
Σ_2	549.05167647	+30.386589	−0.1794266	40.700668	544.228123

E	l	G	G'	G''	θ
°					° ′ ″
0	+1.108078	90.7041702	1.1599384	0.0511977	6 38 1.200
30	+0.783048	90.7038960	0.9248448	0.1403539	6 12 59.177
60	+0.387477	90.7035801	0.621341	0.2322660	5 33 35.601
90	+0.027235	90.7035526	0.3116929	0.2834826	4 38 20.618
120	−0.201603	90.7038159	0.0673334	0.2687363	3 29 4.472
150	−0.238118	90.7041279	0.016452	0.2545245	3 7 43.816
180	−0.072423	90.7041756	0.2137666	0.2855179	4 14 53.293
210	+0.251567	90.7039071	0.5115373	0.2585083	5 16 44.963
240	+0.647420	90.7035910	0.8267173	0.1776657	6 2 4.051
270	+1.008976	90.7035405	1.092685	0.082702	6 32 0.533
300	+1.238859	90.7038135	1.2542286	0.0151565	6 47 36.301
330	+1.275102	90.7041229	1.2780897	0.0029452	6 49 30.415
Σ_1	+3.107808	544.2231463	4.143326	1.0305401	32 45 14.918
Σ_2	+3.107809	544.2231470	4.1353023	1.022516	32 37 19.522

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 549.05167622.$

† $6[a'^2e' - kaa'e\cos K] = +30.386587.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = -0.1794290.$

ACTION OF SATURN ON VENUS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.00437971	0.27883659	0.18265468	6.6396204	3.0027123	4.8644028̄
30	0.00384466	0.27812417	0.18185352	6.6392492	3.0007786	4.8628053
60	0.00307377	0.27709752	0.18069890	6.6400076̄	2.9996349̄	4.8619714̄
90	0.00213852	0.27585161	0.17929758	6.6416827	2.9995754	4.8620009̄
120	0.00120578	0.27460866	0.17789947	6.6438207	3.0006087	4.8628099̄
150	0.00097200	0.27429705	0.17754895	6.6458518	3.0024609̄	4.8645568̄
180	0.00179285	0.27539102	0.17877951	6.6472415̄	3.0046480̄	4.8670287
210	0.00277061	0.27669370	0.18024473	6.6476233	3.0065929	4.8690060̄
240	0.00362228	0.27782805	0.18152050	6.6468912	3.0077706̄	4.8699374̄
270	0.00424801	0.27866126	0.18245753	6.6452313	3.0078525̄	4.8696688̄
300	0.00459387	0.27912172	0.18297533	6.6430830	3.0068083	4.8683601
330	0.00463699	0.27917912	0.18303987	6.6410259	3.0049227	4.8664247̄
Σ_1	0.01866826	1.66288356	1.08452839	9.8606643	8.0221827̄	9.1945101
Σ_2	0.01861079	1.66280691	1.08444218	9.8606642	8.0221829	9.1944623̄

E	log V	J_1'	J_2	J_3	F_2
0°	4.8640969	90.64262161	+0.26581387	−3.1529304	−22.089684
30	4.8619670	90.77821079	+0.41797337	−2.4126056	−32.658241
60	4.8605842	90.92403274	+0.41481456	−1.0167115	−34.435270
90	4.8603076̄	90.98237184	+0.27423234	+0.6607234	−26.944615
120	4.8612040	90.92121232	+0.07649557	+2.1702330	−12.193388
150	4.8630356	90.85355551	−0.10022240	+3.1073439	+ 5.865829
180	4.8653231̄	90.87694181	−0.22507782	+3.2209585	+22.394088
210	4.8674621̄	90.89557561	−0.30595380	+2.4806343	+32.962649
240	4.8688763	90.86858755	−0.34689500	+1.0847396	+34.739680
270	4.8691748̄	90.78219477	−0.32168268	−0.5926962	+27.249019
300	4.8682696̄	90.66900008	−0.19522740	−2.1022048	+12.497792
330	4.8664071̄	90.60246403	+0.02557719	−3.0393154	− 5.561423
Σ_1	9.1883540	544.90239611*	−0.01007622	+0.2040844	+ 0.913218
Σ_2	9.1883541̄	544.89437255	−0.01007598	+0.2040844	+ 0.913218

* $\Sigma_1(J_1' - G'') = 543.87185601.$
$\Sigma_2(J_1' - G'') = 543.87185600.$

ACTION OF SATURN ON VENUS.

E	F_2	$1000 \times R_0$	$100000 \times S_0$	$1000 \times W_0$	$1000 \times R^{(n)}$	$100000 \times S^{(n)}$
0°	−0.15178108	0.21824849	−0.02789044	−0.023072725	0.00000000	−0.03882394
30	−0.76855930	0.21811010	−0.02299898	−0.017634117	+0.15166640	−0.03198543
60	−1.16657105	0.21852784	−0.04315220	−0.007491896	+0.26253554	−0.05986233
90	−0.94659087	0.21909707	−0.07037836	+0.004695343	+0.30289965	−0.09729742
120	−0.32871848	0.21963056	−0.06653491	+0.015732681	+0.26206084	−0.09167023
150	+0.06775179	0.22023472	−0.01412236	+0.022675427	+0.15133936	−0.01940900
180	−0.15599309	0.22103492	+0.06128516	+0.023605169	0.00000000	+0.08415031
210	−0.77884429	0.22186276	+0.10918347	+0.018202966	−0.15245807	+0.15005585
240	−1.18017310	0.22225231	+0.09717617	+0.007900358	−0.26518905	+0.13388702
270	−0.95986467	0.22180036	+0.03944793	−0.004483114	−0.30663694	+0.05453639
300	−0.33810816	0.22057741	−0.01719598	−0.015556236	−0.26499777	−0.02385490
330	+0.06476239	0.21916657	−0.03744378	−0.022338577	−0.15240100	−0.05207429
Σ_1	−3.32134496	1.32027153	+0.00368780	+0.001117351	−0.00559044	+0.00382593
Σ_2	−3.32134495	1.32027158	+0.00368792	+0.001117928	−0.00559070	+0.00382610

E	$1000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$	$1000 \times \left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1\right) \sin v S_0\right]$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$1000 \times -2\frac{r}{a} R_0$
0°	−0.00055781	−0.21824849	−0.013519262	−0.018697064	−0.43351010
30	+0.10930468	−0.18874417	−0.001743111	−0.017547753	−0.43363510
60	+0.18946670	−0.10888722	+0.003103342	−0.006818927	−0.43556030
90	+0.21909678	+0.00009174	−0.003823625	+0.002725094	−0.43819414
120	+0.19022138	+0.10978821	−0.015659442	+0.001516257	−0.44076418
150	+0.10971092	+0.19096268	−0.020662231	−0.009340619	−0.44307990
180	−0.00122570	+0.22103492	−0.013831594	−0.019129016	−0.44509495
210	−0.11216824	+0.19142744	−0.001923256	−0.018101079	−0.44635520
240	−0.19279204	+0.11058265	+0.003187081	−0.007228982	−0.44602556
270	−0.22179790	+0.00072885	−0.003614847	+0.002651640	−0.44360072
300	−0.19184805	−0.10885437	−0.015464964	+0.001682669	−0.43964535
330	−0.11088191	−0.18905102	−0.020417779	−0.009062352	−0.43573540
Σ_1	−0.00673552	+0.00541570	−0.052184839	−0.048675063	−2.64060044
Σ_2	−0.00673567	+0.00541552	−0.052184849	−0.048675069	−2.64060046

$\sin \varphi \cdot \frac{1}{2} A_1^{(s)} + \cos \varphi \cdot B_0^{(c)} = +0.00000000000029.$

Differential Coefficients.

			log coeff.
$[de/dt]_{00}$	$= -$ 2.3648522	m'	n 0.3738040
$[d\chi/dt]_{00}$	$= +277.85744$	m'	p 2.4438220
$[di/dt]_{00}$	$= -$ 18.322835	m'	n 1.2629927
$[d\Omega/dt]_{00}$	$= -288.76199$	m'	n 2.4605400
$[d\pi/dt]_{00}$	$= +277.35124$	m'	p 2.4430301
$[dL/dt]_{00}$	$= -927.63054$	m'	n 2.9673751

Final Values Corresponding to the Above Value of m'.

$$[de/dt]_{00} = -0.00067536338''$$
$$[d\chi/dt]_{00} = +0.079351564$$
$$[di/dt]_{00} = -0.0052327048$$
$$[d\Omega/dt]_{00} = -0.082465731$$
$$[d\pi/dt]_{00} = +0.079207000$$
$$[dL/dt]_{00} = -0.26491624$$

Comparison with Other Results.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$-0.00067''$	$-0.00067''$	$-0.00067536''$
$e[d\pi/dt]_{00}$	$+0.00055$	$+0.00054$	$+0.00054202$
$[di/dt]_{00}$	-0.00523	-0.00523	-0.00523270
$\sin i\,[d\Omega/dt]_{00}$	-0.00489	-0.00488	-0.00488077
$[dL/dt]$	-0.265		-0.26491624

Notes.

As in the previous case, the considerable disagreement of the sums of the functions near the beginning of the computation nearly disappears as the work progresses, showing that the convergence of the expansion of the perturbing function is here very rapid. The greatest error which would have arisen from the neglect of all terms from the 6th to the 11th orders would have here occurred with the coefficient $[d\chi/dt]_{00}$ and would have amounted to 1/70000th part of the remaining terms.

ACTION OF URANUS ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	369.8294733	28.016529	− 8.627090	60.35594	367.496220
30	370.1021929	30.912376	− 2.059265	3.43885	367.496155
60	370.0319613	30.136215	+ 5.076263	20.89679	367.496140
90	369.6376057	25.896024	+10.867539	95.77536	367.496165
120	369.0247981	19.327964	+13.762797	153.60480	367.496245
150	368.3577351	12.191933	+12.986248	136.75991	367.496230
180	367.8151465	6.400029	+ 8.745974	62.03086	367.496275
210	367.5424142	3.504185	+ 2.178150	3.84740	367.496245
240	367.6126214	4.280346	− 4.957378	19.92946	367.496135
270	368.0069649	8.520535	−10.748653	93.69136	367.496165
300	368.6197847	15.088597	−13.643909	150.96247	367.496240
330	369.2868724	22.224622	−12.867361	134.26735	367.496240
Σ_1	2212.9337853*	103.249680†	+ 0.356657‡	467.78032	2204.977255
Σ_2	2212.9337852	103.249675	+ 0.356658	467.78025	2204.977200

E	l	G	G'	G''	θ
					° ′ ″
0°	+1.522310	367.495771	1.6238940	0.1011370	3 55 40.93
30	+1.795095	367.496129	1.8003166	0.0051977	4 1 9.46
60	+1.724880	367.495985	1.7573874	0.0323564	4 0 5.50
90	+1.330495	367.495453	1.5044398	0.1732316	3 52 23.80
120	+0.717605	367.495105	1.0990547	0.3803068	3 38 8.91
150	+0.050560	367.495217	0.6363646	0.5847917	3 18 7.21
180	−0.492075	367.495816	0.2329563	0.7245710	2 55 22.89
210	−0.764775	367.496217	0.0134531	0.7782012	2 39 26.72
240	−0.694455	367.495988	0.0708724	0.7651843	2 43 51.72
270	−0.300145	367.495472	0.3769271	0.6763791	3 3 57.85
300	+0.312600	367.495121	0.8167025	0.5029833	3 25 59.41
330	+0.979685	367.495243	1.2686692	0.2879853	3 43 48.64
Σ_1	+3.090865	2204.973786	5.6008673	2.5065388	20 39 9.36
Σ_2	+3.090915	2204.973731	5.6001704	2.5057866	20 38 53.68

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 2212.9337852.$

† $6[a'^2e' - kaa'e\cos K] = +103.249685.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = +0.356657.$

ACTION OF URANUS ON VENUS.

E	log K_0	log L'	log N_0	log N	log P	log Q
0°	0.00153256	0.27504417	0.17838936	5.7255240	0.8698244	3.3385415
30	0.00160469	0.27514027	0.17849746	5.7265670	0.8711895	3.3398056
60	0.00159052	0.27512139	0.17847622	5.7286906	0.8732303	3.3418760
90	0.00149009	0.27498757	0.17832569	5.7313185	0.8753928	3.3441876
120	0.00131281	0.27475131	0.17805993	5.7337420	0.8770917	3.3461011
150	0.00108264	0.27444453	0.17771485	5.7353151	0.8778752	3.3470878
180	0.00084825	0.27413211	0.17736340	5.7356234	0.8775397	3.3468789
210	0.00070103	0.27393587	0.17714264	5.7345896	0.8761825	3.3455607
240	0.00074042	0.27398839	0.17720173	5.7324870	0.8741636	3.3435328
270	0.00093336	0.27424555	0.17749102	5.7298710	0.8720155	3.3413115
300	0.00117044	0.27456155	0.17784648	5.7274385	0.8703090	3.3394395
330	0.00138191	0.27484340	0.17816353	5.7258446	0.8695041	3.3384162
Σ_1	0.00719500	1.64759892	1.06733712	4.3835054	5.2421585	0.0563696
Σ_2	0.00719372	1.64759719	1.06733519	4.3835056	5.2421594	0.0563691

E	log V	J_1'	J_2	J_3	F_2
0°	3.3383922	367.09942052	−0.77238411	−13.377206	+148.77538
30	3.3397979	366.74089875	−0.18040606	−16.560521	+ 35.51232
60	3.3418282	366.88453611	+0.52246333	−15.273725	− 87.54088
90	3.3439319	367.40160988	+0.87586900	− 9.861620	−187.41225
120	3.3455398	367.86665960	+0.72768333	− 1.774376	−237.34142
150	3.3462248	367.95568703	+0.32642368	+ 6.821041	−223.94969
180	3.3458097	367.72285452	+0.04295564	+13.621489	−150.82559
210	3.3444123	367.51508313	+0.01342241	+16.804806	− 37.56251
240	3.3424036	367.61069788	+0.04551264	+15.518012	+ 85.49071
270	3.3403134	367.89469814	−0.13535069	+10.105905	+185.36209
300	3.3386972	367.98729038	−0.54951786	+ 2.018660	+235.29114
330	3.3379911	367.66272932	−0.88324571	− 6.576756	+221.89949
Σ_1	0.0526706	2205.17145901*	+0.01671297	+ 0.732854	− 6.15066
Σ_2	0.0526714	2205.17070625	+0.01671263	+ 0.732855	− 6.15055

* $\Sigma_1(J_1' - G'') = 2202.66492021.$
$\Sigma_2(J_1' - G'') = 2202.66491965.$

ACTION OF URANUS ON VENUS.

E	F_3	$1000 \times R_0$	$1000000 \times S_0$	$1000000 \times W_0$	$1000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	− 4.0226412	0.02650922	−0.05811066	−2.918780	0.00000000	−0.08089097
30	− 0.1770896	0.02652281	−0.01305234	−3.621493	+0.01844306	−0.01815231
60	− 1.6152186	0.02667654	+0.04940526	−3.356828	+0.03204873	+0.06853681
90	− 6.9332675	0.02691093	+0.05269544	−2.182313	+0.03720412	+0.07285096
120	−10.8407275	0.02710189	−0.01759673	−0.401343	+0.03233767	−0.02424436
150	− 9.4434565	0.02715609	−0.09660941	+1.506695	+0.01866093	−0.13277471
180	− 4.1342719	0.02707758	−0.10424233	+3.017072	0.00000000	−0.14313458
210	− 0.2013093	0.02694790	−0.02527822	+3.713882	−0.01851786	−0.03474102
240	− 1.5455370	0.02684358	+0.07399797	+3.412656	−0.03202946	+0.10195266
270	− 6.7883563	0.02677475	+0.10841711	+2.207472	−0.03701585	+0.14988562
300	−10.6594103	0.02669990	+0.05468656	+0.432404	−0.03207678	+0.07586323
330	− 9.2743255	0.02659629	−0.02803268	−1.439065	−0.01849416	−0.03898596
Σ_1	−32.8178065	0.16090871	−0.00185993	+0.185181	+0.00028016	−0.00191721
Σ_2	−32.8178047	0.16090877	−0.00186010	+0.185178	+0.00028024	−0.00191742

E	$1000\times\left[R_0 \sin v + \left(\cos v + \cos E\right) S_0\right]$	$1000\times\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$1000000 \times W_0 \cos u$	$1000000 \times W_0 \sin u$	$1000 \times -2\frac{r}{a}R_0$
0°	−0.00011622	−0.02650922	−1.710234	− 2.365243	−0.05265561
30	+0.01331757	−0.02293687	−0.357980	− 3.603757	−0.05273125
60	+0.02323048	−0.01311517	+1.390488	− 3.055297	−0.05317053
90	+0.02690995	+0.00028955	+1.777154	− 1.266575	−0.05382187
120	+0.02340803	+0.01365914	+0.399475	− 0.038680	−0.05438924
150	+0.01366523	+0.02346772	−1.372927	− 0.620648	−0.05463405
180	+0.00020848	+0.02707758	−1.767867	− 2.444894	−0.05452574
210	−0.01335043	+0.02340859	−0.392395	− 3.693094	−0.05421519
240	−0.02324178	+0.01343114	+1.376698	− 3.122646	−0.05387086
270	−0.02677488	−0.00003361	+1.779939	− 1.305659	−0.05354950
300	−0.02314722	−0.01330732	+0.429867	− 0.046772	−0.05321706
330	−0.01342562	−0.02295918	−1.315326	− 0.583802	−0.05287735
Σ_1	+0.00034177	+0.00123615	+0.118427	−11.073532	−0.32182904
Σ_2	+0.00034182	+0.00123620	+0.118465	−11.073535	−0.32182921

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = +0.000000000000020.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 0.12000343	m'	p 9.0791936
$[d\chi/dt]_{00}$	$= +$ 63.424159	m'	p 1.8022547
$[di/dt]_{00}$	$= +$ 0.04158807	m'	p 8.6189687
$[d\Omega/dt]_{00}$	$= -$ 65.693091	m'	n 1.8175197
$[d\pi/dt]_{00}$	$= +$ 63.308999	m'	p 1.8014655
$[dL/dt]_{00}$	$= -113.109825$	m'	n 2.0535003

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= +0.0000052633084$
$[d\chi/dt]_{00}$	$= +0.0027817616$
$[di/dt]_{00}$	$= +0.000001824038$
$[d\Omega/dt]_{00}$	$= -0.0028812762$
$[d\pi/dt]_{00}$	$= +0.0027767109$
$[dL/dt]_{00}$	$= -0.0049609570$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	+0.00000	+0.00001	+0.000005263
$e[d\pi/dt]_{00}$	+0.00002	+0.00002	+0.000019001
$[di/dt]_{00}$	+0.00000	+0.00000	+0.000001824
$\sin i\,[d\Omega/dt]_{00}$	−0.000165	−0.00017	−0.000170530

NOTES.

That a division into eight parts is here fully sufficient is shown by the agreement of the final sums. Thus the greatest effect produced by all terms from the 4th to the 7th order is seen to occur with the coefficient $[d\chi/dt]_{00}$ and to amount to but 0″.00000004.

ACTION OF NEPTUNE ON VENUS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	904.77843877	+ 9.109428	+21.528911	30.253556	904.17419
45	904.51260261	− 6.657698	+16.194152	17.117826	904.17356
90	904.39237664	−14.030230	+ 1.286186	0.107979	904.17298
135	904.48820486	− 8.689452	−14.462107	13.651972	904.17345
180	904.74393521	+ 6.236084	−21.825589	31.093114	904.17407
225	905.00974680	+22.003203	−16.490829	17.750770	904.17359
270	905.12994825	+29.375736	− 1.582864	0.163538	904.17281
315	905.03414463	+24.034972	+14.165428	13.097596	904.17334
Σ_1	3619.04469887*	+30.691018†	− 0.593356†	61.618187	3616.69405
Σ_2	3619.04469890	+30.691025	− 0.593356	61.618164	3616.69394

E	l	G	G'	G''	θ
					° ′ ″
0°	0.53898	904.17415	0.5952295	0.0562134	1 32 17.018
45	0.27377	904.17354	0.3309885	0.0571984	1 11 14.024
90	0.15413	904.17298	0.1548950	0.0007710	0 45 6.502
135	0.24949	904.17343	0.2998555	0.0503537	1 7 39.561
180	0.50459	904.17403	0.5654480	0.0608162	1 30 28.921
225	0.77088	904.17357	0.7955805	0.0246764	1 43 33.466
270	0.89187	904.17281	0.8920680	0.0002028	1 48 0.658
315	0.79553	904.17332	0.8133615	0.0178097	1 44 14.706
Σ_1	2.08957	3616.69397	2.2076405	0.1180034	5 35 53.099
Σ_2	2.08967	3616.69386	2.2397860	0.1500382	5 46 41.757

ACTION OF NEPTUNE ON VENUS.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00023476	0.27331427	0.17644338	$5.1378654\bar{4}$	$9.4986215\bar{5}$	$2.3580297\bar{7}$
45	0.00013986	0.27318776	0.17630105	$5.1395214\bar{4}$	$9.5001506\bar{6}$	$2.3595431\bar{1}$
90	0.00005608	0.27307605	0.17617538	$5.1436917\bar{7}$	$9.5042639\bar{9}$	$2.3636152\bar{2}$
135	0.00012618	0.27316951	0.17628052	$5.1479186\bar{6}$	$9.5085363\bar{3}$	$2.3679232\bar{2}$
180	0.00022568	0.27330217	0.17642976	$5.1497409\bar{9}$	9.5104807	2.3698895
225	0.00029563	0.27339543	0.17653468	$5.1481063\bar{3}$	$9.5089743\bar{3}$	$2.3683773\bar{3}$
270	0.00032162	0.27343007	0.17657365	$5.1439578\bar{8}$	$9.5048849\bar{9}$	$2.3642800\bar{0}$
315	0.00029957	0.27340068	0.17654059	$5.1397097\bar{7}$	9.5005900	2.3599901
Σ_1	0.00083814	1.09312256	0.70562217	0.5752556	$8.0182509\bar{9}$	$9.4558143\bar{3}$
Σ_2	0.00086124	1.09315338	0.70565684	0.5752558	8.0182511	$9.4558336\bar{6}$

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee'\cos K] = 3619.04469884.$

† $4[a'^2e' - kaa'e\cos K] = +30.691024.$

‡ $-4k'aa'\cos\varphi' \cdot e\sin K' = -0.593359.$

ACTION OF NEPTUNE ON VENUS.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	2.3579959	902.1174996	+0.0278169	−43.343250	−165.19752
45	2.3595087	903.0233587	+1.1892143	−32.830864	−124.26237
90	2.3636147	904.1640829	+0.1518779	− 2.911684	− 9.86927
135	2.3678929	903.3068137	−1.1744691	+28.888013	+110.97189
180	2.3698530	902.1221024	−0.3409776	+43.940408	+167.47400
225	2.3683624	902.9705135	+0.9081699	+33.428015	+126.53886
270	2.3642798	904.1592634	+0.1562063	+ 3.508843	+ 12.14577
315	2.3599794	903.2945923	−0.9279474	−28.290864	−108.69539
Σ_1	9.4557434	3612.5629483*	−0.0050765	+ 1.194317	+ 4.55298
Σ_2	9.4557434	3612.5952782	−0.0050323	+ 1.194300	+ 4.55299

E	F_3	$100000 \times R_0$	$100000000 \times S_0$	$1000000 \times W_0$	$100000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	+0.5839042	0.6821434	−0.4573134	−0.9883466	0.0000000	−0.006365877
45	−3.9412736	0.6867470	+2.3281642	−0.7513860	+0.6746065	+0.032343148
90	−0.4777116	0.6961065	+0.3193229	−0.0672756	+0.9623607	+0.004414608
135	+4.0782664	0.7008255	−2.3820088	+0.6740551	+0.6818058	−0.032772474
180	+0.6001079	0.7010447	−0.2565237	+1.0297346	0.0000000	−0.003522313
225	−4.0073139	0.7005531	+2.5294517	+0.7805505	−0.6815408	+0.034801040
270	−0.5873116	0.6967370	+0.4002346	+0.0811602	−0.9632323	+0.005533205
315	+3.9893101	0.6879052	−2.4698966	−0.6479491	−0.6757442	−0.034312118
Σ_1	+0.1189889	2.7760316	+0.0057204	+0.0552726	−0.0008716	+0.000059623
Σ_2	+0.1189890	2.7760308	+0.0057105	+0.0552705	−0.0008727	+0.000059596

E	$100000 \times [R_0 \sin v + (\cos v + \cos E)S_0]$	$100000 \times \left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$100000 \times W_0 \cos u$	$100000 \times W_0 \sin u$	$1000 \times -2\frac{r}{a}R_0$
0°	−0.00091463	−0.68214344	−0.057911307	−0.080091019	−0.013549511
45	+0.49123773	−0.47994176	+0.012282960	−0.074127847	−0.013668483
90	+0.69608806	+0.00540218	+0.005478547	−0.003904555	−0.013922131
135	+0.49653747	+0.49458425	−0.066499046	−0.011017258	−0.014084333
180	+0.00051305	+0.70104468	−0.060336403	−0.083444904	−0.014116841
225	−0.49655471	+0.49418261	+0.012013897	−0.077124930	−0.014078857
270	−0.69672347	+0.00396738	+0.006544151	−0.004800407	−0.013934740
315	−0.49226067	−0.48055578	−0.064023059	−0.009971443	−0.013691533
Σ_1	−0.00103699	+0.02827080	−0.106225012	−0.172240885	−0.055523223
Σ_2	−0.00104018	+0.02826932	−0.106225248	−0.172241478	−0.055523206

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = -0.0000000000000085.$

* $\Sigma_1(J_1' - G'') = 3612.4449449.$

$\Sigma_2(J_1' - G'') = 3612.4452400.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= -\ 0''.0054696734$	m'	n 7.7379614
$[d\chi/dt]_{00}$	$= +21.756678$	m'	p 1.3375926
$[di/dt]_{00}$	$= -\ 0.55945727$	m'	n 9.7477669
$[d\Omega/dt]_{00}$	$= -15.327159$	m'	n 1.1854617
$[d\pi/dt]_{00}$	$= +21.729810$	m'	p 1.3370559
$[dL/dt]_{00}$	$= -29.268164$	m'	n 1.4663954

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$$[de/dt]_{00} = -0''.00000027764841$$
$$[d\chi/dt]_{00} = +0.0011044000$$
$$[di/dt]_{00} = -0.000028398849$$
$$[d\Omega/dt]_{00} = -0.00077802855$$
$$[d\pi/dt]_{00} = +0.0011030360$$
$$[dL/dt]_{00} = -0.0014856935$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$-0''.00000$	$-0''.00000$	$-0''.00000028$
$e[d\pi/dt]_{00}$	$+0.00001$	$+0.00001$	$+0.00000755$
$[di/dt]_{00}$	-0.00004	-0.00003	-0.00002840
$\sin i\ [d\Omega/dt]_{00}$	-0.00006	-0.00005	-0.00004605

NOTES.

The large disagreement of the sums of the functions near the beginning of the computation is caused, as in previous cases, by the presence of the term a'^2e'. The greatest disagreement in the final sums occurs in the second column and shows that the effect of all terms from the 4th to the 7th orders is to produce a change of $0''.00000003$ in the value of $[d\chi/dt]_{00}$.

EARTH.

Action of Mercury on the Earth.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$1000\times g$	h
0°	1.25770017	+0.37398164	+0.15686177	0.15586327	1.11042819
30	1.20879374	+0.24429166	+0.30606106	0.59337027	1.06575342
60	1.14343370	+0.05583076	+0.37253457	0.87910918	1.00282212
90	1.07923573	−0.14090326	+0.33847094	0.72569217	0.93669980
120	1.03345314	−0.29319554	+0.21299723	0.28738066	0.88633014
150	1.01830180	−0.36023958	+0.02973421	0.00560044	0.86904450
180	1.03773900	−0.32407097	−0.16221302	0.16667904	0.89145124
210	1.08650443	−0.19438096	−0.31141230	0.61430099	0.94482277
240	1.15158355	−0.00592005	−0.37788591	0.90454667	1.01121409
270	1.21564099	+0.19081399	−0.34382220	0.74882000	1.07228810
300	1.26156411	+0.34310630	−0.21834847	0.30200208	1.11392105
330	1.27699637	+0.41015028	−0.03508545	0.00779765	1.12755062
Σ_1	6.88547367*	+0.14973214†	−0.01605383‡	2.69558090	6.01616683
Σ_2	6.88547306	+0.14973213	−0.01605374	2.69558152	6.01615921

E	l	G	G'	G''	θ		
					°	′	″
0°	0.14093752	1.11028337	0.14207046	0.00098811	21	1	34.45
30	0.13670586	1.06515341	0.14124977	0.00394390	21	37	28.51
60	0.13427712	1.00181061	0.14149060	0.00620197	22	30	21.00
90	0.13620147	0.93572981	0.14260963	0.00543817	23	21	59.90
120	0.14078854	0.88589477	0.14348474	0.00226084	23	53	48.72
150	0.14292284	0.86903562	0.14297679	0.00004507	23	55	58.88
180	0.13995330	0.89120229	0.14152378	0.00132152	23	34	54.52
210	0.13534720	0.94401808	0.14077439	0.00462250	23	2	51.29
240	0.13403500	1.01019211	0.14138996	0.00633298	22	24	31.27
270	0.13701843	1.07154031	0.14266461	0.00489838	21	43	51.55
300	0.14130860	1.11364215	0.14347758	0.00189008	21	9	39.20
330	0.14311129	1.12754360	0.1431666$\bar{3}$	0.00004830	20	52	41.84
Σ_1	0.83130008	6.01302530	0.85343712	0.01899550	134	34	49.16
Σ_2	0.83130709	6.01302083	0.8534418$\bar{2}$	0.01899632	134	34	51.97

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 6.8854738.$

† $6[a'^2e' - kaa'e\cos K] = +0.14973211.$

‡ $-6k'aa'\cos\varphi'\cdot e\sin K' = -0.01605375.$

ACTION OF MERCURY ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.04527343	0.33291492	0.24335185	9.9618524	0.2031270	0.1593841
30	0.04797642	0.33646399	0.24732687	9.9917424	0.2701720	0.2100521
60	0.05211516	0.34189221	0.25340453	0.039602$\bar{0}$	0.374562$\bar{4}$	0.289540$\bar{6}$
90	0.05634203	0.34742859	0.25960067	0.0958412	0.4959354	0.3817747
120	0.05903754	0.35095528	0.26354624	0.1435571	0.5975343	0.4586143
150	0.05922392	0.35119901	0.26381890	0.163158$\bar{6}$	0.636237$\bar{4}$	0.487917$\bar{4}$
180	0.05742739	0.34884899	0.26118992	0.1459441	0.5935535	0.4565142
210	0.05475392	0.34534936	0.25727397	0.1016262	0.4927722	0.3817985
240	0.05164961	0.34128199	0.25272142	0.048225$\bar{7}$	0.375271$\bar{5}$	0.293829$\bar{0}$
270	0.04846610	0.33710662	0.24804652	0.000482$\bar{2}$	0.273610$\bar{2}$	0.2165394
300	0.04587421	0.33370403	0.24423576	9.967336$\bar{7}$	0.206076$\bar{5}$	0.164090$\bar{4}$
330	0.04461832	0.33205428	0.24238775	9.9536821	0.1814324	0.143917$\bar{9}$
Σ_1	0.31137734	2.04959742	1.51844973	0.3065179	2.350125$\bar{1}$	1.821972$\bar{5}$
Σ_2	0.31138070	2.04960185	1.51845468	0.3065326	2.3501595	1.8219999

E	$\log V$	J_1'	J_2	J_3	F_2
0°	0.1589092	0.14310992	+0.032517759	+0.009293505	−0.004694216
30	0.2080805	0.14534318	+0.064629382	+0.012521451	−0.009159128
60	0.2862522	0.14795099	+0.079427870	+0.011088166	−0.011148401
90	0.3786919	0.14823938	+0.072184328	+0.005377209	−0.010129019
120	0.4572602	0.14575314	+0.044711897	+0.003081410	−0.006374115
150	0.4878898	0.14319523	+0.004967377	−0.012020967	−0.000889820
180	0.4557266	0.14344333	−0.035694279	−0.019045640	+0.004854357
210	0.3791987	0.14602945	−0.066230415	−0.022272938	+0.009319268
240	0.2904985	0.14803624	−0.078997000	−0.020838344	+0.011308544
270	0.2141065	0.14763142	−0.071296738	−0.015126732	+0.010289159
300	0.1631852	0.14536909	−0.045398479	−0.006668768	+0.006534256
330	0.1438950	0.14322488	−0.007686344	+0.002269483	+0.001049961
Σ_1	1.8118319	0.87366271*	−0.003432232	−0.029252491	+0.000480425
Σ_2	1.8118624	0.87366354	−0.003432410	−0.029252494	+0.000480421

* $\Sigma_1(J_1' - G'') = 0.85466721.$
$\Sigma_2(J_1' - G'') = 0.85466722.$

ACTION OF MERCURY ON THE EARTH.

E	$1000 \times F_3$	R_0	S_0	W_0	$R^{(n)}$	$S^{(n)}$
0°	−0.3347370	−0.9146346	+0.03939096	+0.012865148	0.0000000	+0.04006286
30	−0.1102483	−0.9755942	+0.08729250	+0.020012484	−0.4949864	+0.08857904
60	+0.5661785	−1.0850638	+0.12713088	+0.022775691	−0.9476313	+0.12820596
90	+1.0162060	−1.2358930	+0.14090497	+0.016043819	−1.2358930	+0.14090497
120	+0.7859945	−1.3865198	+0.10290744	−0.005719611	−1.1907761	+0.10205179
150	+0.1010653	−1.4553467	+0.01142556	−0.036530916	−0.7172557	+0.01126198
180	−0.3579653	−1.3946588	−0.08289463	−0.055794474	0.0000000	−0.08152732
210	−0.1348464	−1.2530893	−0.12959913	−0.053749951	+0.6175749	−0.12774376
240	+0.5468018	−1.1065943	−0.12737525	−0.039380718	+0.9503695	−0.12631602
270	+1.0072426	−0.9942912	−0.09740907	−0.022874526	+0.9942912	−0.09740907
300	+0.7898462	−0.9252302	−0.05560180	−0.008440800	+0.8080489	−0.05607200
330	+0.1166998	−0.8987688	−0.00911126	+0.003338197	+0.4560076	−0.00924555
Σ_1	+1.9961187	−6.8127015	+0.00355760	−0.073694764	−0.3799890	+0.00640527
Σ_2	+1.9961190	−6.8129832	+0.00350357	−0.073760893	−0.3802614	+0.00634761

E	$R_0 \sin v + (\cos v + \cos E)S_0$	$-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$W_0 \cos(v+\pi)$	$W_0 \sin(v+\pi)$	$-2\frac{r}{a}R_0$
0°	+0.0787819	+0.9146346	−0.002313800	+0.012655370	1.7985901
30	−0.3440930	+0.9286742	−0.013088473	+0.015139066	1.9228487
60	−0.8219877	+0.7498962	−0.021560174	+0.007341046	2.1519297
90	−1.2380823	+0.2610826	−0.015731601	−0.003149764	2.4717860
120	−1.2947997	−0.5330552	+0.004304192	+0.003766681	2.7962929
150	−0.7369917	−1.2550382	+0.011990880	+0.034506913	2.9529690
180	+0.1657893	−1.3946588	−0.010034649	+0.054884684	2.8360980
210	+0.8424959	−0.9617143	−0.034466377	+0.041244714	2.5425788
240	+1.0791999	−0.3473975	−0.036894585	+0.013770630	2.2317474
270	+0.9957850	+0.1781428	−0.022567368	−0.003736033	1.9885826
300	+0.7530386	+0.5475911	−0.006510755	−0.005371886	1.8349430
330	+0.4402010	+0.7837111	+0.001148460	+0.003134422	1.7714297
Σ_1	−0.0399777	−0.0629896	−0.073009771	+0.087046525	13.6496011
Σ_2	−0.0406851	−0.0651418	−0.072714479	+0.087139318	13.6501948

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = +0.00000006.$

DIFFERENTIAL COEFFICIENTS.

				log coeff.
	$[de/dt]_{00}$	$= -$ 8710.1780	m'	n 3.9400270
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= -$ 824986.23	m'	n 5.9164467
	$[dp/dt]_{00}$	$= +$ 18814.333	m'	p 4.2744888
	$[dq/dt]_{00}$	$= -$ 15740.112	m'	n 4.1970078
	$[dL/dt]_{00}$	$= +2948201.7$	m'	p 6.4695572

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= -0.0011613570$
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= -0.10999815$
	$[dp/dt]_{00}$	$= +0.0025085775$
	$[dq/dt]_{00}$	$= -0.0020986812$
	$[dL/dt]_{00}$	$= +0.39309355$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	-0.00116	-0.00116	-0.00116136
$e[d\pi/dt]_{00}$	-0.00184	-0.00184	-0.00184479
$[dp/dt]_{00}$	$+0.00250$	$+0.00251$	$+0.00250858$
$[dq/dt]_{00}$	-0.00209	-0.00210	-0.00209868
$[dL/dt]_{00}$	$+0.3931$		$+0.39309355$

NOTES.

Although I and e' are here very large, the error in the approximate test with ϵ, G, G', G'' and θ is small in consequence of the smallness of the factor a'. As we approach the end of the computation, however, the difference of the sums steadily increases, indicating the rather slow convergence of the perturbing function. The greatest difference is in the coefficient $[d\pi/dt]_{00}$ where terms from the fifth to the eleventh orders inclusive amount to one sixtieth part of the remaining terms and produce an effect of $0''.0018$ in the value of $[d\pi/dt]_{00}$. A division into twelve parts is thus necessary in this case, but a comparison with the computation of the action of Mars on Mercury, and especially with the similar case of Mercury on Venus, where twenty-four points of division are employed, renders it evident that more than twelve points are in the present case unnecessary.

The agreement with previous values is exact. The results obtained by HILL in the "*New Theory*," pages 511 and 512, are,

$$[dp/dt]_{00} = +0''.0025049 \quad [dq/dt]_{00} = -0''.0020956$$

These are, however, but provisional values. (See the note to the computation of Venus on the Earth.)

ACTION OF VENUS ON THE EARTH.

E	A	$\log B$	ϵ			q^2	θ'		
°			°	′	″		°	′	″
0	1.49844749	9.8537612	331	0	3.89	0.07950559	7	0	55.62
30	1.50411382	9.8546442	1	19	46.170	0.08070333	5	48	28.79
60	1.51484369	9.856764$\bar{5}$	31	34	37.20	0.08262793	4	19	13.62
90	1.52786583	9.8597314	61	38	57.20	0.08465948	3	12	16.55
120	1.5397420$\bar{1}$	9.8626537	91	29	35.875	0.08634022	2	33	43.11
150	1.54723847	9.8644870	121	8	43.24	0.0874100$\bar{9}$	2	9	29.44
180	1.54824407	9.8645724	150	43	29.14	0.08768573	1	52	5.43
210	1.54243719	9.862968$\bar{3}$	180	22	35.176	0.08700647	1	54	16.46
240	1.53142595	9.860356$\bar{8}$	210	12	5.09	0.08536569	2	37	52.10
270	1.51826327	9.8576205	240	13	17.66	0.08309965	4	8	28.42
300	1.5065276$\bar{4}$	9.8554195	270	23	50.222	0.08090038	5	56	51.93
330	1.49931255	9.8540767	300	40	21.95	0.07954648	7	7	42.75
Σ_1	9.13923084*	9.1535280	1085	23	41.42	0.5024255$\bar{4}$	24	20	41.81
Σ_2	9.13923113	9.153528$\bar{1}$	905	23	41.40	0.5024255$\bar{0}$	24	20	42.41

E	$1000 \times r'$	$1000 \times g$	G	G'	$1000000 \times G''$	θ		
°						°	′	″
0	2.7380635	0.002936418	0.97594422	0.522484505	5.79	47	1	40.84
30	2.3200483	0.000006753	0.98353171	0.520557558	0.04	46	40	41.61
60	1.7893068	0.003473631	0.99543723	0.519388564	6.67	46	14	52.33
90	1.3770128	0.009946420	1.00773136	0.520128988	18.98	45	55	31.27
120	1.1340363	0.013007536	1.01774388	0.521998108	24.54	45	44	23.02
150	0.9732049	0.009615238	1.02406979	0.523162060	17.95	45	37	23.13
180	0.8464697	0.003140184	1.02571566	0.522509836	5.92	45	32	21.65
210	0.8529468	0.00000056$\bar{3}$	1.02173849	0.520674148	0.00	45	32	59.47
240	1.1449685	0.003259143	1.01199593	0.519411649	6.09	45	45	34.89
270	1.7299249	0.009581104	0.99828868	0.519968638	18.53	46	11	45.77
300	2.3843853	0.012589278	0.98467983	0.521847851	24.49	46	43	7.27
330	2.7841227	0.00925611$\bar{5}$	0.97615654	0.523149586	18.07	47	3	38.83
Σ_1	10.0372301	0.038406190	6.01151675	3.127640513	73.50	277	2	0.00
Σ_2	10.0372604	0.038406192	6.01151657	3.127640978	73.57	277	2	0.08

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 9.13923110.$

ACTION OF VENUS ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.26147483	0.60779696	0.54775634	0.2626427	0.8915846	$0.8209715\bar{5}$
30	0.25683878	0.60207812	0.54149917	0.2549482	0.8714495	0.8036590
60	0.25122074	0.59513848	0.53390180	0.2468811	0.8459860	0.7827661
90	0.24707162	0.59000660	0.52828039	0.2420422	0.8253429	$0.7669697\bar{7}$
120	0.24470700	0.58707937	0.52507272	0.2404872	0.8122693	$0.7579113\bar{3}$
150	0.24322990	0.58524987	0.52306751	0.2402489	0.8048244	0.7529792
180	0.24217345	0.58394092	0.52163264	0.2400756	0.8019574	$0.7506787\bar{7}$
210	0.24230579	0.58410492	0.52181242	0.2408209	0.8062462	0.7532935
240	0.24496050	0.58739327	0.52541673	0.2444416	0.8214721	0.7646769
270	0.25055055	0.59430993	0.53299440	0.2516543	0.8474359	$0.7853846\bar{6}$
300	0.25737182	0.60273603	0.54221917	0.2600985	0.8762226	$0.8090118\bar{8}$
330	0.26191246	0.60833646	0.54834645	$0.2649133\bar{3}$	$0.8941948\bar{8}$	$0.8237322\bar{2}$
Σ_1	1.50190834	3.56408503	3.19599940	1.4946267	5.0494920	4.6860161
Σ_2	1.50190910	3.56408590	3.19600034	$1.4946278\bar{8}$	$5.0494937\bar{7}$	$4.6860181\bar{1}$

E	$\log V$	J_1'	$1000 \times J_2$	$1000 \times J_3$	$1000 \times F_2$
0°	0.8209685	0.522862677	$-3.063273\bar{1}$	$+12.629050\bar{0}$	+1.2372987
30	0.8036589	0.521939851	$-0.733650\bar{4}$	$+24.877829\bar{9}$	−0.0593369
60	$0.782762\bar{8}$	0.521368465	+2.4504401	+30.338229	−1.3457284
90	$0.766960\bar{3}$	0.521723193	$+5.083579\bar{7}$	+27.547130	−2.2771885
120	$0.757899\bar{2}$	0.522632205	+5.8345100	+17.252379	−2.6041325
150	0.7529704	$0.52319164\bar{2}$	+4.4234178	+ 2.21247$\bar{1}$	−2.2389562
180	$0.750675\bar{8}$	0.522862807	+1.7386808	$-13.542647\bar{7}$	−1.2795084
210	0.7532935	0.521968683	−0.9069972	−25.791387	$+0.017127\bar{5}$
240	0.7646739	0.521377068	$-2.685926\bar{9}$	−31.251723	+1.3035189
270	$0.785375\bar{3}$	0.521675619	−3.5987100	−28.460585	+2.2349789
300	$0.808999\bar{4}$	$0.52258215\bar{0}$	−4.0331250	-18.165873	+2.5619225
330	$0.823723\bar{0}$	$0.52318645\bar{2}$	−4.0263352	− 3.126030	$+2.196746\bar{5}$
Σ_1	4.6859794	3.133685372*	+0.2413060	− 2.740585	−0.1266292
Σ_2	$4.685981\bar{3}$	3.133685439	+0.2413047	− 2.740573	−0.1266288

* $\Sigma_1(J_1' - G'') = 3.13361187\bar{2}$.
$\Sigma_2(J_1' - G'') = 3.133611869$.

ACTION OF VENUS ON THE EARTH.

E	$1000 \times F_3$	R_0	$100 \times S_0$	W_0	$-\frac{r}{a}R_0$	$R^{(n)}$
0°	+0.06535471	− 1.8283274	−1.0644424	+0.08413477	1.7976643	0.0000000
30	−0.00196302	− 1.7898617	−0.5109526	+0.15828193	1.7638656	−0.9081206
60	−0.00570449	− 1.7535729	+0.5420211	+0.18393250	1.7388680	−1.5314810
90	+0.05916590	− 1.7367356	+1.4494077	+0.16147390	1.7367356	−1.7367356
120	+0.12885901	− 1.7362036	+1.6510300	+0.09963433	1.7507629	−1.4910928
150	+0.13426042	− 1.7386892	+1.0760758	+0.01338361	1.7639423	−0.8568990
180	+0.06988985	− 1.7361351	+0.1682926	−0.07583153	1.7652520	0.0000000
210	−0.0005974$\bar{8}$	− 1.7337538	−0.5029615	−0.14614352	1.7589352	+0.8544665
240	−0.00787443	− 1.7442502	−0.6981624	−0.18183297	1.7588768	+1.4980034
270	+0.05404190	− 1.7747204	−0.6225129	−0.17324740	1.7747204	+1.7747204
300	+0.12215392	− 1.8154792	−0.6714245	−0.11610019	1.8002552	+1.5855471
330	+0.12777090	− 1.8402326	−0.9613070	−0.01982985	1.8135046	+0.9336770
Σ_1	+0.37267857	−10.6139684	−0.0726856	−0.00606309	10.6116792	+0.0609767
Σ_2	+0.3726786$\bar{2}$	−10.6139933	−0.0722505	−0.00608133	10.6117037	+0.0611087

E	$S^{(n)}$	$R_0 \sin v$ $+(\cos v+\cos E)S_0$	$-R_0 \cos v$ $+\left(\frac{r}{a}\sec^2\varphi+1\right)\sin v S_0$	$W_0 \cos(v+\pi)$	$W_0 \sin(v+\pi)$
0°	−0.010825990	−0.0212888$\bar{9}$	+1.8283274	−0.01513166	+0.08276286
30	−0.005184831	−0.9168211	+1.5373039	−0.10351881	+0.11973727
60	+0.005466048	−1.5259137	+0.8639706	−0.17411620	+0.05928500
90	+0.014494077	−1.7367344	−0.0001389	−0.15833153	−0.03170097
120	+0.016373004	−1.5075995	−0.8612810	−0.07497804	−0.06561473
150	+0.010606704	−0.8754611	−1.5022509	−0.00439303	−0.01264209
180	+0.001655167	−0.0033365$\bar{9}$	−1.7361351	−0.01363832	+0.07459502
210	−0.004957609	+0.8630786	−1.5036464	−0.09371241	+0.11214236
240	−0.006923567	+1.5048615	−0.8818403	−0.17035373	+0.06358327
270	−0.006225129	+1.7745751	−0.0173138	−0.17092102	−0.02829602
300	−0.006771025	+1.5786949	+0.8963895	−0.08955312	−0.07388837
330	−0.009754749	+0.9169364	+1.5955426	−0.00682218	−0.01861936
Σ_1	−0.001026363	+0.0253885	+0.1094311	−0.53777107	+0.14072305
Σ_2	−0.001021537	+0.0255735	+0.1094965	−0.53769898	+0.14062119

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = -0.0000000083.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
	$[de/dt]_{00}$	$= +\ 5503''.0089\ m'$	p 3.7406002
$[d\chi/dt] =$	$[d\pi/dt]$	$= +1409586.4\ m'$	p 6.1490917
	$[dp/dt]_{00}$	$= +\ 30388.832\ m'$	p 4.4827140
	$[dq/dt]_{00}$	$= -\ 116164.73\ m'$	n 5.0650743
	$[dL/dt]_{00}$	$= +4584354.6\ m'$	p 6.6612782

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= +\ 0''.013483339$
$[d\chi/dt] =$	$[d\pi/dt]_{00}$	$= +\ 3.4537341$
	$[dp/dt]_{00}$	$= +\ 0.074457966$
	$[dq/dt]_{00}$	$= -\ 0.28462399$
	$[dL/dt]_{00}$	$= +11.232473$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$+\ 0''.01344$	$+0''.01348$	$+\ 0''.0134833$
$e[d\pi/dt]$	$+\ 0.05796$	$+0.05792$	$+\ 0.0579231$
$[dp/dt]_{00}$	$+\ 0.07450$	$+0.07446$	$+\ 0.0744580$
$[dq/dt]_{00}$	$-\ 0.28454$	-0.28462	$-\ 0.2846240$
$[dL/dt]_{00}$	$+11.2298$		$+11.232473$

NOTES.

This computation is of special interest because, notwithstanding the low eccentricities of both the Earth and Venus, the perturbing function is but slowly convergent for this case. In 1893, the computation was effected by MR. R. T. A. INNES who employed HILL'S second modification of GAUSS'S method, using in the work manuscript tables prepared by himself. (See *M. N.*, Vol. LIII, No. 6. The tables were afterward published in *M. N.*, Vol. LIV, No. 5.) The values of $[dp/dt]_{00}$ and $[dq/dt]_{00}$ were also obtained by HILL in the "*New Theory*," pages 511 and 512.

As the results of INNES differed considerably in some cases from those hitherto obtained, particularly in the case of $[de/dt]_{00}$, which agreed to the first two figures only with the values of LEVERRIER and NEWCOMB, and in the case of $[dq/dt]_{00}$, which

differed in the fourth figure from the value given by HILL, and in order to make the comparison more exact, the roots in the present paper were computed by the formulas of the second method, their values being afterward verified by those of the first. It was found that the functions tabulated by MR. INNES are substantially correct, though the last two significant figures of all functions from R_0 to the end usually differ, doubtless owing to the inaccuracy of the tables employed by MR. INNES. Using the values as given by him, all of this part of his computation was duplicated, with the result that an error was found in his value of $[de/dt]_{00}$, while for $[dq/dt]_{00}$ and the other coefficients his values were found to be substantially correct. The various values here referred to are as follows:

	Innes.	Hill.
	"	
$[de/dt]_{00}$	+ 0.013476*	
$e[d\pi/dt]_{00}$	+ 0.057915	"
$[dp/dt]_{00}$	+ 0.074459	+0.0744329
$[dq/dt]_{00}$	− 0.284623	−0.2845280
$[dL/dt]_{00}$	+11.232490	

It will be noticed that the results of INNES are in almost exact accordance with those here given. The disagreement of the value of $[dq/dt]_{00}$ as derived by GAUSS'S method with that found by HILL is, however, a more serious matter, and is almost the sole cause of the considerable disagreement of the values of this variation in the complete perturbations of the Earth's orbit, the values of $[dq/dt]_{00}$ from the action of all of the other planets except Venus agreeing with those obtained by HILL very exactly. Using the values tabulated on page 510 of the "*New Theory*" and the formulas of page 511, I have duplicated the computation by HILL'S methods and find the same results as he obtained. It is to be noticed that the theory of the motion of the ecliptic here given by HILL was to serve a temporary purpose only, the numerical values of the coefficients stated by LEVERRIER in the *Annales*, Vol. II, pages 94 to 96, being employed without a re-computation of them.

* The uncorrected value was + 0″.013156.

ACTION OF MARS ON THE EARTH.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	3.12005845	−0.6857946	+1.1901000	0.028604007	$2.3106194\overline{4}$
22.5	3.04885416	−1.0809801	+0.7480691	0.011301685	$2.305935\overline{8}$
45	3.01959381	−1.2762880	+0.1890610	0.000721878	2.3030821
67.5	3.03677529	−1.2419833	−0.4018201	0.003260792	2.3034537
90	3.09782583	−0.9832901	−0.9346186	0.017641228	2.3070230
112.5	3.19346899	−0.5395909	−1.3282200	0.035628705	2.3122967
135	3.30912609	+0.0215645	−1.5227032	0.046826348	2.3167796
157.5	3.42714623	+0.6147456	−1.4884581	0.044743825	2.3179851
180	3.52951891	+1.1496460	−1.2306997	0.030588915	2.3148501
202.5	3.60064084	+1.5448315	−0.7886687	0.012561720	2.3089820
225	3.62970226	+1.7401394	−0.2296607	0.001065206	2.3043625
247.5	3.61232195	+1.7058351	+0.3612206	0.002635147	2.3044350
270	3.55118902	+1.4471417	+0.8940191	0.016141855	2.3088270
292.5	3.45562818	+1.0034423	+1.2876203	0.033483868	2.3139342
315	3.34017005	+0.4422870	+1.4821032	0.044362582	$2.3163\overline{15}3$
337.5	3.22234876	−0.1508942	+1.4478582	0.042336214	$2.31484\overline{21}$
Σ_1	26.59718442*	+1.8554059†	−0.1623989‡	0.185952019	18.4818589
Σ_2	26.59718440	+1.8554060	−0.1623987	0.185951956	18.4818645

* $8a^2 + 4a^2e^2 + 8[a'^2 - 2kaa'ee'\cos K] = 26.59718442.$
† $8[a'^2e' - kaa'e\cos K] = +1.8554056.$
‡ $-8k'aa'\cos\varphi' \cdot e\sin K' = -0.1623983.$

ACTION OF MARS ON THE EARTH.

E	l	G	G'	G''	θ		
					°	′	″
0°	0.7892434	2.3024091	0.8127396	0.0152860	36	42	22.96
22.5	0.7227227	2.3028298	0.7325284	0.0066997	34	27	16.75
45	0.6963160	2.3028870	0.6969608	0.0004498	33	23	4.72
67.5	0.7131259	2.3025627	0.7159948	0.0019779	33	55	44.11
90	0.7706071	2.3020189	0.7853689	0.0097577	35	54	24.65
112.5	0.8609765	2.3015508	0.8891330	0.0174106	38	41	59.61
135	0.9721508	2.3014739	1.0076482	0.0201918	41	42	38.47
157.5	1.0889654	2.3019609	1.1223087	0.0173190	44	30	20.12
180	1.1944730	2.3028661	1.2173683	0.0109112	46	46	8.30
202.5	1.2714631	2.3036994	1.2810025	0.0042567	48	15	58.20
225	1.3051440	2.3038996	1.3059610	0.0003540	48	50	43.66
247.5	1.2876912	2.3033085	1.2897047	0.0008871	48	27	7.65
270	1.2221663	2.3023363	1.2343370	0.0056800	47	8	13.64
292.5	1.1214982	2.3016065	1.1465149	0.0126889	45	3	3.25
315	1.0036591	2.3014626	1.0370980	0.0185863	42	25	11.06
337.5	0.8873110	2.3018396	0.9202987	0.0199852	39	31	20.00
Σ_1	7.9537596	18.4193535	8.0974817	0.0812168	332	52	47.46
Σ_2	7.9537539	18.4193582	8.0974855	0.0812251	332	52	49.69

ACTION OF MARS ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.14793515	0.46562072	0.39121913	9.5856600	9.3211682	9.6118228
22.5	0.12867513	0.44104101	0.36395436	9.5698260	9.2838199	9.5702568
45	0.12011431	0.43007010	0.35176641	9.5662159	9.2715711	9.5556249
67.5	0.12442259	0.43559482	0.35790544	9.5749532	9.2853792	9.5702743
90	0.14089818	0.45665643	0.38128232	9.5949794	9.3237440	9.6123158
112.5	0.16647531	0.48915019	0.41726416	9.6240919	9.3826551	9.6760625
135	0.19735020	0.52805532	0.46020914	9.6588907	9.4553466	9.7533001
157.5	0.22941061	0.56809820	0.50425204	9.6947360	9.5321279	9.8336349
180	0.25803549	0.60355500	0.54311538	9.7259997	9.6009117	9.9047936
202.5	0.27840311	0.62861983	0.57051120	9.7469169	9.6490816	9.9542006
225	0.28660968	0.63868139	0.58149048	9.7530543	9.6666751	9.9720145
247.5	0.28101715	0.63182707	0.57401211	9.7427949	9.6495832	9.9542876
270	0.26293394	0.60959543	0.54972342	9.7180756	9.6011932	9.9045601
292.5	0.23607945	0.57638319	0.51334444	9.6838585	9.5314041	9.8327841
315	0.20515940	0.53784176	0.47098807	9.6465515	9.4533990	9.7520425
337.5	0.17455362	0.49936282	0.42855178	9.6122456	9.3799495	9.6749680
Σ_1	1.61903635	4.27007615	3.72979435	7.2494269	5.6940086	8.0664741
Σ_2	1.61903696	4.27007713	3.72979553	7.2494229	5.6940004	8.0664686

ACTION OF MARS ON THE EARTH.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	9.6084099	2.3152378	+0.11029083	+0.061812671	−0.25644059
22.5	$9.5687497\bar{7}$	2.3059045	+0.06946998	+0.074883922	−0.16119255
45	$9.5555233\bar{3}$	2.2995328	+0.01805514	+0.077080482	−0.04073852
67.5	9.5698283	2.3016313	−0.03654213	+0.068068172	+0.08658346
90	$9.6101298\bar{8}$	2.3103260	−0.08632858	+0.049219187	+0.20138993
112.5	$9.6721978\bar{8}$	2.3186977	−0.12376087	+0.023403207	+0.28620250
135	9.7488617	2.3215901	−0.14288822	−0.005449606	+0.32810925
157.5	9.8298619	2.3181659	−0.14041555	−0.032946848	+0.32073022
180	9.9024333	2.3108630	−0.11642269	−0.054902532	+0.26518896
202.5	9.9532839	2.3034774	−0.07451078	−0.067974156	+0.16994090
225	9.9719383	2.2994233	−0.02127963	−0.070171605	+0.04948689
247.5	9.9540965	2.3004768	+0.03479475	−0.061160169	−0.07783516
270	9.9033316	2.3061702	+0.08485465	−0.042311544	−0.19264158
292.5	9.8300222	2.3139376	+0.12119708	−0.016495194	−0.27745411
315	$9.7479652\bar{2}$	2.3199983	+0.13850882	+0.012358484	−0.31936096
337.5	9.6705453	2.3208585	+0.13455789	+0.039856619	−0.31198183
Σ_1	8.0485930	18.4831415*	−0.01520968	+0.027635537	+0.03499338
Σ_2	$8.048585\bar{5}$	18.4831497	−0.01520963	+0.027635553	+0.03499343

* $\Sigma_1(J_1' - G'') = 18.4019247.$
$\Sigma_2(J_1' - G'') = 18.4019246.$

ACTION OF MARS ON THE EARTH.

E	$1000 \times F_3$	R_0	S_0	W_0	$R^{(n)}$	$S^{(n)}$
0°	− 5.019874	0.22207195	−0.00895625	+0.02403761	0.00000000	−0.00910901
22.5	− 1.341522	0.21055435	−0.00524961	+0.02748415	+0.08184381	−0.00533223
45	+ 0.173061	0.20752103	−0.00112516	+0.02773157	+0.14850060	−0.00113867
67.5	− 1.404619	0.21281662	+0.00313244	+0.02500875	+0.19788699	+0.00315267
90	− 5.191525	0.22627677	+0.00726155	+0.01896286	+0.22627677	+0.00726155
112.5	− 9.004074	0.24749832	+0.01089521	+0.00882887	+0.22720045	+0.01082573
135	−10.631982	0.27522212	+0.01347761	−0.00609013	+0.19233060	+0.01331965
157.5	− 9.129495	0.30647387	+0.01430898	−0.02537639	+0.11549298	+0.01409065
180	− 5.368216	0.33606721	+0.01279803	−0.04599728	0.00000000	+0.01258694
202.5	− 1.527816	0.35740451	+0.00883760	−0.06172287	−0.13468590	+0.00870276
225	+ 0.177177	0.36477534	+0.00302208	−0.06569865	−0.25491209	+0.00298666
247.5	− 1.210722	0.35578971	−0.00342959	−0.05556595	−0.32661056	−0.00340772
270	− 4.837361	0.33265813	−0.00898150	−0.03579916	−0.33265813	−0.00898150
292.5	− 8.543560	0.30146830	−0.01237466	−0.01405700	−0.28031952	−0.01245460
315	−10.135232	0.26942963	−0.01319021	+0.00403826	−0.19280199	−0.01334851
337.5	− 8.672134	0.24201044	−0.01181400	+0.01658570	−0.09407098	−0.01199994
Σ_1	−40.833952	2.23402218	+0.00430615	−0.07881492	−0.21326424	+0.00357711
Σ_2	−40.833942	2.23401612	+0.00430637	−0.07881474	−0.21326273	+0.00357732

$$\sin\varphi \cdot \tfrac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = +\,0.0000000050.$$

ACTION OF MARS ON THE EARTH.

E	$R_0 \sin v + (\cos v + \cos E) S_0$	$-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1\right) \sin v S_0$	$W_0 \cos (v + \pi)$	$W_0 \sin (v + \pi)$	$-2 \frac{r}{a} R_0$
0°	−0.01791249	−0.22207195	−0.00432317	+0.02364565	−0.43669510
22.5	+0.07214538	−0.19805102	−0.01506202	+0.02298943	−0.41458381
45	+0.14689806	−0.14657925	−0.02300258	+0.01548939	−0.41012014
67.5	+0.20021147	−0.07256855	−0.02452836	+0.00487823	−0.42290136
90	+0.22612318	+0.01831801	−0.01859384	−0.00372284	−0.45255354
112.5	+0.21867465	+0.11830141	−0.00734127	−0.00490454	−0.49817351
135	+0.17313163	+0.21584084	+0.00340234	+0.00505112	−0.55697199
157.5	+0.08900258	+0.29475425	+0.00517812	+0.02484247	−0.62244500
180	−0.02559607	+0.33606721	−0.00827262	+0.04524724	−0.68340683
202.5	−0.15101812	+0.32435071	−0.03316015	+0.05205879	−0.72588458
225	−0.25917515	+0.25670929	−0.05360955	+0.03797800	−0.73820246
247.5	−0.32389097	+0.14753236	−0.05413656	+0.01252224	−0.71614631
270	−0.33246073	+0.02354205	−0.03531845	−0.00584698	−0.66531623
292.5	−0.28957300	−0.08808428	−0.01192486	−0.00744291	−0.59906678
315	−0.21131668	−0.16946344	+0.00233483	+0.00329486	−0.53246901
337.5	−0.11585773	−0.21387153	+0.00359251	+0.01619196	−0.47652121
Σ_1	−0.30030825	+0.31236276	−0.13738304	+0.12113644	−4.47573530
Σ_2	−0.30030574	+0.31236335	−0.13738259	+0.12113567	−4.47572256

DIFFERENTIAL COEFFICIENTS.

			log coeff.
	$[de/dt]_{00}$	$= -\ 48641.893''\ m'$	$n\ 4.6870105\bar{5}$
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +3016769.1\ m'$	$p\ 6.479542\bar{1}$
	$[dp/dt]_{00}$	$= +\ 19626.398\ m'$	$p\ 4.2928406$
	$[dq/dt]_{00}$	$= -\ 22258.695\ m'$	$n\ 4.347499\bar{7}$
	$[dL/dt]_{00}$	$= -\ 724628.93\ m'$	$n\ 5.8601157$

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= -0.015723904''$
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +0.97519611$
	$[dp/dt]_{00}$	$= +0.0063443986$
	$[dq/dt]_{00}$	$= -0.0071953108$
	$[dL/dt]_{00}$	$= -0.23424243$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Innes.	Hall.	Newcomb.	Method of Gauss.
	"	"	"	"	"
$[de/dt]_{00}$	−0.01573	−0.015722	−0.0157232	−0.01572	−0.0157239
$[d\pi/dt]_{00}$	+0.9754	+0.975224	+0.9751387	+0.9755	+0.9751961
$[dp/dt]_{00}$	+0.00635	+0.0063401	+0.0063444	+0.00634	+0.0063444
$[dq/dt]_{00}$	−0.00721	−0.0071898	−0.0071952	−0.00719	−0.0071953
$[dL/dt]_{00}$	−0.2337	−0.23469	−0.2342416		−0.2342424

NOTES.

In the "*New Theory*," Page 511, HILL points out that the convergence of the expansion of the perturbing function is slow in this case, the terms of the fifth order in the inclinations and eccentricities amounting to one per cent. of those of the first order. He stated that a computation by GAUSS's method would be very desirable and consequently this was effected by DR. ASAPH HALL, JR., in July, 1891 (*A. J.* No. 244), and by INNES in November, 1891 (*M. N.*, Vol. LII, Nos. 2 and 7). HALL's computation is the first application of GAUSS's method made after the publication of HILL's memoir.

Both HALL and INNES employed the values of the elements stated by LEVERRIER; Hall divided the orbit of the Earth into twelve parts and INNES into sixteen. The values of $[dp/dt]_{00}$ and $[dq/dt]_{00}$ given by the latter were however in error owing to a misprint that occurred in HILL's original paper in the value of J_3; in *M. N.*, Vol. LII, No. 7, INNES pointed out this error but did not re-compute the variations.

The final results of the present paper were printed in *A. J.*, No. 518, but the values there given are all slightly incorrect owing to errors in some of the preliminary constants, which remained undetected even in the duplication. Upon devising new test equations these were always applied to all computations previously made and in this way the errors affecting practically every figure of the present computation were discovered. The work was then both repeated and duplicated so that it is hardly possible that any errors can yet remain in it.

The latter part of INNES' computation was also duplicated, the values of J_3, W_0, $[dp/dt]_0$ and $[dq/dt]_0$ being freed from the errors referred to by him. It is these corrected values which are given above.

It will be noticed that the agreement of the results here given with those of HALL is very exact notwithstanding the difference of the original elements used in the computation. The divergences from those of INNES are more considerable, probably because the latter computer did not employ the accurate tables of HILL.

The values obtained in the "*New Theory*" for the motion of the plane of the ecliptic are,

$$[dp/dt]_{00} = +0.''0063362$$
$$[dq/dt]_{00} = -0.0072112$$

ACTION OF JUPITER ON THE EARTH.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	28.04923872	+1.4444306	+5.1077212	1.6444314	27.008467
30	27.80097551	−1.1738070	+4.4818820	1.2661417	27.008061
60	27.62449781	−3.1281533	+2.6317812	0.4365772	27.007154
90	27.56719541	−3.8949449	+0.0531514	0.0001781	27.006666
120	27.6444742	−3.2687173	−2.5630644	0.4140764	27.007100
150	27.83557589	−1.4172691	−4.5158555	1.2854086	27.008066
180	28.08919180	+1.1633052	−5.2819683	1.7585429	27.008637
210	28.33731437	+3.7815432	−4.6561285	1.3665053	27.008243
240	28.5135108	+5.7358901	−2.8060283	0.4963014	27.007275
270	28.57067253	+6.5026807	−0.2273984	0.0032594	27.006689
300	28.49353432	+5.8764527	+2.3888176	0.3596892	27.007056
330	28.30271399	+4.0250047	+4.3416085	1.1881266	27.007962
Σ_1	168.41444773*	+7.8232080†	−0.5227409‡	5.1096185	162.045689
Σ_2	168.41444770	+7.8232076	−0.522740	5.1096197	162.045687

E	l	G	G'	G''	θ
°					° ′ ″
0	0.977739	27.0061276	1.0387013	0.0586223	11 36 58.64
30	0.729882	27.0062768	0.7909419	0.0592752	10 12 32.08
60	0.554312	27.0065429	0.5826671	0.0277442	8 38 31.93
90	0.497498	27.0066658	0.497510	0.000013	7 48 2.72
120	0.574342	27.0065199	0.600457	0.025534	8 45 10.87
150	0.764478	27.0062522	0.8240511	0.0577594	10 23 57.2
180	1.017523	27.0061314	1.0803044	0.0602760	11 50 45.30
210	1.266039	27.0062772	1.3067275	0.0387224	12 53 16.35
240	1.443204	27.0065561	1.4565395	0.0126169	13 29 4.47
270	1.500952	27.0066843	1.5010367	0.0000804	13 38 11.24
300	1.423446	27.0065354	1.4332595	0.0092925	13 21 37.74
330	1.231720	27.0062551	1.2681196	0.0346927	12 40 46.32
Σ_1	5.990566	162.0384133	6.191928	0.194086	67 42 8.95
Σ_2	5.990569	162.0384114	6.188387	0.190543	67 36 45.94

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 168.41444773.$

† $6[a'^2e' - kaa'e \cos K] = +7.8232074.$

‡ $-6k'aa' \cos \varphi' \cdot e \sin K' = -0.5227409.$

ACTION OF JUPITER ON THE EARTH.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.01351621	0.29098220	0.19630730	7.8502195	5.2763937	6.6141228
30	0.01041684	0.28686620	0.19168189	7.8490834	5.2711158	6.6083484
60	0.00744942	0.28292144	0.18724767	7.8522627	5.2713543	6.6075955
90	0.00606351	0.28107775	0.18517479	7.8588567	5.2769921	6.6125603
120	0.00764272	0.28317853	0.18753669	7.8670773	5.2864976	6.6227349
150	0.01081185	0.28739101	0.19227173	7.8747484	5.2973550	6.6346279
180	0.01406123	0.29170555	0.19712004	7.8798617	5.3067059	6.6445510
210	0.01667410	0.29517157	0.20101379	7.8810685	5.3120661	6.6499953
240	0.01827428	0.29729273	0.20339625	7.8780194	5.3119679	6.6497435
270	0.01869373	0.29784858	0.20402051	7.8714850	5.3063886	6.6440330
300	0.01793523	0.29684340	0.20289160	7.8631935	5.2968004	6.6344668
330	0.01613325	0.29445437	0.20020816	7.8553923	5.2858029	6.6235786
Σ_1	0.07887909	1.74292385	1.17449955	7.1906339	1.7497196	9.7732144
Σ_2	0.07879328	1.74280948	1.17437087	7.1906342	1.7497203	9.7731434

E	log V	J_1'	J_2	J_3	F_2
0°	6.6129514	27.0647415822	+0.24634548	+0.02063093	−6.6623169
30	6.6071626	27.061450021	+0.21012324	+0.32748544	−5.8459973
60	6.6070396	27.022786983	+0.12128740	+0.54805936	−3.4327952
90	6.6125600	26.992014082	+0.00316191	+0.62325071	−0.0693287
120	6.6222233	27.021540830	−0.11740027	+0.53291210	+3.3431631
150	6.6334726	27.060747670	−0.21222668	+0.30124976	+5.8903093
180	6.6433468	27.0663952822	−0.25546365	−0.00966336	+6.8895984
210	6.6492222	27.041108427	−0.23112984	−0.31651859	+6.0732768
240	6.6494917	27.0080050022	−0.14158677	−0.53709407	+3.6600761
270	6.6440314	26.992057684	−0.01087596	−0.61228634	+0.2966099
300	6.6342813	27.004932957	+0.12157000	−0.52194700	−3.1158827
330	6.6228858	27.037481542	+0.21569955	−0.29028297	−5.6630276
Σ_1	9.7693340	162.188402634*	−0.02524781	+0.03289796	+0.6818428
Σ_2	9.7693345	162.184859426	−0.02524778	+0.03289801	+0.6818424

* $\Sigma_1(J_1' - G'') = 161.994316084.$
$\Sigma_2]J_1' - G'') = 161.994316076.$

ACTION OF JUPITER ON THE EARTH.

E	F_3	$1000 \times R_0$	$1000 \times S_0$	$1000 \times W_0$	$1000 \times R^{(n)}$	$1000 \times S^{(n)}$
0°	−0.14994500	3.5906058	−0.02485742	+0.00562843	0.0000000	−0.02528143
30	−0.11198359	3.5669877	−0.02409495	+0.13045170	+1.8097792	−0.02445007
60	−0.03628181	3.5812719	−0.01504685	+0.22107406	+3.1277000	−0.01517409
90	+0.00006601	3.6318358	−0.00001620	+0.25540271	+3.6318358	−0.00001620
120	−0.04119166	3.7069905	+0.01547114	+0.22249821	+3.1836522	+0.01534248
150	−0.12070192	3.7863426	+0.02555514	+0.12714491	+1.8660682	+0.02518930
180	−0.16035007	3.8464128	+0.02722801	−0.00750003	0.0000000	+0.02677889
210	−0.12100025	3.8690539	+0.02153453	−0.14361296	−1.9068325	+0.02122624
240	−0.04149417	3.8484336	+0.01189802	−0.24048156	−3.3051260	+0.01179908
270	+0.00005467	3.7925083	+0.00121404	−0.26976404	−3.7925083	+0.00121404
300	−0.03599894	3.7183323	−0.00934064	−0.22557067	−3.2473996	−0.00941964
330	−0.11169659	3.6453213	−0.01884026	−0.12397381	−1.8495240	−0.01911793
Σ_1	−0.46526165	22.2920469	+0.00535226	−0.02435156	−0.2411734	+0.00404529
Σ_2	−0.46526167	22.2920496	+0.00535230	−0.02435149	−0.2411816	+0.00404538

E	$1000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000\times\left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v S_0\right]$	$1000\times W_0 \cos(v+\pi)$	$1000\times W_0 \sin(v+\pi)$	$1000\times -2\frac{r}{a}R_0$
0°	−0.0497148	−3.5906058	−0.00101227	+0.00553665	− 7.0607738
30	+1.7678935	−3.0981989	−0.08531744	+0.09868428	− 7.0303590
60	+3.1124039	−1.7713812	−0.20927556	+0.07125644	− 7.1024820
90	+3.6313250	+0.0608776	−0.25043240	−0.05014132	− 7.2636717
120	+3.1675401	+1.9264211	−0.16743708	−0.14652740	− 7.4761534
150	+1.8214372	+3.3200889	−0.04173395	−0.12010043	− 7.6826719
180	−0.0544560	+3.8464128	−0.00134888	+0.00737774	− 7.8218420
210	−1.9439518	+3.3453092	−0.09208972	+0.11020057	− 7.8505000
240	−3.3167069	+1.9516989	−0.22529979	+0.08409148	− 7.7614089
270	−3.7919952	+0.0611766	−0.26614166	−0.04405982	− 7.5850167
300	−3.2561646	−1.7957525	−0.17399247	−0.14355750	− 7.3742992
330	−1.8818157	−3.1224540	−0.04265147	−0.11640599	− 7.1847533
Σ_1	−0.3970983	+0.5667933	−0.77836605	−0.12182259	−44.5969593
Σ_2	−0.3971070	+0.5667994	−0.77836664	−0.12182271	−44.5969726

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = -0.0000000000093.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
	$[de/dt]_{00}$	$= -\ 85.''760340\ m'$	n 1.9332865
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +7298.7450\ m'$	p 3.8632482
	$[dp/dt]_{00}$	$= -\ 26.316855\ m'$	n 1.4202340
	$[dq/dt]_{00}$	$= -\ 168.14734\ m'$	n 2.2256900
	$[dL/dt]_{00}$	$= -9631.7202\ m'$	n 3.9837038

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= -0.''081841849$
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +6.9652565$
	$[dp/dt]_{00}$	$= -0.025114405$
	$[dq/dt]_{00}$	$= -0.16046446$
	$[dL/dt]_{00}$	$= -9.1916336$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	-0.08182	-0.08182	-0.0818418
$e[d\pi/dt]_{00}$	$+0.11679$	$+0.11677$	$+0.1168153$
$[dp/dt]_{00}$	-0.02501	-0.02511	-0.0251144
$[dq/dt]_{00}$	-0.16041	-0.16047	-0.1604644
$[dL/dt]_{00}$	-9.1916		-9.1916336

NOTES.

The very close agreement of the sums toward the end of this computation is owing to the circularity of the two orbits and to their small mutual inclination. It is evident that a division into eight parts would have been sufficient, while the errors arising from a division into only six parts would have been almost inappreciable.

In this, as in several other cases, the divergence from the last figure of NEWCOMB's results is rather larger than was to have been expected. The values stated by NEWCOMB were computed to one more significant figure than was published to insure the accuracy of the final figure given. The uncertainty of this figure is evidently due to neglected terms in the series employed by LEVERRIER and NEWCOMB. In the present case we obtain for $[de/dt]_{00}$,

Computed from the six even points of division . . . . . $-0''.0818428$
Computed from the six odd points of division . . . . . $-0\ .0818409$,

and the difference between any two corresponding values for any other coefficient is even less than this.

The values of the coefficients which define the motion of the plane of the ecliptic are stated by HILL as follows:

$$[dp/dt]_{00} = -0.''0251149$$
$$[dq/dt]_{00} = -0.''1604628$$

ACTION OF SATURN ON THE EARTH.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	92.9909218	+14.3293908	+1.6679163	0.795517	90.703702
30	92.7594069	+12.2250618	+6.1218324	10.716773	90.705031
60	92.3168471	+ 8.1696352	+8.9277816	22.792314	90.707414
90	91.7819295	+ 3.2497638	+9.3339213	24.913194	$90.70842\bar{4}$
120	$91.298036\bar{4}$	− 1.2162762	+7.2314183	14.953684	90.707081
150	90.9947748	− 4.0318145	+3.1836393	2.898337	90.704732
180	$90.953300\bar{6}$	− 4.4424293	−1.7248166	0.850721	90.703708
210	91.1846750	− 2.3380978	−6.1787321	10.916915	90.705020
240	$91.626953\bar{4}$	+ 1.7173265	−8.9846857	23.083782	90.707394
270	92.1617302	+ 6.6371970	−9.3908217	25.217872	90.708453
300	92.6457640	+11.1032373	−7.2883175	15.189927	90.707143
330	92.9493070	+13.9187769	−3.2405388	3.002863	90.704751
Σ_1	$551.831823\bar{2}$*	+29.6608843†	−0.1707036‡	77.665945	544.236442
Σ_2	551.8318234	+29.6608873	−0.1706996	77.665954	$544.23641\bar{1}$

E	l	G	G'	G''	θ
°					° ′ ″
0	+2.001263	90.703603	2.0057342	0.0043727	8 33 39.757
30	+1.768419	$90.70370\bar{3}$	$1.834164\bar{2}$	0.0644170	8 18 56.347
60	+1.323475	90.704603	1.4944314	0.1681447	7 46 25.450
90	$+0.7875\bar{49}$	$90.70537\bar{0}$	$1.051749\bar{3}$	0.2611465	6 53 59.891
120	+0.304998	90.705257	0.5874560	0.2806341	5 36 19.643
150	+0.004086	90.704380	0.1809884	0.1765509	3 35 46.006
180	−0.036365	90.703605	0.0803976	0.1166593	2 40 11.401
210	+0.193697	90.703690	0.4578844	0.2628566	5 6 24.385
240	+0.633602	$90.7045\bar{69}$	$0.914665\bar{2}$	0.2782376	6 34 30.181
270	+1.167320	90.705348	1.3729264	0.2025015	7 33 52.413
300	+1.652663	$90.70526\bar{3}$	$1.750226\bar{0}$	0.0956818	8 11 49.887
330	+1.958598	90.704378	$1.975728\bar{1}$	0.0167564	8 31 21.103
Σ_1	+5.879636	544.226899	6.8329103	0.9437302	39 22 56.319
Σ_2	$+5.87966\bar{9}$	544.226868	6.8734407	0.9842289	40 0 20.145

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 551.8318229.$

† $6[a'^2e' - kaa'e \cos K] = +29.6608842.$

‡ $-6k'aa' \cos \varphi' \cdot e \sin K' = -0.1707000.$

ACTION OF SATURN ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00730944	0.28273527	0.18703836	7.0561506	3.4235951	5.2855436
30	0.00689441	0.28218320	0.18641769	7.0572862	3.4236026	5.2857705
60	0.00602140	0.28102171	0.18511178	7.0610565	3.4252108	5.2877346
90	0.00473960	0.27931572	0.18319348	7.0664172	3.4279697	5.2907291
120	0.00312447	0.27716505	0.18077485	7.0719166	3.4311334	5.2937173
150	0.00128428	0.27471328	0.17801715	7.0761000	3.4338677	5.2956443
180	0.00070759	0.27394462	0.17715249	7.0778799	3.4354590	5.2968496
210	0.00259230	0.27645617	0.17997757	7.0767946	3.4354874	5.2978905
240	0.00430241	0.27873368	0.18253896	7.0731165	3.4339314	5.2966961
270	0.00570043	0.28059458	0.18463152	7.0677983	3.4311901	5.2938284
300	0.00669846	0.28192252	0.18612461	7.0622487	3.4279906	5.2902830
330	0.00724349	0.28264754	0.18693973	7.0579727	3.4252033	5.2872040
Σ_1	0.02816377	1.67552285	1.09874105	2.4023686	6.1773201	1.7508241
Σ_2	0.02845451	1.67591049	1.09917714	2.4023690	6.1773208	1.7510668

E	$\log V$	J_1'	J_2	J_3	F_2
0°	5.2855175	90.7004946	+0.12827478	−0.7968027	− 8.486434
30	5.2853861	90.7506534	+0.29200243	+1.2416991	−31.148158
60	5.2867312	90.7742572	+0.41637229	+2.9535293	−45.424968
90	5.2891707	90.7991434	+0.49225830	+3.8800058	−47.491413
120	5.2920417	90.8302700	+0.45906255	+3.7728791	−36.793788
150	5.2945889	90.8046167	+0.26449000	+2.6608537	−16.198502
180	5.2961519	90.8127812	−0.06165231	+0.8418949	+ 8.775944
210	5.2963206	90.9507883	−0.39625124	−1.1966086	+31.437674
240	5.2950355	90.8893132	−0.59035199	−2.9084413	+45.714495
270	5.2926201	90.7428396	−0.56521208	−3.8349184	+47.780934
300	5.2897120	90.6423828	−0.36124653	−3.7277897	+37.083286
330	5.2871040	90.6419564	−0.09682848	−2.6157627	+16.488008
Σ_1	1.7451898	544.6494990*	−0.00954121	+0.1352696	+ 0.868535
Σ_2	1.7451904	544.6899978	−0.00954107	+0.1352689	+ 0.868543

* $\Sigma_1(J_1' - G'') = 543.7057688.$

$\Sigma_2(J_1' - G'') = 543.7057689.$

ACTION OF SATURN ON THE EARTH.

E	F_3	$1000 \times R_0$	$100000 \times S_0$	$100000 \times W_0$	$10000 \times R^{(n)}$	$100000 \times S^{(n)}$
0°	−0.3555837	0.57362783	+0.02247623	−1.5471185	0.0000000	+0.0228596
30	−1.2681621	0.57437868	−0.26276188	+2.3619011	+2.9142195	−0.2666346
60	−1.2912829	0.57674453	−0.40344654	+5.6791610	+5.0370180	−0.4068583
90	−0.3956591	0.58127800	−0.31429014	+7.5403759	+5.8127800	−0.3142901
120	+0.5220384	0.58774791	−0.09358683	+7.4052797	+5.0477180	−0.0928086
150	+0.5361282	0.59426527	+0.08130415	+5.2579217	+2.9287882	+0.0801402
180	−0.3802586	0.59796705	+0.11726800	+1.6546186	0.0000000	+0.1153337
210	−1.3248861	0.59688219	+0.07295990	−2.4035193	−2.9416851	+0.0719154
240	−1.3648579	0.59139747	+0.07709136	−5.7742147	−5.0790610	+0.0764503
270	−0.4663692	0.58408733	+0.18082475	−7.5353293	−5.8408733	+0.1808248
300	+0.4731390	0.57795107	+0.28959440	−7.2511167	−5.0475302	+0.2920434
330	+0.5221422	0.57454813	+0.25136029	−5.0525337	−2.9150792	+0.2550648
Σ_1	−2.3968057	3.50543586	+0.00939762	+0.1666094	−0.0418552	+0.0070201
Σ_2	−2.3968061	3.50543960	+0.00939707	+0.1688164	−0.0418499	+0.0070205

E	$1000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000\times\left[-R_0\cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v S_0\right]$	$1000\times W_0\cos(v+\pi)$	$1000\times W_0\sin(v+\pi)$	$-2\frac{r}{a}R_0$
0°	+0.00044952	−0.57362783	+0.002782496	−0.015218912	−0.0011280149
30	+0.28684099	−0.49762977	−0.015447198	+0.017867337	−0.0011320724
60	+0.49964759	−0.28807540	−0.053760700	+0.018305034	−0.0011438222
90	+0.58124893	+0.00346289	−0.073936353	−0.014803463	−0.0011625560
120	+0.50564826	+0.29959114	−0.055727122	−0.048767876	−0.0011853529
150	+0.29142597	+0.51791202	−0.017258560	−0.049666046	−0.0012057929
180	−0.00234536	+0.59796705	+0.002975836	−0.016276382	−0.0012159913
210	−0.29539381	+0.51865750	−0.015412218	+0.018443264	−0.0012111028
240	−0.50861524	+0.30174597	−0.054096837	+0.020191247	−0.0011927133
270	−0.58403554	+0.00617932	−0.074341448	−0.012307244	−0.0011681747
300	−0.50182267	−0.28668159	−0.055931013	−0.046147489	−0.0011462092
330	−0.28712395	−0.49766091	−0.017382540	−0.047441087	−0.0011324063
Σ_1	−0.00703790	+0.05091934	−0.213757340	−0.087914378	−0.0070121038
Σ_2	−0.00703741	+0.05092105	−0.213778317	−0.087907239	−0.0070121051

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = +0.00000000000028.$

DIFFERENTIAL COEFFICIENTS.

				log coeff.
	$[de/dt]_{00}$	$= -$	$''$ 1.5163927 m'	n 0.1808117
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +$	655.70924 m'	p 2.8167113
	$[dp/dt]_{00}$	$= -$	18.991017 m'	n 1.2785482
	$[dq/dt]_{00}$	$= -$	46.179399 m'	n 1.6644483
	$[dL/dt]_{00}$	$= -$	1514.4911 m'	n 3.1802667

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= -0.''00043305713$
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00}$	$= +0.18725991$
	$[dp/dt]_{00}$	$= -0.0054235259$
	$[dq/dt]_{00}$	$= -0.013188086$
	$[dL/dt]_{00}$	$= -0.43251400$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	$''$	$''$	$''$
$[de/dt]_{00}$	-0.00044	-0.00043	-0.00043306
$e[d\pi/dt]_{00}$	$+0.00315$	$+0.00314$	$+0.00314056$
$[dp/dt]_{00}$	-0.00542	-0.00542	-0.00542353
$[dq/dt]_{00}$	-0.01317	-0.01318	-0.01318809
$[dL/dt]_{00}$	-0.4325		-0.43251400

NOTES.

Here, as in the previous case, the approximate tests completely fail with the angle ϵ, the roots G, G', G'', and with the functions which immediately depend upon these quantities. The close agreement of the final sums shows, however, that the expansion of the perturbing function is quite rapidly convergent for this case.

The values obtained by HILL in the "*New Theory*" are:

$$[dp/dt]_{00} = -0.''0054237 \qquad [dq/dt]_{00} = -0.''0131883$$

The agreement of the final results here obtained with all other values is satisfactory.

ACTION OF URANUS ON THE EARTH.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	369.9391833	+24.383407	−17.462615	247.29194	367.49698
45	370.9299673	+34.837638	− 7.159449	41.56718	367.49556
90	370.9628887	+34.937206	+ 7.512108	45.76306	367.49557
135	370.0188613	+24.623780	+17.957661	261.51163	367.4970$\bar{8}$
180	368.6506847	+ 9.938828	+18.058344	264.45220	367.4970$\bar{6}$
225	367.6596194	− 0.515405	+ 7.755178	48.77249	367.49561
270	367.6264169	− 0.614971	− 6.916382	38.79263	367.49553
315	368.5707253	+ 9.698453	−17.361932	244.44864	367.49688
Σ_1	1477.179173$\bar{6}$*	+68.644470†	+ 1.191455‡	596.29983	1469.9851$\bar{4}$
Σ_2	1477.1791733	+68.644466	+ 1.191458	596.29994	1469.9851$\bar{3}$

E	l	G	G'	G''	θ
					° ′ ″
0°	+1.63126	367.495141	1.9739873	0.3408899	4 33 0.174
45	+2.62346	367.495250	2.6661957	0.0424235	4 55 28.953
90	+2.65637	367.495229	2.7027882	0.0460735	4 57 40.407
135	+1.7108$\bar{5}$	367.495130	2.0584810	0.3456945	4 38 13.695
180	+0.3426$\bar{9}$	367.495095	1.037945$\bar{1}$	0.6933002	3 55 55.015
225	−0.64694	367.495249	0.163776$\bar{1}$	0.8103506	2 56 52.565
270	−0.68006	367.495243	0.1303078	0.8100787	2 53 46.998
315	+0.26290	367.495069	0.9586081	0.6938970	3 50 28.866
Σ_1	+3.9502$\bar{6}$	1469.980708	5.845028$\bar{4}$	1.8903423	16 20 22.594
Σ_2	+3.9502$\bar{7}$	1469.980698	5.847060$\bar{9}$	1.8923656	16 21 4.079

ACTION OF URANUS ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00205713	0.27574316	0.17917560	6.1388849	1.2833195	3.7524062
45	0.00241045	0.27621390	0.17970508	6.1440956	1.2897058	3.758498$\bar{9}$
90	0.00244638	0.27626178	0.17975893	6.154487$\bar{3}$	1.3001369	3.768940$\bar{1}$
135	0.00213675	0.27584925	0.17929492	6.163886$\bar{8}$	1.3084161	3.7775217
180	0.00153562	0.27504825	0.17839395	6.166877$\bar{0}$	1.3097851	3.779200$\bar{9}$
225	0.00086277	0.27415148	0.17738519	6.1617903	1.3035252	3.7729672
270	0.00083285	0.27411159	0.17734031	6.151520$\bar{9}$	1.293216$\bar{5}$	3.762653$\bar{2}$
315	0.00146560	0.27495493	0.17828898	6.1419973	1.2848106	3.7542155
Σ_1	0.00687198	1.10116478	0.71466879	4.611770$\bar{0}$	5.1864579	3.063200$\bar{3}$
Σ_2	0.00687557	1.10116956	0.71467417	4.611770$\bar{0}$	5.1864577	5.063203$\bar{3}$

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee' \cos K] = 1477.1791732.$

† $4[a'^2e' - kaa'e \cos K] = +68.644468.$

‡ $+ 4k'aa' \cos \varphi' \cdot e \sin K' = +1.191454.$

ACTION OF URANUS ON THE EARTH.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	3.7519032	367.8221780	−0.84734235	+2.2088876	+301.43340
45	3.7584363	367.4766488	−0.35515557	+4.6641774	+123.58384
90	$3.768872\bar{1}$	367.4892672	+0.38057614	+4.3579828	−129.67133
135	3.7770116	367.8350259	+0.86234922	+1.4696662	−309.97871
180	3.7781780	368.1745883	+0.82112849	−2.3088335	−311.71664
225	3.7717715	368.2455011	+0.34414218	−4.7641198	−133.86711
270	3.7614579	368.2514548	−0.29842856	−4.4579225	+119.38809
315	3.7531918	368.1823028	−0.79535630	−1.5696087	+299.69549
Σ_1	5.0604111	1471.7374883*	+0.05593372	−0.1998856	− 20.56648
Σ_2	$5.060411\bar{2}$	1471.7394786	+0.05597953	−0.1998849	− 20.56649

E	F_3	$1000000 \times R_0$	$1000000 \times S_0$	$1000000 \times W_0$	$1000000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	+3.5560766	68.949325	+0.10018820	+1.2544323	0.000000	+0.10189713
45	+0.4865778	69.825413	+0.03716997	+2.6752556	+49.966575	+0.03761606
90	+0.8230258	71.523975	−0.03529126	+2.5611571	+71.523975	−0.03529126
135	+4.0393764	73.045932	−0.11453773	+0.8877061	+51.045918	−0.11319535
180	+3.8028425	73.443068	−0.14342145	−1.3776247	0.000000	−0.14105578
225	+0.5809413	72.487200	−0.06580265	−2.8156195	−50.655465	−0.06503154
270	+0.7097098	70.794418	+0.06221357	−2.5724979	−70.794418	+0.06221357
315	+3.7847601	69.352325	+0.12685775	−0.8818747	−49.628040	+0.12838021
Σ_1	+8.8916547	284.710786	−0.01631094	−0.1345332	+ 0.729557	−0.01223634
Σ_2	+8.8916556	284.710870	−0.01631266	−0.1345325	+ 0.728988	−0.01223053

E	$1000000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000000\times\left[-R_0\cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v\, S_0\right]$	$1000000 \times W_0 \cos(v+\pi)$	$1000000 \times W_0 \sin(v+\pi)$	$1000\times -2\frac{r}{a}R_0$
0°	+ 0.200376	−68.949325	−0.2256099	+1.2339773	−0.13558593
45	+50.011805	−48.728590	−2.2190515	+1.4942570	−0.13799470
90	+71.514500	+ 1.128956	−2.5113156	−0.5028130	−0.14304794
135	+51.201671	+52.095595	−0.4959295	−0.7362581	−0.14782435
180	+ 0.286843	+73.443068	−0.2477661	+1.3551611	−0.14934959
225	−50.554733	+51.949429	−2.2975222	+1.6276075	−0.14669365
270	−70.785500	+ 1.062876	−2.5379547	−0.4201590	−0.14158884
315	−49.442739	−48.631449	−0.5098809	−0.7195307	−0.13705976
Σ_1	+ 1.216219	+ 6.685575	−5.5226463	+1.6661664	−0.56957230
Σ_2	+ 1.216004	+ 6.684985	−5.5223841	+1.6660757	−0.56957246

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = -0.00000000000025.$

* $\Sigma_1(J_1' - G'') = 1469.8471460.$
$\Sigma_2(J_1' - G'') = 1469.8471130.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
	$[de/dt]_{00}$	$= +\ 0.''39395664\ m'$	$p\ 9.5954484$
$[d\chi/dt]_{00}$	$= [d\pi/dt]_{00}$	$= +129.13143\ m'$	$p\ 2.1110320$
	$[dp/dt]_{00}$	$= +\ 0.53988815\ m'$	$p\ 9.7323038$
	$[dq/dt]_{00}$	$= -\ 1.7895101\ m'$	$n\ 0.252734\bar{2}$
	$[dL/dt]_{00}$	$= -184.51950\ m'$	$n\ 2.2660422$

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00}$	$= +0.''000017278801$
$[d\chi/dt]_{00}$	$= [d\pi/dt]_{00}$	$= +0.0056636605$
	$[dp/dt]_{00}$	$= +0.000023679306$
	$[dq/dt]_{00}$	$= -0.000078487295$
	$[dL/dt]_{00}$	$= -0.0080929604$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$+0.''00002$	$+0.''00002$	$+0.''0000172788$
$e[d\pi/dt]_{00}$	$+0.00009$	$+0.00010$	$+0.0000949860$
$[dp/dt]_{00}$	$+0.00002$	$+0.00002$	$+0.0000236793$
$[dq/dt]_{00}$	-0.00008	-0.00008	-0.0000784873
$[dL/dt]_{00}$	-0.0081		-0.0080929604

NOTES.

It will be noticed that, owing to the very small mutual inclination, the approximate tests are here more exactly satisfied than even in the case of Saturn, where twelve points of division were employed. It is therefore evident that eight points are fully sufficient and that the greatest error arising from a division into only four points (which occurs with the coefficient $[d\pi/dt]_{00}$), could not amount to more than 1/20,000th of the whole.

The results obtained by HILL are:

$$[dp/dt]_{00} = +0.''0000237 \qquad [dq/dt]_{00} = -0.''0000785$$

exactly agreeing with those here given.

ACTION OF NEPTUNE ON THE EARTH.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	905.47785591	+23.74841$\bar{1}$	+24.815277	40.194861	904.17306
45	905.10315254	+ 1.12729$\bar{7}$	+28.979673	54.817494	904.1730$\bar{6}$
90	904.80486595	−17.81434$\bar{5}$	+15.920217	16.543607	904.17365
135	904.75792710	−21.98076$\bar{2}$	− 6.713038	2.941510	904.17372
180	904.98963355	− 8.93131$\bar{3}$	−25.661837	42.984089	904.17317
225	905.36405558	+13.68980$\bar{2}$	−29.826236	58.066960	904.1730$\bar{4}$
270	905.66206098	+32.63145$\bar{0}$	−16.766778	18.349806	904.1736$\bar{1}$
315	905.70928102	+36.79786$\bar{0}$	+ 5.866476	2.246398	904.17374
Σ_1	3620.93441639*	+29.634203†	− 1.693121†	118.072363	3616.6934$\bar{9}$
Σ_2	3620.93441624	+29.634196	− 1.693125	118.072362	3616.69355

E	l	G	G'	G''	θ
					° ′ ″
0°	1.23952	904.17301	1.27445$\bar{2}$	0.03488$\bar{2}$	2 10 50.926
45	0.86483$\bar{3}$	904.17299	0.93007$\bar{6}$	0.06518$\bar{6}$	1 54 4.344
90	0.56594	904.17363	0.596627	0.030667	1 30 33.476
135	0.51893	904.17372	0.525125	0.006195	1 23 20.558
180	0.75119	904.17312	0.809936	0.058696	1 46 33.999
225	1.12575$\bar{5}$	904.17297$\bar{7}$	1.180229	0.054414	2 7 3.526
270	1.42319$\bar{9}$	904.17359$\bar{9}$	1.437325	0.014120	2 17 46.322
315	1.47027	904.17374	1.471958	0.001688	2 18 49.394
Σ_1	3.97984$\bar{4}$	3616.69335$\bar{5}$	4.11834$\bar{0}$	0.13836$\bar{5}$	7 45 44.723
Σ_2	3.97977	3616.69342$\bar{2}$	4.10738$\bar{8}$	0.12748$\bar{3}$	7 43 17.822

ACTION OF NEPTUNE ON THE EARTH.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00047205	0.27363061	0.17679925	5.551378$\bar{9}$	9.912472$\bar{9}$	2.7719098
45	0.00035874	0.27347955	0.17662931	5.555572$\bar{5}$	9.9164864	2.775919$\bar{0}$
90	0.00022606	0.27330267	0.17643033	5.565826$\bar{2}$	9.926595$\bar{7}$	2.785989$\bar{9}$
135	0.00019146	0.27325655	0.17637844	5.5760491	9.936796$\bar{0}$	2.7961727
180	0.00031307	0.27341867	0.17656083	5.5803398	9.9411989	2.8006208
225	0.00044509	0.27359467	0.17675882	5.576268$\bar{6}$	9.937308$\bar{0}$	2.7967497
270	0.00052334	0.27369899	0.17687617	5.566135$\bar{5}$	9.927317$\bar{3}$	2.786753$\bar{1}$
315	0.00053136	0.27370968	0.17688820	5.5557902	9.9169945	2.7764257
Σ_1	0.00153452	1.09405094	0.70666658	2.263680$\bar{3}$	9.707584$\bar{7}$	1.1452735
Σ_2	0.00152666	1.09404045	0.70665477	2.2636803	9.7075848	1.145267$\bar{1}$

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee'\cos K] = 3620.93441628.$

† $4[a'^2e' - kaa'e\cos K] = +29.634198.$

‡ $-4k'aa'\cos\varphi'.\ e\sin K' = -1.693118.$

ACTION OF NEPTUNE ON THE EARTH.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	2.7718889	903.9916758	+0.58828709	--13.723633	−190.54625
45	2.7758798	904.1719863	+0.01502688	+ 7.632175	−222.52292
90	2.7859714	903.5306465	−0.23467770	+24.659009	−122.24477
135	2.7961690	903.3686400	+0.17398234	+27.382788	+ 51.54664
180	2.8005856	904.0154903	+0.15962945	+14.207956	+197.04666
225	2.7967170	904.1717548	−0.46647484	− 7.147853	+229.02334
270	$2.786744\bar{6}$	903.5394237	−0.52719220	−24.174694	+128.74518
315	$2.776424\bar{7}$	903.3535982	+0.26287497	−26.898466	− 45.04625
Σ_1	$1.145190\bar{5}$	3615.0772363*	−0.01395336	+ 0.968638	+ 13.00082
Σ_2	$1.145190\bar{5}$	3615.0659793	−0.01459065	+ 0.968644	+ 13.00081

E	F_3	$100000 \times R_0$	$10000000 \times S_0$	$100000 \times W_0$	$100000 \times R^{(n)}$	$10000000 \times S^{(n)}$
0°	− 5.0649535	1.7793453	+0.19215285	−0.08120439	0.0000000	+0.19543047
45	− 6.5852856	1.7972262	−0.17462602	+0.04549983	+1.2860826	−0.17672179
90	− 1.8342882	1.8362237	−0.24659977	+0.15062685	+1.8362237	−0.24659977
135	-- 0.4453113	1.8790643	+0.15337661	+0.17125249	+1.3131267	+0.15157902
180	− 5.4164244	1.9016607	+0.27295016	+0.08971979	0.0000000	+0.26844799
225	− 6.9853355	1.8852158	−0.09387083	−0.04482073	−1.3174254	−0.09277065
270	− 2.0485736	1.8383392	−0.21373143	−0.14796412	−1.8383392	−0.21373143
315	− 0.3483062	1.7940628	+0.11988971	−0.16075330	−1.2838186	+0.12132856
Σ_1	−14.3642397	7.3555689	+0.00477181	+0.01117813	−0.0021155	+0.00354726
Σ_2	−14.3642386	7.3555691	+0.00476947	+0.01117829	−0.0020347	+0.00341514

E	$100000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$	$100000 \times \left[-R_0 \cos v + \left(\frac{r}{a} \sec^2 \varphi + 1 \right) \sin v S_0 \right]$	$100000 \times W_0 \cos (v+\pi)$	$100000 \times W_0 \sin (v+\pi)$	$100000 \times -2\frac{r}{a} R_0$
0°	+0.0038431	−1.7793453	+0.01460463	−0.07988026	− 3.4990076
45	+1.2834469	−1.2580636	−0.03774087	+0.02541381	− 3.5518258
90	+1.8360068	+0.0258636	−0.14769558	−0.02957146	− 3.6724475
135	+1.3107603	+1.3464279	−0.09567261	−0.14203579	− 3.8026961
180	−0.0054590	+1.9016607	+0.01613612	−0.08825681	− 3.8671076
225	−1.3159048	+1.3499919	−0.03657334	+0.02590924	− 3.8151452
270	−1.8380449	+0.0351057	−0.14597725	−0.02416657	− 3.6766784
315	−1.2819528	−1.2550750	−0.09294409	−0.13116027	− 3.5455738
Σ_1	−0.0036540	+0.1832847	−0.26293208	−0.22187510	−14.7152411
Σ_2	−0.0036504	+0.1832812	−0.26293091	−0.22187301	−14.7152409

$\sin \varphi \cdot \frac{1}{2} A_1^{(a)} + \cos \varphi \cdot B_0^{(c)} = +0.0000000000000014.$

* $\Sigma_1(J_1' - G'') = 3614.9388718.$
$\Sigma_2(J_1' - G'') = 3614.9384968.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
	$[de/dt]_{00} = -0''.011831221$	m'	n 8.0730296
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00} = +35.402545$	m'	p 1.5490345
	$[dp/dt]_{00} = -0.71895833$	m'	n 9.8567037
	$[dq/dt]_{00} = -0.85200049$	m'	n 9.9304399
	$[dL/dt]_{00} = -47.671428$	m'	n 1.6782582

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

	$[de/dt]_{00} = -0''.00000060056972$	
$[d\chi/dt]_{00} =$	$[d\pi/dt]_{00} = +0.0017970838$	
	$[dp/dt]_{00} = -0.000036495344$	
	$[dq/dt]_{00} = -0.000043248757$	
	$[dL/dt]_{00} = -0.0024198698$	

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$0''.00000$	$0''.00000$	$-0''.00000060057$
$e[d\pi/dt]_{00}$	$+0.00003$	$+0.00003$	$+0.00003013915$
$[dp/dt]_{00}$	-0.00004	-0.00004	-0.00003649534
$[dq/dt]_{00}$	-0.00004	-0.00004	-0.00004324876

NOTES.

The mutual inclination is here nearly twice as great as in the case of Uranus, and yet the convergence of the perturbing function is more rapid because the eccentricity of Neptune is so much smaller than that of Uranus. Hence, although the sums of ϵ, G, G', G'', etc., are in great disagreement, the final sums from which the differential coefficients are obtained are almost identical. The greatest error arising from a division into only four parts occurs with the coefficient $[dp/dt]_{00}$ and amounts to but $0''.000000000002$

The results of HILL are:

$$[dp/dt]_{00} = -0''.0000366 \qquad [dq/dt]_{00} = -0''.0000435$$

MARS.

Action of Mercury on Mars.

E	A	$B \sin \epsilon$	$B \cos \epsilon$	$1000 \times g$	h
0°	2.01395126	−0.51051059	−0.07779114	1.6508943	1.87197670
30	2.19162773	−0.49427471	+0.22547307	1.5475600	2.04795479
60	2.44453125	−0.33152771	+0.47957033	0.6962245	2.29696077
90	2.71228886	+0.06587754	+0.61641571	0.0274906	2.56269605
120	2.92685114	+0.23149508	+0.59934132	0.3394636	2.77862654
150	3.02703005	+0.48090911	+0.43292228	1.4649936	2.88207295
180	2.97859089	+0.61553465	+0.16175058	2.4000193	2.83609064
210	2.79081655	+0.59929870	−0.14151362	2.2750785	2.64795925
240	2.51771727	+0.43655177	−0.39561101	1.2072061	2.37176681
270	2.23986183	+0.17090155	−0.53245622	0.1850128	2.09074126
300	2.03539750	−0.12647099	−0.51538178	0.1013191	1.88662892
330	1.95541421	−0.37588517	−0.34896288	0.8949940	1.81063180
Σ_1	14.91703931*	+0.31507221†	+0.25187830‡	6.9351269	14.04205038
Σ_2	14.91703923	+0.31507194	+0.25187834	6.9351295	14.04205609

E	l	G	G'	G''	θ
°					° ′ ″
0	0.13564010	1.87146851	0.14234545	0.00619717	16 20 8.69
30	0.13733849	2.04755912	0.14301881	0.00528466	15 35 30.16
60	0.14123602	2.29682015	0.14348918	0.00211253	14 34 33.48
90	0.14325835	2.56269162	0.14333762	0.00007484	13 41 0.67
120	0.14189014	2.77858020	0.14279207	0.00085559	13 8 25.31
150	0.13862265	2.88188764	0.14237834	0.00357038	12 59 49.56
180	0.13616579	2.83577714	0.14242175	0.00594246	13 12 30.22
210	0.13652285	2.64761705	0.14287917	0.00601413	13 42 7.60
240	0.13961601	2.37153873	0.14339402	0.00354993	14 24 9.49
270	0.14278611	2.09069583	0.14344844	0.00061690	15 12 58.81
300	0.14243412	1.88659813	0.14284089	0.00037598	15 59 29.51
330	0.13844795	1.81033610	0.14221981	0.00347617	16 27 51.54
Σ_1	0.83698218	14.04078286	0.85728335	0.01903366	87 39 16.70
Σ_2	0.83697639	14.04078736	0.85728218	0.01903708	87 39 18.34

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 14.91703924.$

† $-6k'aa' \cos \varphi' \cdot e \sin K' = +0.31507212.$

‡ $6[a'^2e' - kaa'e \cos K] = +0.25187831.$

ACTION OF MERCURY ON MARS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.02698253	0.30881667	0.21633345	0.0802043̄	9.8417843	0.0229194
30	0.02453876	0.30558607	0.21270775	0.0315423̄	9.7124165	9.9318941
60	0.02139874	0.30143126	0.20804354	9.9863199	9.5646986̄	9.8328372̄
90	0.01882471	0.29802213	0.20421541	9.9544525	9.4350567̄	9.7499589
120	0.01734193	0.29605696	0.20200830	9.9396879	9.3478315̄	9.6977395
150	0.01696133	0.29555239	0.20144156	9.9427992̄	9.3179221̄	9.6840260̄
180	0.01752419	0.29629857	0.20227967	9.9632970̄	9.3524331	9.7119954
210	0.01887658	0.29809086	0.20429260	9.9992756	9.4496853̄	9.7797276
240	0.02088535	0.30075154	0.20728036	0.0456467̄	9.5950386	9.8772472
270	0.02335208	0.30401640	0.21094579	0.0914150	9.7545935	9.9819419̄
300	0.02583676	0.30730232	0.21463400	0.1193985̄	9.8751689̄	0.0582665
330	0.02741722	0.30939106	0.21697800	0.1150661	9.9072726̄	0.0734518
Σ_1	0.12996950	1.81065732	1.25057932	0.1345541	7.5769549̄	9.2010052̄
Σ_2	0.12997068	1.81065891	1.25058111	0.1345506	7.5769465	9.2010002

E	log V	J_1'	J_2	J_3	F_2
0°	0.0211422	0.148654076	−0.10722959	−0.015479687	+0.015330048
30	9.9305073	0.148321182	−0.10335718	−0.012048361	+0.014842503
60	9.8323421̄	0.145601872	−0.06932482	−0.006505466	+0.009955397
90	9.7499432	0.143412552	−0.01386724	−0.000353153	+0.001978227
120	9.6975735̄	0.143648964	+0.04858051	+0.004751607	−0.006951532
150	9.6833581̄	0.145958760	+0.10133761	+0.007449454	−0.014441148
180	9.7108663̄	0.148399366	+0.13001805	+0.007034457	−0.018483798
210	9.7785043	0.148942861	+0.12663289	+0.003626261	−0.017996253
240	9.8764415̄	0.146960494	+0.09193479	−0.001870362	−0.013109151
270	9.9817831	0.144077345	−0.03534552	−0.007999542	−0.005131979
300	0.0581594	0.143355345	−0.02759166	−0.013127432	+0.003797779
330	0.0724209	0.145910972	0.07970449	−0.015871553	+0.011287401
Σ_1	9.1965248	0.876620117*	+0.06638728	−0.025196883	−0.009461257
Σ_2	9.1965168	0.876623672	+0.06638711	−0.025196894	−0.009461249

* $\Sigma_1(J_1' - G'') = 0.857586462.$
$\Sigma_2(J_1' - G'') = 0.857586592.$

ACTION OF MERCURY ON MARS.

E	$1000 \times F_3$	R_0	S_0	W_0	$R^{(n)}$	$S^{(n)}$
0°	+0.3749543	−1.1968194	−0.10192940	−0.015991438	0.00000000	−0.07377742
30	+0.9451901	−1.0712026	−0.08041942	−0.009779348	−0.38240323	−0.05741705
60	+0.8478648	−0.9676664	−0.04346887	−0.004110828	−0.57689880	−0.02992414
90	+0.1645035	−0.9003960	−0.00725842	−0.000153771	−0.5909306$\bar{8}$	−0.00476371
120	−0.4045215	−0.8699380	+0.02266377	+0.002278066	−0.47241848	+0.01421151
150	−0.2449197	−0.8749740	+0.04587694	+0.003542283	−0.26566469	+0.02785885
180	+0.5450969	−0.9160775	+0.06265312	+0.003737622	0.00000000	+0.03761136
210	+1.2365773	−0.9949389	+0.07097316	+0.002525788	+0.30208776	+0.04309857
240	+1.1824210	−1.1083446	+0.06401101	−0.000941853	+0.60188486	+0.04013864
270	+0.4525829	−1.2337308	+0.03097697	−0.007413721	+0.8096985$\bar{9}$	+0.02033021
300	−0.2401087	−1.3158841	−0.02869644	−0.015188691	+0.78449755	−0.01975474
330	−0.2482279	−1.2994033	−0.08505077	−0.018952193	+0.46386745	−0.06072369
Σ_1	+2.3057068	−6.3747320	−0.02476681	−0.030217122	+0.33706513	−0.03149479
Σ_2	+2.3057062	6.3746456	−0.02490154	−0.030230962	+0.33665520	−0.03161682

E	$R_0 \sin v + (\cos v + \cos E)S_0$	$-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	−0.2038588	+1.1968194	−0.004111709	+0.015453801	2.1703888$\bar{8}$
30	−0.7173754	+0.8165787	−0.007231890	+0.006582964	1.9693580$\bar{0}$
60	−0.9154636	+0.3357086	−0.004043881	+0.000738872	1.8450804
90	−0.8957943	−0.0984951	−0.000144266	−0.000053225	1.8007921
120	−0.7408612	−0.4547261	+0.001481632	+0.001730419	1.8210138
150	−0.4834776	−0.7324563	+0.000768358	+0.003457946	1.8912954
180	−0.1253062	−0.9160795	−0.000961015	+0.003611962	2.0030408
210	+0.3338218	−0.9514367	−0.001700736	+0.001867379	2.150595$\bar{1}$
240	+0.8448003	−0.7366597	+0.000887111	−0.000316419	2.3200626
270	+1.2254640	−0.1770222	+0.007311027	+0.001229687	2.4674617$\bar{7}$
300	+1.1635310	+0.6123066	+0.011609114	+0.009794123	2.5090379
330	+0.5585553	+1.1811278	+0.005822238	+0.018035720	2.3888945
Σ_1	+0.0228415	+0.0373693	+0.004861252	+0.031012758	12.6686243$\bar{3}$
Σ_2	+0.0211938	+0.0382962	+0.004824731	+0.031120471	12.6683967$\bar{7}$

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = +0''.000000008.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 2517.5250	m'	p 3.4009738
$[d\chi/dt]_{00}$	$= +$ 46380.761	m'	p 4.6663379
$[di/dt]_{00}$	$= +$ 558.61256	m'	p 2.7471107
$[d\Omega/dt]_{00}$	$= +$ 110961.28	m'	p 5.0451714
$[d\pi/dt]_{00}$	$= +$ 46438.628	m'	p 4.6668794
$[dL/dt]_{00}$	$= +1455134.1$	m'	p 6.1629030

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= +0.00033567000$
$[d\chi/dt]_{00}$	$= +0.0061841007$
$[di/dt]_{00}$	$= +0.000074481672$
$[d\Omega/dt]_{00}$	$= +0.014794833$
$[d\pi/dt]_{00}$	$= +0.0061918174$
$[dL/dt]_{00}$	$= +0.19401785$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	+0.00036	+0.00033	+0.0003357
$e[d\pi/dt]_{00}$	+0.00058	+0.00057	+0.0005775
$[di/dt]_{00}$	+0.00008	+0.00007	+0.0000745
$\sin i\,[d\Omega/dt]_{00}$	+0.00047	+0.00048	+0.0004778

NOTES.

On account of the large eccentricities of both orbits and the high mutual inclination, the coefficients of the expansion diminish but slowly. Thus the combined effect of all terms from the 6th to the 11th orders is 1/30th of the whole with $[de/dt]_{00}$, 1/90th with $[d\pi/dt]_{00}$, and 1/200th with $[di/dt]_{00}$. Yet all of the variations are very small on account of the smallness of the mass of Mercury. A comparison with the computation of Mars on Mercury renders it evident that a division into twelve parts is sufficient and that terms of orders above the eleventh are wholly inappreciable.

ACTION OF VENUS ON MARS.

E	A	B cos ϵ	B sin ϵ	$1000 \times g$	h
0°	2.41946745	−0.9101348	−0.4038413	0.003995803	1.8967820
30	2.47732532	−0.5530947	−0.8457642	0.017525935	1.9549495
60	2.63301427	−0.0217107	−1.0499345	0.027008907	2.1104659
90	2.85220984	+0.5416339	−0.9616453	0.022657521	2.3292431
120	3.07987480	+0.9859909	−0.6045536	0.008954722	$2.556675\bar{9}$
150	3.25131035	+1.1922952	−0.0743414	0.000135408	2.7282304
180	3.31318850	+1.1052676	+0.4869209	0.005808973	2.7904410
210	3.24523275	+0.7482272	+0.9288436	0.021138190	2.7227529
240	3.06934808	+0.2168441	+1.1330145	0.031452377	2.5468467
270	2.84005464	−0.3465004	+1.0447250	0.026741552	2.3172324
300	2.62248756	−0.7908574	+0.6876330	0.011585000	$2.099329\bar{4}$
330	2.47124772	−0.9971620	+0.1574211	0.000607167	1.9481304
Σ_1	17.13738066*	+0.5853997†	+0.2492390‡	0.088805782	14.0005408
Σ_2	17.13738062	+0.5854002	+0.2492388	0.088805773	14.0005387

E	l	G	G'	$10000 \times G''$	θ		
					°	′	″
0°	$0.522661\bar{0}$	1.8967805	0.5226665	0.0403053	31	39	49.77
30	0.5223513	$1.954943\bar{3}$	$0.522374\bar{8}$	0.1716188	31	7	34.86
60	0.5225239	$2.110457\bar{9}$	0.5225564	0.2449047	29	50	30.35
90	0.5229423	2.3292377	0.5229663	0.1860051	28	17	2.93
120	$0.523174\bar{5}$	2.5566741	0.5231829	0.0669458	26	53	44.43
150	0.5230555	2.7282304	$0.523055\bar{5}$	0.0009489	25	58	2.26
180	0.5227230	2.7904401	$0.522727\bar{9}$	0.0398246	25	38	46.94
210	0.5224554	2.7227494	0.5224737	0.1485921	25	58	48.56
240	$0.522476\bar{9}$	2.5468406	0.5225067	0.2363523	26	55	59.83
270	0.5227977	2.3172260	0.5228262	0.2207297	28	21	35.72
300	0.5231337	$2.099325\bar{9}$	$0.523147\bar{8}$	0.1054853	29	56	49.75
330	0.5230928	1.9481302	0.5230936	$0.005958\bar{2}$	31	12	36.97
Σ_1	$3.136692\bar{9}$	14.0005190	3.1367881	0.7338180	170	55	41.07
Σ_2	3.1366950	$14.000517\bar{0}$	3.1367900	$0.733852\bar{8}$	170	55	41.30

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = 17.13738065.$

† $6[a'^2e' - kaa'e \cos K] = +0.5854002.$

‡ $-6k'aa' \cos \varphi' \cdot e \sin K' = +0.2492389.$

ACTION OF VENUS ON MARS.

E	log K_0	log L_0'	log N_0	log N	log P	log Q
0°	0.10710693	0.41334627	0.33316554	0.1537292	0.0110396	0.2088768
30	0.10323050	0.40834940	0.32760279	0.1420610	9.9681345	0.1785258
60	0.09431348	0.39683240	0.31477271	0.1149515	9.8630204	0.1053425
90	0.08414014	0.38365383	0.30007678	0.0820056	9.7312249	0.0148651
120	0.07564158	0.37216263	0.28775224	0.0524079	9.6096675	9.9324836
150	0.07024983	0.36559243	0.27991042	0.0325881	9.5264184	9.8766175
180	0.06843912	0.36323215	0.27727293	0.0260737	9.4979593	9.8576733
210	0.07032296	0.36568773	0.28001691	0.0339679	9.5296356	9.8789748
240	0.07586487	0.37290310	0.28807661	0.0551373	9.6160289	9.9372082
270	0.08461922	0.38427537	0.30077024	0.0858518	9.7401822	0.0216495
300	0.09502686	0.39775494	0.31580088	0.1191144	9.8727052	0.1128332
330	0.10383003	0.40912260	0.32846370	0.1449403	9.9748268	0.1837860
Σ_1	0.51639284	2.31668149	1.81684091	0.5214139	8.4704208	0.1544175
Σ_2	0.51639268	2.31668136	1.81684084	0.5214145	8.4704222	0.1544186

E	log V	J_1'	J_2	J_3	F_2
0°	0.2088756	0.52281610	−0.0030557479	−0.012760938	+0.0014450479
30	0.1785212	0.52261080	−0.0058421642	−0.016264386	+0.0030263615
60	0.1053364	0.52273704	−0.0069459413	−0.015093265	+0.0037569353
90	0.0148609	0.52303324	−0.0063117580	−0.009561780	+0.0034410135
120	9.9324822	0.52319006	−0.0041023613	−0.001152260	+0.0021632475
150	9.8766174	0.5230798	−0.0007324438	+0.007882145	+0.0002660128
180	9.8576725	0.52281605	+0.0030484592	+0.015121070	−0.0017423280
210	9.8789719	0.52262308	+0.0062562314	+0.018625039	−0.0033236408
240	9.9372033	0.5226684	+0.0079298727	+0.017454980	−0.0040542164
270	0.0216445	0.52292962	+0.0074476695	+0.011924014	−0.0037382940
300	0.1128306	0.52316808	+0.0048298806	+0.003513972	−0.0024605273
330	0.1837858	0.52311928	+0.0008867096	−0.005521489	−0.0005632929
Σ_1	0.1544005	3.13739580*	+0.0017041620	+0.007083559	−0.0008918410
Σ_2	0.1544017	3.13739582	+0.0017042445	+0.007083543	−0.0008918399

* $\Sigma_1(J_1' - G'') = 3.1373224\bar{2}$.
$\Sigma_2(J_1' - G'') = 3.13732243$.

ACTION OF VENUS ON MARS.

E	$100000 \times F_3$	R_0	$1000 \times S_0$	$1000 \times W_0$	$R^{(a)}$	$S^{(a)}$
0°	− 2.705324	−1.4244790	−3.460778	−20.67006	0.0000000	−0.002504942
30	− 0.803003	−1.3866027	−6.000157	−24.54092	−0.4949963	−0.004283931
60	+ 5.500746	−1.3028006	−6.111878	−19.19609	−0.7766973	−0.004207440
90	+ 9.816975	−1.2077652	−4.678320	− 9.84176	−0.7926572	−0.003070385
120	+ 7.633806	−1.1282518	−2.631099	− 0.95528	−0.6126954	−0.001649853
150	+ 0.880725	−1.0779056	−0.461908	+ 5.93579	−0.3272800	−0.000280495
180	− 3.932916	−1.0618130	+1.648220	+10.88332	0.0000000	+0.000989445
210	− 2.161955	−1.0812437	+3.609363	+14.08781	+0.3282934	+0.002191792
240	+ 4.374565	−1.1352348	+5.187600	+15.12313	+0.6164876	+0.003252928
270	+ 9.225323	−1.2184712	+5.773047	+12.58406	+0.7996835	+0.003788855
300	+ 7.735212	−1.3155488	+4.427365	+ 4.61417	+0.7842976	+0.003047815
330	+ 1.648024	−1.3961376	+0.822269	− 8.41473	+0.4984001	+0.000587075
Σ_1	+18.606089	−7.3681280	−0.940570	−10.20081	+0.0113925	−0.001072047
Σ_2	+18.606089	−7.3681260	−0.935706	−10.18975	+0.0114435	−0.001067089

E	$R_0 \sin v$ $+(\cos v + \cos E)S_0$	$-R_0 \cos v$ $+\left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	−0.0069216	+1.4244790	−0.005314674	+0.019975127	2.5832417
30	−0.7611745	+1.1593980	−0.018148167	+0.016519714	2.5492068
60	−1.1839523	+0.5449664	−0.018883467	+0.003450266	2.4840909
90	−1.2020643	−0.1220026	−0.009233404	−0.003406539	2.4155306
120	−0.9266826	−0.6439879	−0.000621304	−0.000725630	2.3617340
150	−0.4956901	−0.9571936	+0.001287534	+0.005794466	2.3299411
180	−0.0032964	−1.0618130	−0.002798312	+0.010517416	2.3216920
210	+0.4917082	−0.9631868	−0.009486007	+0.010415471	2.3371562
240	+0.9297081	−0.6522758	−0.014244152	+0.005080676	2.3763511
270	+1.2126214	−0.1251906	−0.012409750	−0.002087273	2.4369424
300	+1.1939212	+0.5533939	−0.003526734	−0.002975358	2.5083988
330	+0.7575012	+1.1728178	+0.002585060	+0.008007817	2.5667361
Σ_1	+0.0027764	+0.1647626	−0.045388643	+0.035322497	14.6355085
Σ_2	+0.0029019	+0.1646422	−0.045404734	+0.035243656	14.6355132

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(c)} = 0.''0000000073.$

Differential Coefficients.

$$
\begin{aligned}
[de/dt]_{00} &= + \quad 324.6318'' \, m' \quad p\ 2.511391\bar{1} \\
[d\chi/dt]_{00} &= + 201915.56 \quad m' \quad p\ 5.3051698 \\
[di/dt]_{00} &= - \quad 5236.2608 \, m' \quad n\ 3.719021\bar{3} \\
[d\Omega/dt]_{00} &= + 126021.28 \quad m' \quad p\ 5.1004439 \\
[d\pi/dt]_{00} &= + 201981.28 \quad m' \quad p\ 5.305311\bar{2} \\
[dL/dt]_{00} &= +1681713.6 \quad m' \quad p\ 6.2257520
\end{aligned}
$$

Final Values Corresponding to the Above Value of m'.

$$
\begin{aligned}
[de/dt]_{00} &= +0.0007954049'' \\
[d\chi/dt]_{00} &= +0.49472856 \\
[di/dt]_{00} &= -0.012829757 \\
[d\Omega/dt]_{00} &= +0.30877426 \\
[d\pi/dt]_{00} &= +0.49488961 \\
[dL/dt]_{00} &= +4.1204933
\end{aligned}
$$

Comparison with Other Results.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	+0.00080″	+0.00079″	+0.000795″
$e[d\pi/dt]_{00}$	+0.04618	+0.04614	+0.0461574
$[di/dt]_{00}$	−0.01280	−0.01284	−0.012830
$\sin i\,[d\Omega/dt]_{00}$	+0.00993	+0.00998	+0.009972
$[dL/dt]_{00}$	+4.117		+4.120493

Notes.

The very close agreement of the sums of the functions near the beginning of the computation is caused by the great circularity of the orbit of Venus. The discrepancies increase however as the work proceeds because of the high eccentricity of Mars and the rather large mutual inclination. All terms from the 6th to the 11th orders, inclusive, produce an effect equal to 1/30th of the whole in the very small coefficient $[de/dt]_{00}$, and 1/1000th of the whole in $[d\Omega/dt]_{00}$. Yet it is evident that terms of the twelfth and higher orders are wholly inappreciable.

ACTION OF THE EARTH ON MARS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	$1000\times g$	h
0°	2.88085183	−0.8153552	−1.1018750	0.34149937	1.8833414
30	2.95824702	−0.0872040	−1.3962265	0.54832354	1.9597848
60	3.13279096	+0.6917421	−1.2860904	0.46523043	2.133265$\bar{1}$
90	3.36510700	+1.3127653	−0.8009781	0.18045398	2.3654511
120	3.59664230	+1.6094626	−0.0708746	0.00141289	2.5975998
150	3.76166099	+1.5023341	+0.7085891	0.14122581	2.7632686
180	3.80855449	+1.0200854	+1.3285566	0.49646115	2.8103218
210	3.72106141	+0.2919341	+1.6229077	0.74082034	2.7223893
240	3.52632175	−0.4870119	+1.5127725	0.64368353	2.526938$\bar{6}$
270	3.28390784	−1.1080347	+1.0276597	0.29704612	2.284191$\bar{0}$
300	3.06247042	−1.4047322	+0.2975561	0.02490366	2.0633533
330	2.91764743	−1.2976042	−0.4819076	0.06532098	1.9197321
Σ_1	20.00763175*	+0.6141908†	+0.6800452‡	1.97319103	14.0148199
Σ_2	20.00763169	+0.6141906	+0.6800443	1.97319077	14.014816$\bar{9}$

E	l	G	G'	G''	θ
°					° ′ ″
0	0.9972292	1.8831367	0.9976157	0.00018178	46 42 32.02
30	0.9981810	1.9594937	0.9987522	0.00028018	45 33 35.1$\bar{5}$
60	0.999244$\bar{7}$	2.1330727	0.9996552	0.00021818	43 12 19.69
90	0.999374$\bar{7}$	2.365395$\bar{3}$	0.9995068	0.00007633	40 32 44.90
120	0.9987612	2.597599$\bar{5}$	0.9987622	0.00000054	38 19 18.09
150	0.9981111	2.763239$\bar{7}$	0.9981913	0.00005120	36 56 40.43
180	0.9979514	2.8102243	0.998225$\bar{9}$	0.00017698	36 35 10.16
210	0.9983909	2.7222314	0.9988211	0.00027246	37 17 7.3$\bar{7}$
240	0.999102$\bar{0}$	2.5267718	0.9995236	0.00025487	38 58 33.58
270	0.9994356	2.2840897	0.999667$\bar{1}$	0.00013009	41 25 15.69
300	0.9988359	2.0633420	0.9988593	0.00001208	44 5 20.00
330	0.9976341	1.9196952	0.9977050	0.00003411	46 7 50.4$\bar{5}$
Σ_1	5.9911243	14.014147$\bar{0}$	5.992641$\bar{9}$	0.00084443	247 53 13.54
Σ_2	5.991127$\bar{4}$	14.0141449	5.992643$\bar{5}$	0.00084437	247 53 13.9$\bar{8}$

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 20.00763172.$

† $6[a'^2e' - kaa'e\cos K] = +0.6141907.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = +0.6800448.$

ACTION OF THE EARTH ON MARS.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.25724275	0.60257672	0.54204483	0.3085062	0.3612353	0.5756272
30	0.24243067	0.58425966	0.52198206	$0.279659\bar{0}$	0.2795065	$0.509435\bar{0}$
60	0.21406633	0.54897788	0.48324184	0.2277018	0.1185796	0.3818936
90	0.18497596	0.51250382	0.44306081	$0.172790\bar{7}$	$9.937459\bar{0}$	0.2419337
120	0.16284593	0.48455413	0.41218099	$0.129268\bar{5}$	$9.784678\bar{0}$	$0.126877\bar{2}$
150	0.15007482	0.46834273	0.39423491	0.1040949	9.6895845	$0.056903\bar{3}$
180	0.14686278	0.46425584	0.38970669	0.0998549	$9.666574\bar{2}$	0.0407933
210	0.15317152	0.47227918	0.39859494	$0.116878\bar{7}$	$9.719220\bar{9}$	0.0805051
240	0.16915889	0.49254542	0.42101792	0.1535252	9.8408510	0.1719333
270	0.19422097	0.52412777	0.45588060	0.2048055	0.0114576	$0.301948\bar{3}$
300	0.22440852	0.56187400	0.49741669	0.2597585	0.1924850	$0.442601\bar{5}$
330	0.24970710	0.59326698	0.53185208	$0.300384\bar{5}$	$0.327171\bar{5}$	0.5489966
Σ_1	1.17458520	3.15478399	2.74560896	$1.178615\bar{1}$	9.9644030	1.7397260
Σ_2	1.17458104	3.15478014	2.74560540	1.1786131	$9.964399\bar{9}$	$1.739721\bar{9}$

E	$\log V$	J_1'	J_2	J_3	F_2
0°	0.5755790	0.99892684	−0.018227992	+0.028809050	+0.018467466
30	0.5093633	0.99952648	−0.022898545	+0.020515873	+0.023400805
60	0.3818419	0.99990323	−0.021379458	+0.006120062	+0.021554920
90	$0.241917\bar{3}$	0.99967015	−0.013761084	−0.010518960	+0.013424423
120	$0.126877\bar{1}$	0.99911770	−0.001690224	−0.024941756	+0.001187862
150	0.0568937	0.99877639	+0.011675876	−0.033284760	−0.011875978
180	0.0407608	0.99892204	+0.022549333	−0.033314577	−0.022266656
210	0.0804536	0.99942130	+0.027739796	−0.025024224	−0.027199994
240	$0.171881\bar{7}$	0.99985593	+0.025696938	−0.010634071	−0.025354120
270	0.3019193	0.99982014	+0.017055202	+0.006002126	−0.017223615
300	0.4425985	0.99929729	+0.004464507	+0.020427743	−0.004987052
330	0.5489877	0.99880864	−0.008398230	+0.028776417	+0.008076789
Σ_1	1.7395389	5.99602303*	+0.011413104	−0.013533549	−0.011397580
Σ_2	$1.739534\bar{9}$	5.99602310	+0.011413015	−0.013533528	−0.011397570

* $\Sigma_1(J_1' - G'') = 5.99517860.$
$\Sigma_2(J_1' - G'') = 5.99517873.$

ACTION OF THE EARTH ON MARS.

E	$1000 \times F_3$	R_0	S_0	W_0	$R^{(n)}$	$S^{(n)}$
0°	−0.13911285	−2.0302096	−0.026171976	+0.10810004	0.0000000	−0.018943511
30	−0.51397488	−1.9019956	−0.029451014	+0.06531256	−0.6789837	−0.021027135
60	−0.65316258	−1.6889696	−0.023181583	+0.01388518	−1.0069217	−0.015958290
90	−0.40694037	−1.4884239	−0.012395858	−0.01871310	−0.9768538	−0.008135411
120	−0.02612649	−1.3462165	−0.001540204	−0.03342035	−0.7310608	−0.000965798
150	+0.08995598	−1.2702099	+0.007499119	−0.03789964	−0.3856685	+0.004553852
180	−0.20223795	−1.2578222	+0.014435140	−0.03668660	0.0000000	+0.008665574
210	−0.63957154	−1.3082364	+0.019136251	−0.03045222	+0.3972143	+0.011620521
240	−0.80757787	−1.4237326	+0.020598419	−0.01635704	+0.7731558	+0.012916412
270	−0.54879816	−1.6023546	+0.016496265	+0.01146539	+1.0516264	+0.010826511
300	−0.11741630	−1.8174946	+0.004601712	+0.05641742	+1.0835451	+0.003167836
330	+0.07369507	−1.9931947	−0.012572900	+0.10202141	+0.7115406	−0.008976672
Σ_1	−1.94563404	−9.5644451	−0.011258492	+0.09193865	+0.1187184	−0.011117777
Σ_2	−1.94563390	−9.5644151	−0.011288137	+0.09173440	+0.1188753	−0.011138334

E	$R_0 \sin v + (\cos v + \cos E)S_0$	$-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$W_0 \cos u$	$W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	−0.0523440	+2.0302096	+0.027794619	−0.10446568	3.6817114
30	−1.0803151	+1.5681913	+0.048299056	−0.04396512	3.4967331
60	−1.5490313	+0.6794308	+0.013659050	−0.00249569	3.2204119
90	−1.4807799	−0.1636143	−0.017556368	−0.00647718	2.9768478
120	−1.1074128	−0.7656902	−0.021736270	−0.02538610	2.8179922
150	−0.5982290	−1.1202178	−0.008220822	−0.03699731	2.7456152
180	−0.0288703	−1.2578222	+0.009432839	−0.03545319	2.7502731
210	+0.5690362	−1.1796147	+0.020504958	−0.02251409	2.8278111
240	+1.1509410	−0.8419068	+0.015406346	−0.00549521	2.9802545
270	+1.5938315	−0.1824413	−0.011306572	−0.00190172	3.2047092
300	+1.6480563	+0.7672282	−0.043121307	−0.03637964	3.4654744
330	+1.0579845	+1.6887208	−0.031341647	−0.09708793	3.6643987
Σ_1	+0.0613389	+0.6114494	+0.001435277	−0.20967551	18.9161075
Σ_2	+0.0615274	+0.6110240	+0.000378605	−0.20894335	18.9161151

$\sin\varphi \cdot \frac{1}{2}A_1^{(a)} + \cos\varphi \cdot B_0^{(a)} = +0.000000102.$

DIFFERENTIAL COEFFICIENTS.

				log coeff.
		″		
$[de/dt]_{00}$	= +	7024.3393	m'	p 3.8466055
$[d\chi/dt]_{00}$	= +	749340.69	m'	p 5.8746793
$[di/dt]_{00}$	= +	104.61082	m'	p 2.0195766
$[d\Omega/dt]_{00}$	= −	747594.66	m'	n 5.8736662
$[d\pi/dt]_{00}$	= +	748950.76	m'	p 5.8744532
$[dL/dt]_{00}$	= +	2175235.9	m'	p 6.3375064

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

		″
$[de/dt]_{00}$	=	+0.021481158
$[d\chi/dt]_{00}$	=	+2.2915614
$[di/dt]_{00}$	=	+0.00031991074
$[d\Omega/dt]_{00}$	=	−2.2862242
$[d\pi/dt]_{00}$	=	+2.2903688
$[dL/dt]_{00}$	=	+6.6520970

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	″	″	″
$[de/dt]_{00}$	+0.02151	+0.02148	+0.02148116
$e[d\pi/dt]_{00}$	+0.21276	+0.21374	+0.21361818
$[di/dt]_{00}$	+0.00030	+0.00032	+0.00031991
$\sin i\,[d\Omega/dt]_{00}$	−0.07391	−0.07379	−0.07383093
$[dL/dt]_{00}$	+6.638		+6.6520970

NOTES.

As in all cases in which Mars is the disturbed body, the gradual increase in the discrepancies in the sums of the functions as the computation proceeds is caused principally by the large value of e. The greatest effect which is here produced by the inclusion of all terms from the fifth to the eleventh orders occurs with the coefficient $[d\pi/dt]_{00}$ and amounts to 0″.0007. It is evident that a division into twelve parts is fully sufficient.

ACTION OF JUPITER ON MARS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	29.52014024	+6.924444	−4.477639	1.2637451	27.008366
30	29.73057269	+8.555992	−0.736229	0.0341655	27.006182
60	29.83402815	+8.090126	+3.325863	0.6972206	$27.00766\bar{9}$
90	29.81017790	+5.651672	+6.620206	2.7625134	27.011679
120	29.66910846	+1.894010	+8.264081	4.3047762	27.014415
150	29.44492347	$-2.17599\bar{5}$	+7.817013	3.8516159	27.013253
180	29.19030067	−5.467791	+5.398794	1.8371936	27.009343
210	28.96977017	−7.099343	+1.657384	0.1731441	27.006389
240	28.84611897	−6.633477	−2.404708	0.3644965	$27.00697\bar{7}$
270	28.85987156	−4.195019	−5.699050	2.0472288	27.010168
300	29.01103882	$-0.43735\bar{8}$	−7.342925	3.3985953	27.012567
330	29.25541959	+3.632647	−6.895857	2.9973538	27.011662
Σ_1	176.07073531*	$+4.36995\bar{5}$†	+2.763466‡	11.8660213	162.059336
Σ_2	176.07073538	$+4.36995\bar{5}$	+2.763467	11.8660215	162.059333

E	l	G	G'	G''	θ
°					° ′ ″
0	2.448742	27.0064605	2.4695960	0.0189481	17 39 53.68
30	2.661358	27.0061300	2.6618860	0.0004753	18 17 56.75
60	$2.76332\bar{8}$	27.0066036	2.7737001	0.0093077	18 43 15.01
90	2.735467	27.0074648	2.7765210	0.0368400	18 48 58.35
120	2.591662	27.0078870	2.6581522	0.0599626	18 28 29.57
150	2.368639	27.0074649	$2.433041\bar{6}$	0.0586151	17 39 45.34
180	2.117926	27.0066097	2.1522663	0.0316074	16 30 39.82
210	1.900349	27.0061336	$1.903971\bar{8}$	0.0033673	15 24 39.18
240	$1.77611\bar{1}$	27.0064416	$1.784209\bar{8}$	0.0075644	14 55 27.23
270	1.786671	27.0071624	1.8310752	0.0413982	15 15 16.64
300	1.935440	27.0075479	2.0032755	0.0628165	16 2 14.95
330	2.180725	27.0071916	2.2348561	0.0496603	16 53 32.61
Σ_1	13.633208	162.0415503	$13.841199\bar{9}$	0.1902067	102 20 0.26
Σ_2	13.633209	162.0415473	13.8413516	0.1903562	102 20 8.87

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee'\cos K] = 176.07073528.$

† $6[a'^2e' - kaa'e\cos K] = +4.369954.$

‡ $-6k'aa'\cos\varphi' \cdot e\sin K' = +2.763466.$

ACTION OF JUPITER ON MARS.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.03165483	0.31498612	0.22325507	8.3476456	5.7990872	7.1391284
30	0.03402451	0.31811150	0.22676030	8.3623570	5.8175286	7.1576473
60	0.03565117	0.32025551	0.22916442	8.3954324	5.8524487	7.1929772
90	0.03602468	0.32074766	0.22971622	8.4366023	5.8931984	7.2342427
120	0.03469756	0.31899875	0.22775524	8.4742980	5.9283892	7.2695995
150	0.03164635	0.31497492	0.22324250	8.4991683	5.9492925	7.2899854
180	0.02757619	0.30960108	0.21721367	8.5057537	5.9513987	7.2909894
210	0.02396325	0.30482489	0.21185335	8.4928484	5.9346402	7.2731852
240	0.02244999	0.30282273	0.20960576	8.4633475	5.9029922	7.2413643
270	0.02347170	0.30417466	0.21112344	8.4239469	5.8638333	7.2029262
300	0.02598821	0.30750252	0.21485868	8.3844577	5.8269721	7.1668224
330	0.02889111	0.31133796	0.21916251	8.3560126	5.8027960	7.1428978
Σ_1	0.17801795	1.87416671	1.32185284	0.5709348	5.2612881	3.3008810
Σ_2	0.17802160	1.87417159	1.32185832	0.5709353	5.2612888	3.3008846

E	$\log V$	J_1'	J_2	J_3	F_2
0°	7.1387522	27.0207890	−0.20899481	+0.30968500	+ 5.840160
30	7.1576379	27.0065721	−0.03630747	−0.02674093	+ 0.960260
60	7.1927926	27.0099958	+0.15284461	−0.36413185	− 4.337906
90	7.2335125	27.0290264	+0.31348417	−0.61207993	− 8.634698
120	7.2684108	27.0492502	+0.40064241	−0.70414548	−10.778797
150	7.2888221	27.0526590	+0.38540712	−0.61566134	−10.195688
180	7.2903609	27.0334483	+0.26823852	−0.37034175	− 7.041617
210	7.2731181	27.0094582	+0.08088102	−0.03392190	− 2.161717
240	7.2412135	27.0106120	−0.12278062	+0.30345694	+ 3.136448
270	7.2021019	27.0363209	−0.28345023	+0.55139880	+ 7.433241
300	7.1655732	27.0521350	−0.35632057	+0.64347029	+ 9.577337
330	7.1419111	27.0423420	−0.32638726	+0.55499853	+ 8.994231
Σ_1	3.2971031	162.1762303*	+0.13362954	−0.18200685	− 3.604375
Σ_2	3.2971036	162.1763786	+0.13362735	−0.18200677	− 3.604371

* $\Sigma_1(J_1' - G'') = 161.9860236.$
$\Sigma_2(J_1' - G'') = 161.9860224.$

ACTION OF JUPITER ON MARS.

E	F_3	R_0	$1000 \times S_0$	$1000 \times W_0$	$1000 \times R^{(n)}$	$1000 \times S^{(n)}$
0°	−0.11464084	0.011523729	+0.08005004	+0.4190397	0.000000	+0.05794093
30	−0.02209150	0.011964482	+0.01088811	−0.0398940	+4.271140	+0.00777378
60	+0.08535418	0.012921829	−0.07058125	−0.5615348	+7.703673	−0.04858841
90	+0.09095508	0.014185469	−0.13852471	−1.0407895	+9.309934	−0.09091387
120	−0.02126155	0.015433331	−0.17072189	−1.3081943	+8.381048	−0.10705259
150	−0.14774803	0.016299442	−0.15776374	−1.2103390	+4.948932	−0.09580229
180	−0.16666131	0.016504164	−0.10615120	−0.7376107	0.000000	−0.06372374
210	−0.05846157	0.015978601	−0.03427562	−0.0686500	−4.851517	−0.02081393
240	+0.07437981	0.014896574	+0.03689409	+0.5347724	−8.089562	+0.02313475
270	+0.10831696	0.013593080	+0.09184683	+0.8860645	−8.921148	+0.06027915
300	+0.01978452	0.012429971	+0.12131684	+0.9434385	−7.410440	+0.08351498
330	−0.09401607	0.011688531	+0.11863615	+0.7635196	−4.172629	+0.08470265
Σ_1	−0.12304519	0.083709598	−0.10919337	−0.7100892	+0.584719	−0.05477408
Σ_2	−0.12304513	0.083709605	−0.10919298	−0.7100884	+0.584712	−0.05477451

E	$R_0 \sin v + (\cos v + \cos E)S_0$	$-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$-2\frac{r}{a}R_0$
0°	+0.000160100	−0.011523729	+0.10774325	−0.4049515	−0.020897865
30	+0.006498115	−0.010046690	−0.02950189	+0.0268547	−0.021996162
60	+0.011621455	−0.005638033	−0.55239000	+0.1009292	−0.024638457
90	+0.014136555	+0.001045999	−0.97645422	−0.3602495	−0.028370938
120	+0.012896599	+0.008458998	−0.85083667	−0.9937043	−0.032306104
150	+0.007784436	+0.014315488	−0.26253500	−1.1815227	−0.035231972
180	+0.000212302	+0.016504164	+0.18965406	−0.7128120	−0.036086942
210	−0.007299886	+0.014215603	+0.04622538	−0.0507547	−0.034538456
240	−0.012311629	+0.008381401	−0.50369057	+0.1796589	−0.031182525
270	−0.013542396	+0.001084104	−0.87379071	−0.1469683	−0.027186159
300	−0.011129593	−0.005518211	−0.72109467	−0.6083574	−0.023700620
330	−0.006127613	−0.009949901	−0.23455823	−0.7265980	−0.021488839
Σ_1	+0.001449234	+0.010664590	−2.33061460	−2.4392371	−0.168812513
Σ_2	+0.001449211	+0.010664603	−2.33061467	−2.4392385	−0.168812526

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = -0.00000000019.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +$ 165.70584	m'	p 2.2193378
$[d\chi/dt]_{00}$	$= +13074.175$	m'	p 4.1164143
$[di/dt]_{00}$	$= -$ 268.82366	m'	n 2.4294675
$[d\Omega/dt]_{00}$	$= -$ 8712.2760	m'	n 3.9401316
$[d\pi/dt]_{00}$	$= +13069.631$	m'	p 4.1162634
$[dL/dt]_{00}$	$= -19334.282$	m'	n 4.2863281

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= +$ 0.15813453
$[d\chi/dt]_{00}$	$= +12.476799$
$[di/dt]_{00}$	$= -$ 0.25654077
$[d\Omega/dt]_{00}$	$= -$ 8.3142000
$[d\pi/dt]_{00}$	$= +12.472464$
$[dL/dt]_{00}$	$= -18.450874$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$+$ 0.15810	$+0.15818$	$+$ 0.1581345
$e[d\pi/dt]_{00}$	$+$ 1.16323	$+1.16372$	$+$ 1.1632822
$[di/dt]_{00}$	$-$ 0.25648	-0.25655	$-$ 0.2565408
$\sin i\,[d\Omega/dt]_{00}$	$-$ 0.26864	-0.26850	$-$ 0.2684974
$[dL/dt]_{00}$	-18.450		-18.450874

NOTES.

The very exact agreement of the final sums shows that for this case the expansion of the perturbing function is highly convergent. The greatest effect of all terms from the sixth to the eleventh orders inclusive occurs with $[de/dt]_{00}$, and amounts to but 1/100000th of the whole variation.

This computation has been twice effected by DR. ARTHUR B. TURNER from the same elements as are here employed. His first computation was made by HILL'S first method, exactly as here, and was presented as a *Thesis to the Faculty of the*

Graduate School of the University of Pennsylvania, 1902. The values of the functions in this computation agree practically throughout with those here given, but the last two figures usually differ because eight place logarithms are here employed in certain parts of the work.

DR. TURNER'S second computation, (*A. N.*, 3065), was made according to the method developed by DR. LOUIS ARNDT[30]. The two papers taken together are of high value since they afford a means of comparing the labor and accuracy appertaining to the two very different methods. It is DR. TURNER'S opinion that while the formulas of ARNDT'S method are presented in a more symmetric form yet they are less accurate in application than those of DR. HILL. This is confirmed by the circumstance that the residual from the equation $[da/dt]_{00} = 0$ is 300 times larger with the former method than with the latter.

DR. TURNER'S results from ARNDT'S method, which agree almost exactly with his earlier values and with those here obtained, are as follows:

$[de/dt]_{00}$	$= +\ 0.''1581330$	$[d\Omega/dt]_{00}$	$= -\ 8.''314194$
$[d\chi/dt]_{00}$	$= +12.47677$	$[d\pi/dt]_{00}$	$= +12.47244$
$[di/dt]_{00}$	$= -\ 0.2565480$	$[dL/dt]_{00}$	$= -18.45083$

ACTION OF SATURN ON MARS.

E	A	$B \sin \epsilon$	$B \cos \epsilon$	g	h
0°	92.2326164	−11.747198	− 0.835808	39.461155	90.708401$\bar{1}$
30	93.1075498	−13.263728	+ 6.495252	50.307460	90.7105920
60	94.0543836	−10.902482	+13.616320	33.990063	90.710653$\bar{2}$
90	94.8268069	− 5.296150	+18.619311	8.020876	90.7086748
120	95.2215463	+ 2.053054	+20.163679	1.205319	90.707034$\bar{8}$
150	95.1291338	+ 9.175915	+17.835613	24.076867	90.707883$\bar{3}$
180	94.5669406	+14.163873	+12.258914	57.367408	90.710808$\bar{2}$
210	93.6819094	+15.680401	+ 4.927852	70.309758	90.7131088
240	92.7148797	+13.319160	− 2.193216	50.728826	90.7125327
270	91.9323586	+ 7.712824	− 7.196207	17.010933	90.709523$\bar{7}$
300	91.5477178	+ 0.363621	− 8.740573	0.037809	90.706660$\bar{8}$
330	91.6603255	− 6.759241	− 6.412507	13.064630	90.7063166
Σ_1	560.3380844*	+ 7.250028†	+34.269316‡	182.790580	544.256090$\bar{6}$
Σ_2	560.3380840	+ 7.250021	+34.269314	182.790524	544.2560991

* $6a^2 + 3a^2e^2 + 6[a'^2 - 2kaa'ee' \cos K] = + 560.3380843.$

† $- 6k'aa' \cos \varphi' \cdot e \sin K' = + 7.250024.$

‡ $6[a'^2e' - kaa'e \cos K] = + 34.269314.$

ACTION OF SATURN ON MARS.

E	l	G	G'	G''	θ		
°					°	′	″
0	1.238258$\bar{1}$	90.7035382	1.5278682	0.2847474	8	6	50.425
30	2.1110005	90.7043316	2.3529760	0.2357148	9	42	47.930
60	3.057773$\bar{2}$	90.7063778	3.1798913	0.1178425	10	59	5.400
90	3.8321748	90.7076570	3.8561238	0.0229312	11	55	59.135
120	4.228554$\bar{3}$	90.7068811	4.2318481	0.0031400	12	28	42.340
150	4.135293$\bar{3}$	90.7048170	4.2015371	0.0631774	12	31	7.643
180	3.570175$\bar{2}$	90.7035495	3.7462608	0.1688274	11	58	46.967
210	2.6828433	90.7043024	2.9540530	0.2624034	10	50	18.088
240	1.7163897	90.7062481	2.0020241	0.2793497	9	6	39.626
270	0.936877$\bar{7}$	90.7074346	1.1081936	0.1692270	6	48	32.560
300	0.555099$\bar{8}$	90.7066561	0.5558543	0.0007499	4	29	34.176
330	0.6680516	90.7047169	0.8409314	0.1712800	6	3	29.499
Σ_1	14.366250$\bar{1}$	544.2332508	15.2437468	0.8546569	57	9	38.934
Σ_2	14.3662411	544.2332595	15.3138149	0.9247338	57	52	14.855

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00656257	0.28174175	0.18592136	7.5317335	3.895504$\bar{3}$	5.758669$\bar{4}$
30	0.00942345	0.28554606	0.19019807	7.5468281	3.914863$\bar{9}$	5.778271$\bar{1}$
60	0.01207449	0.28906810	0.19415647	7.5819826	3.9546474	5.817937$\bar{5}$
90	0.01427105	0.29198398	0.19743287	7.6263318	4.0028084	5.8660110
120	0.01561994	0.29377356	0.19944338	7.6674180	4.045881$\bar{0}$	5.9092061
150	0.01572227	0.29390930	0.19959586	7.6949830	4.0730267	5.936646$\bar{1}$
180	0.01438391	0.29213375	0.19760114	7.7028806	4.0781506	5.942049$\bar{9}$
210	0.01175197	0.28863978	0.19367515	7.6895882	4.060463$\bar{0}$	5.9243808
240	0.00828439	0.28403181	0.18849593	7.6581064	4.024192$\bar{8}$	5.8876296
270	0.00461510	0.27914998	0.18300711	7.6156278	3.9778730	5.8401825
300	0.00200564	0.27567456	0.17909843	7.5727522	3.9331412	5.7942078
330	0.00365091	0.27786617	0.18156337	7.5415145	3.902482$\bar{3}$	5.764628$\bar{7}$
Σ_1	0.05893094	1.71642353	1.14471671	5.7148732	3.931517$\bar{2}$	5.109700$\bar{1}$
Σ_2	0.05943475	1.71709527	1.14547243	5.7148733	3.931517$\bar{2}$	5.110120$\bar{1}$

ACTION OF SATURN ON MARS.

E	log V	J_1'	J_2	J_3	F_2	F_3
0°	5.7569714	90.9876364	−0.64905242	+0.2442109	+59.776028	−2.2333582
30	5.7768667	90.9027545	−0.80993896	−1.6448282	+67.492953	−2.2386206
60	$5.8172358\bar{}$	90.7031717	−0.67748734	−3.0908929	+55.477667	−1.0572468
90	5.8658745	90.5722076	−0.29321392	−3.7064154	+26.949646	+0.0320703
120	5.9091874	90.5928360	+0.18262899	−3.3264146	−10.447038	−0.2313819
150	5.9362703	90.7254544	+0.58278840	−2.0527628	−46.691968	−1.7836963
180	5.9410456	90.8717164	+0.80450796	−0.2268316	−72.073367	−3.2467872
210	5.9228190	90.9409399	+0.82304924	+1.6620728	−79.790278	−3.2597888
240	5.8859649	90.8876171	+0.67261531	+3.1078694	−67.775016	−1.8125358
270	5.8391722	90.7221965	+0.40924575	+3.7232565	−39.246977	−0.2549584
300	5.7942033	90.5707339	+0.07363159	+3.3433908	− 1.850296	+0.0267586
330	5.7636057	90.8202137	−0.30508990	+2.0700081	+34.394639	−1.0495556
Σ_1	$5.1046084\bar{}$	544.6137115*	+0.40684409	+0.0513320	−36.892022	−8.5545513
Σ_2	5.1046084	544.6837666	+0.40684061	+0.0513310	−36.891985	−8.5545494

E	R_0	$1000000 \times S_0$	$1000 \times W_0$	$1000 \times R^{(n)}$	$100000 \times S^{(n)}$	$1000 \times [R_0 \sin v + (\cos v + \cos E) S_0]$
0°	0.0017097618	+ 9.903284	+0.01219946	0.0000000	+0.7168085	+0.0198066
30	0.0017745243	+ 7.025377	−0.10023833	+0.6334785	+0.5015908	+0.9730086
60	0.0019262900	+ 5.499323	−0.20387007	+1.1484062	+0.3785755	+1.7472855
90	0.0021375714	+ 5.593738	−0.27213044	+1.4028896	+0.3671175	+2.1277319
120	0.0023569201	+ 3.205770	−0.27013289	+1.2799221	+0.2010205	+1.9382857
150	0.0025168546	− 4.917236	−0.17936963	+0.7641820	−0.2986000	+1.1679255
180	0.0025606191	−16.044422	−0.02369074	0.0000000	−0.9631644	+0.0320888
210	0.0024720520	−22.805150	+0.13539869	−0.7505788	−1.3848465	−1.0986739
240	0.0022843230	−19.928906	+0.23709997	−1.2404980	−1.2496588	−1.8606368
270	0.0020608533	− 9.038484	+0.25685358	−1.3525394	−0.5931965	−2.0510270
300	0.0018652798	+ 2.997952	+0.20817962	−1.1120336	+0.2063802	−1.6842325
330	0.0017413378	+ 9.774776	+0.11927113	−0.6216313	+0.6978895	−0.9263634
Σ_1	0.0127031938	−14.366999	−0.04021465	+0.0757967	−0.7100385	+0.1925973
Σ_2	0.0127031934	−14.366879	−0.04021500	+0.0758006	−0.7100452	+0.1926017

* $\Sigma_1(J_1' - G'') = 543.7590546.$
$\Sigma_2(J_1' - G'') = 543.7590328.$

E	$1000 \times \left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$1000 \times W_0 \cos u$	$1000 \times W_0 \sin u$	$1000 \times -2\frac{r}{a}R_0$
0°	−1.7097618	+0.00313672	−0.01178931	− 3.1005914
30	−1.4844381	−0.07412689	+0.06747540	− 3.2623831
60	−0.8120508	−0.20054993	+0.03664321	− 3.6729182
90	+0.2105546	−0.25530900	−0.09419278	− 4.2751426
120	+1.3414125	−0.17569179	−0.20519295	− 4.9336670
150	+2.2292256	−0.03890712	−0.17509913	− 5.4402943
180	+2.5606191	+0.00609135	−0.02289425	− 5.5988851
210	+2.2161495	−0.09117052	+0.10010366	− 5.3434506
240	+1.3285850	−0.22331936	+0.07965468	− 4.7817011
270	+0.2102889	−0.25329570	−0.04260337	− 4.1217062
300	−0.8010985	−0.15911711	−0.13424046	− 3.5565885
330	−1.4740745	−0.03664087	−0.11350351	− 3.2013711
Σ_1	+1.9077055	−0.74945012	−0.25781908	−25.6443513
Σ_2	+1.9077060	−0.74945010	−0.25781973	−25.6443479

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = +0.000000000031.$

Differential Coefficients.

			log coeff.
$[de/dt]_{00}$	$= +$	$22''.022051\ m'$	p 1.3428578
$[d\chi/dt]_{00}$	$= +2338.7360$	m'	p 3.3689812
$[di/dt]_{00}$	$= -$	$86.444970\ m'$	n 1.9367398
$[d\Omega/dt]_{00}$	$= -$	$920.85894\ m'$	n 2.9641931
$[d\pi/dt]_{00}$	$= +2338.2557$	m'	p 3.3688920
$[dL/dt]_{00}$	$= -2935.3283$	m'	n 3.4676567

Final Values Corresponding to the Above Value of m'.

$$[de/dt]_{00} = +0''.0062891406$$
$$[d\chi/dt]_{00} = +0.66790508$$
$$[di/dt]_{00} = -0.024687281$$
$$[d\Omega/dt]_{00} = -0.26298236$$
$$[d\pi/dt]_{00} = +0.66776785$$
$$[dL/dt]_{00} = -0.83828212$$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.00627	+0.00629	+0.0062891
$e[d\pi/dt]_{00}$	+0.06226	+0.06226	+0.0622814
$[di/dt]_{00}$	−0.02467	−0.02468	−0.0246873
$\sin i\,[d\Omega/dt]_{00}$	−0.00852	−0.00849	−0.0084927
$[dL/dt]_{00}$	−0.838		−0.8382821

NOTES.

As in other similar cases, the great disagreement of the sums of the functions near the beginning of the computation arises principally from the term a'^2e', but the remarkably close agreement of the final sums shows that the expansion of the perturbing function for this case is very convergent. The greatest effect produced by all terms from the sixth to the eleventh orders inclusive here occurs with $[de/dt]_{00}$ and amounts to but 1/100000th of the value of this coefficient.

DR. SAMUEL G. BARTON has effected this computation from the same elements as are here employed, making use of the formulas developed by DR. ARNDT[30]. (*A Thesis presented to the Faculty of the Graduate School of the University of Pennsylvania, 1906*). The results obtained by him are as follows:

$$
\begin{aligned}
[de/dt]_{00} &= +0.0062897'' \\
e[d\pi/dt]_{00} &= +0.0622817 \\
[di/dt]_{00} &= -0.0246873 \\
\sin i\,[d\Omega/dt]_{00} &= -0.0084927 \\
[dL/dt]_{00} &= -0.8382857
\end{aligned}
$$

The agreement is thus practically exact.

It is the conclusion of DR. BARTON that in spite of the greater symmetry of the equations employed in the method of ARNDT, computations effected by them are somewhat less accurate than when the methods of HILL are employed. His residual arising from the equation derived from the constancy of the major axis is eight times greater than that here obtained. (See the notes to the computation of the action of Jupiter on Mars, where it is shown that DR. TURNER came to the same conclusion.)

ACTION OF URANUS ON MARS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
0°	367.8110283	− 8.331256	+ 6.828502	37.81310	367.49598
45	368.2057075	− 5.369694	−15.243795	188.44191	367.49649
90	370.1698856	+12.404411	−28.797937	672.53375	367.50044
135	372.5672534	+34.579222	−25.894092	543.74175	367.50030
180	373.9791848	+48.165033	− 8.233286	54.97149	367.49639
225	373.5643097	+45.203469	+13.839014	155.31082	367.49635
270	371.5799361	+27.429370	+27.393154	608.52085	367.49979
315	369.2027636	+ 5.254555	+24.489302	486.34483	367.49949
Σ_1	1483.5400347*	+79.667558†	− 2.809567‡	1373.83919	1469.99259
Σ_2	1483.5400342	+79.667552	− 2.809571	1373.83931	1469.99263

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee'\cos K] = 1483.5400348.$

† $4[a'^2e' - kaa'e\cos K] = +79.667564.$

‡ $-4k'aa'\cos\varphi' \cdot e\sin K' = -2.809567.$

E	l	G	G'	G''	θ
					° ′ ″
0°	− 0.49589	367.49570	0.157536	0.653148	2 41 22.746
45	− 0.10173	367.49510	0.667671	0.768004	3 34 47.145
90	+ 1.85851	367.49543	2.574380	0.710869	5 25 12.441
135	+ 4.25601	367.49622	4.582935	0.322847	6 37 54.392
180	+ 5.67185	367.49598	5.698515	0.026250	7 10 10.482
225	+ 5.25702	367.49519	5.337362	0.079182	6 58 20.716
270	+ 3.26920	367.49525	3.718991	0.445245	6 6 25.010
315	+ 0.89233	367.49588	1.682505	0.786567	4 41 47.833
Σ_1	+10.30367	1469.98235	12.149421	1.835512	21 23 10.679
Σ_2	+10.30363	1469.98238	12.270473	1.956599	21 52 50.086

ACTION OF URANUS ON MARS.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00071814	0.27395868	0.17716832	6.6153321	1.7572439	4.226477
45	0.00127262	0.27469773	0.17799967	6.6414665	1.7838479	4.253308
90	0.00292086	0.27689384	0.18046982	6.702475	1.8471866	4.3168538
135	0.00437720	0.27883326	0.18265093	6.7600912	1.907656	4.3771079
180	0.00511859	0.27982020	0.18376077	6.7833377	1.9325907	4.4018149
225	0.00484000	0.27944936	0.18334377	6.7609874	1.909746	4.378986
270	0.00371007	0.27794494	0.18165196	6.7037350	1.850124	4.319609
315	0.00219200	0.27592287	0.17937772	6.6423516	1.7859125	4.2555483
Σ_1	0.01246766	1.10861766	0.72305087	6.804880	7.387146	7.2647550
Σ_2	0.01268182	1.10890322	0.72337209	6.8048967	7.3871627	7.2649502

ACTION OF URANUS ON MARS.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	4.2255130	368.030126̄	+0.3910957	+5.9910041	−117.85624
45	4.2521750	368.257943̄	−0.6874254	+1.3122864	+263.09961
90	4.3158056	368.150829	−1.4287110	−4.4833753	+497.03672
135	4.3766319	367.663776	−1.2456453	−8.0008361	+446.91789
180	4.4017762	367.403228̄	−0.3170169	−7.1797383	+142.10202
225	4.3788692	367.559090̄	+0.6967658	−2.5011942	−238.85387
270	4.3189529̄	367.911230̄	+1.2249811	+3.2942931	−472.79092
315	4.2543883	368.137688	+1.1030033	+6.8119278	−422.67209
Σ_1	7.2620477̄	1471.495412̄*	−0.1296511	−2.3778169	+ 48.49158
Σ_2	7.2620644	1471.618496̄	−0.1333016	−2.3778161	+ 48.49154

E	F_3	$1000 \times R_0$	$1000000 \times S_0$	$1000000 \times W_0$	$1000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	+ 1.1362797	0.2059026	−0.0165525	+10.07610	0.0000000	−0.0119808
45	− 5.0397791	0.2189646	+0.3708684	+ 2.31469	+0.1087907	+0.2605870
90	− 8.3838519	0.2523915	+0.5397194	− 9.33604	+0.1656447	+0.3542182
135	− 1.8404722	0.2886690	+0.6481209	−19.05928	+0.1256756	+0.3990450
180	+ 1.6518880	0.3050752	+0.4171470	−18.09445	0.0000000	+0.2504179
225	− 5.0490913	0.2898966	−0.2732646	− 6.02536	−0.1262101	−0.1682478
270	− 8.9126276	0.2535451	−0.7948804	+ 6.80306	−0.1664018	−0.5216806
315	− 2.5789640	0.2193839	−0.6003932	+12.22078	−0.1089990	−0.4218605
Σ_1	−14.5083118	1.0169144	+0.1454335	−10.55133	−0.0007571	+0.0709747
Σ_2	−14.5083066	1.0169141	+0.1453315	−10.54917	−0.0007428	+0.0695237

E	$1000 \times [R_0 \sin v + (\cos v + \cos E)S_0]$	$1000 \times \left[-R_0 \cos v + \left(\frac{r}{a}\sec^2\varphi + 1\right)\sin v S_0\right]$	$1000000 \times W_0 \cos u$	$1000000 \times W_0 \sin u$	$1000 \times -2\frac{r}{a}R_0$
0°	−0.00003310	−0.20590256	+2.590763	− 9.737343	−0.37339681
45	+0.16554695	−0.14335624	+2.077126	− 1.021440	−0.40904755
90	+0.25124093	+0.02461951	−8.758941	− 3.231492	−0.50478291
135	+0.18971131	+0.21763709	−8.485224	−17.066255	−0.61541380
180	−0.00083429	+0.30507521	+4.652436	−17.486113	−0.66705785
225	−0.19106859	+0.21804505	+5.009009	− 3.348852	−0.61803079
270	−0.25236576	+0.02523742	−6.708828	− 1.128399	−0.50709023
315	−0.16617611	−0.14329578	−6.836536	−10.129621	−0.40983085
Σ_1	−0.00199222	+0.14902958	−8.224570	−31.583347	−2.05232780
Σ_2	−0.00198644	+0.14903012	−8.235625	−31.566168	−2.05232299

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = -0.0000000000013.$

* $\Sigma_1(J_1' - G'') = 1469.659900̄.$
$\Sigma_2(J_1' - G'') = 1469.661897̄.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= -\ 0''.34119354$	m'	n 9.5330008
$[d\chi/dt]_{00}$	$= +274.05283$	m'	p 2.4378343
$[di/dt]_{00}$	$= -\ 1.4239452$	m'	n 0.1534933
$[d\Omega/dt]_{00}$	$= -169.16430$	m'	n 2.2283087
$[d\pi/dt]_{00}$	$= +273.96460$	m'	p 2.4376945
$[dL/dt]_{00}$	$= -352.43262$	m'	n 2.5470761

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= -0''.000014964631$
$[d\chi/dt]_{00}$	$= +0.012019862$
$[di/dt]_{00}$	$= -0.000062453743$
$[d\Omega/dt]_{00}$	$= -0.0074194879$
$[d\pi/dt]_{00}$	$= +0.012015994$
$[dL/dt]_{00}$	$= -0.015457573$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$-0''.00001$	$-0''.00001$	$-0''.000014964631$
$e[d\pi/dt]_{00}$	$+0.00112$	$+0.00112$	$+0.0011207080$
$[di/dt]_{00}$	-0.00007	-0.00006	-0.000062453743
$\sin i\ [d\Omega/dt]_{00}$	-0.00023	-0.00024	-0.00023960370
$[dL/dt]_{00}$	-0.015		-0.015457573

NOTES.

The greatest error produced in this case by a division into only four parts occurs with the coefficient $[d\chi/dt]_{00}$ and amounts to but $0''.0000001$. It is evident that, notwithstanding the disagreement of the sums of the functions in the first part of the computation, a division into eight parts is fully sufficient.

ACTION OF NEPTUNE ON MARS.

E	A	$B\cos\epsilon$	$B\sin\epsilon$	g	h
$0_\circ$	906.38891911	+21.92636	−39.01745	99.36880	904.17365
45	906.94215394	+47.61371	−15.38170	15.44333	904.17339
90	907.26271705	+49.05172	+19.61543	25.11471	904.17633
135	907.17710690	+25.39796	+45.47311	134.97137	904.17696
180	906.72119273	− 9.49146	+47.04426	144.45927	904.17395
225	906.14776209	−35.17881	+23.40851	35.76678	904.17317
270	905.80700342	−36.61681	−11.58863	8.76590	904.17576
315	905.91280912	−12.96307	−37.44631	91.52721	904.17627
Σ_1	3626.17983231*	+24.86981†	+16.05361‡	277.70868	3616.69969
Σ_2	3626.17983205	+24.86979	+16.05361	277.70869	3616.69979

E	l	G	G'	G''	θ
					° ′ ″
0°	2.15000	904.17353	2.20007	0.049953	2 51 33.458
45	2.70349	904.17337	2.70981	0.006303	3 8 30.703
90	3.02112	904.17630	3.03031	0.009166	3 19 25.738
135	2.93488	904.1768$\bar{0}$	2.98505	0.050008	3 19 16.746
180	2.48197	904.17377	2.54493	0.062779	3 4 42.112
225	1.90932	904.1731$\bar{3}$	1.92986	0.020498	2 39 43.146
270	1.56597	904.17575	1.57215	0.006167	2 23 40.258
315	1.67127	904.17616	1.7299$\bar{0}$	0.058516	2 32 56.175
Σ_1	9.21906	3616.69935	9.34746	0.128065	11 39 21.566
Σ_2	9.21896	3616.69945	9.3546$\bar{2}$	0.135325	11 40 26.770

ACTION OF NEPTUNE ON MARS.

E	$\log K_0$	$\log L_0'$	$\log N_0$	$\log N$	$\log P$	$\log Q$
0°	0.00081164	0.27408332	0.17730851	6.030047$\bar{1}$	0.391578$\bar{8}$	3.251079$\bar{8}$
45	0.00098011	0.27430787	0.17756111	6.0560291	0.417827$\bar{5}$	3.277335$\bar{5}$
90	0.00109700	0.27446368	0.17773639	6.115402$\bar{1}$	0.477350$\bar{8}$	3.336881$\bar{0}$
135	0.00109536	0.27446148	0.17773391	6.1708446	0.5327514	3.392301$\bar{2}$
180	0.00094087	0.27425556	0.17750228	6.1926623	0.554353$\bar{8}$	3.413882$\bar{6}$
225	0.00070344	0.27393908	0.17714626	6.1704766	0.5318928	3.3913614
270	0.00056913	0.27376004	0.17694484	6.114876$\bar{8}$	0.4761252	3.3355658
315	0.00064494	0.27386109	0.17705853	6.0556543	0.4169531	3.2764317
Σ_1	0.00341864	1.09656260	0.70949202	4.452988$\bar{2}$	1.899408$\bar{5}$	3.337409$\bar{1}$
Σ_2	0.00342385	1.09656952	0.70949981	4.4530046	1.899424$\bar{8}$	3.3374297

* $4a^2 + 2a^2e^2 + 4[a'^2 - 2kaa'ee'\cos K] = 3626.17983218.$

† $4[a'^2e' - kaa'e\cos K] = +24.869793.$

‡ $-4k'aa'\cos\varphi' \cdot e\sin K' = +16.05361.$

ACTION OF NEPTUNE ON MARS.

E	$\log V$	J_1'	J_2	J_3	F_2
0°	3.2510498	903.898094	+0.3009875	+15.528734	+299.48600
45	3.2773317	903.965295	−0.6661748	−12.963466	+118.06523
90	3.3368755	902.840060	−0.3701497	−34.790528	−150.56206
135	3.3922712	902.874950	+0.9173304	−37.166389	−349.03766
180	3.4138449	903.910920	+1.0320306	−18.699362	−361.09725
225	3.3913491	904.099942	−0.1708692	+ 9.792741	−179.67656
270	3.3355621	903.071970	−0.8044972	+31.619710	+ 88.95077
315	3.2763966	902.764465	+0.0451253	+33.995664	+287.42636
Σ_1	3.3373322	3613.721044*	+0.1583712	− 6.341446	−123.22254
Σ_2	3.3373486	3613.704652	+0.1254117	− 6.341450	−123.22263

E	F_3	$1000 \times R_0$	$1000000 \times S_0$	$1000000 \times W_0$	$1000 \times R^{(n)}$	$1000000 \times S^{(n)}$
0°	−10.033551	0.05357158	+0.12743633	+2.7656535	0.00000000	+0.09223957
45	− 4.253985	0.05690845	−0.09526013	−2.4561209	+0.02827449	−0.06693360
90	+ 2.323772	0.06500834	−0.12559190	−7.5560767	+0.04266503	−0.08242607
135	− 5.662284	0.07385617	+0.10733754	−9.1730340	+0.03215420	+0.06608723
180	−14.586465	0.07791030	+0.13821863	−4.8544449	0.00000000	+0.08297417
225	− 7.712269	0.07407227	−0.10322243	+2.4086872	−0.03224828	−0.06355357
270	+ 1.985936	0.06494182	−0.14759132	+6.8479047	−0.04262137	−0.09686429
315	− 2.681773	0.05659771	+0.08359968	+6.4235154	−0.02812010	+0.05874051
Σ_1	−20.310308	0.26143204	−0.00752826	−2.7969634	+0.00004366	−0.00407662
Σ_2	−20.310311	0.26143460	−0.00754534	−2.7969523	+0.00006031	−0.00565943

E	$1000\times[R_0 \sin v + (\cos v+\cos E)S_0]$	$1000\times\left[-R_0\cos v + \left(\frac{r}{a}\sec^2\varphi+1\right)\sin v\cdot S_0\right]$	$1000000 \times W_0 \cos u$	$1000000 \times W_0 \sin u$	$1000\times -2\frac{r}{a}R_0$
0°	+0.000254873	−0.053571580	+ 0.7111033	− 2.6726716	−0.09715013
45	+0.042763861	−0.037538560	− 2.2040406	+ 1.0838516	−0.10631064
90	+0.064736687	+0.005812013	− 7.0890057	− 2.6153925	−0.13001668
135	+0.048623034	+0.055602449	− 4.0838509	− 8.2138132	−0.15745404
180	−0.000276437	+0.077910304	+ 1.2481721	− 4.6912376	−0.17035369
225	−0.048771753	+0.055759058	− 2.0023922	+ 1.3387309	−0.15791475
270	−0.064644985	+0.006352181	− 6.7530485	− 1.1358370	−0.12988365
315	−0.042545549	−0.037317274	− 3.5934362	− 5.3243555	−0.10573015
Σ_1	+0.000070138	+0.036502918	−11.8827788	−11.1151387	−0.52740415
Σ_2	+0.000069593	+0.036505673	−11.8837299	−11.1155862	−0.52740958

$\sin\varphi \cdot \frac{1}{2}A_1^{(s)} + \cos\varphi \cdot B_0^{(c)} = +0.0000000000012.$

* $\Sigma_1(J_1' - G'') = 3613.592979.$
$\Sigma_2(J_1' - G'') = 3613.569327.$

DIFFERENTIAL COEFFICIENTS.

			log coeff.
$[de/dt]_{00}$	$= +\ 0.''011982$	m'	$p\ 8.0785350$
$[d\chi/dt]_{00}$	$= +67.128215$	m'	$p\ 1.8269051$
$[di/dt]_{00}$	$= -\ 2.0560028$	m'	$n\ 0.3130237$
$[d\Omega/dt]_{00}$	$= -59.551438$	m'	$n\ 1.7748923$
$[d\pi/dt]_{00}$	$= +67.097154$	m'	$p\ 1.8267041$
$[dL/dt]_{00}$	$= -90.590942$	m'	$n\ 1.9570848$

FINAL VALUES CORRESPONDING TO THE ABOVE VALUE OF m'.

$[de/dt]_{00}$	$= +0.''00000060823$
$[d\chi/dt]_{00}$	$= +0.0034075236$
$[di/dt]_{00}$	$= -0.00010436562$
$[d\Omega/dt]_{00}$	$= -0.0030229161$
$[d\pi/dt]_{00}$	$= +0.0034059472$
$[dL/dt]_{00}$	$= -0.0045985255$

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
$[de/dt]_{00}$	$+0.''00000$	$+0.''00000$	$+0.''00000060823$
$e[d\pi/dt]_{00}$	$+0.00032$	$+0.00032$	$+0.00031766599$
$[di/dt]_{00}$	-0.00011	-0.00011	-0.00010436562
$\sin i\ [d\Omega/dt]_{00}$	-0.00009	-0.00010	-0.000097621545
$[dL/dt]_{00}$	-0.004		-0.0045985255

NOTES.

The agreement of the sums of the functions is much more exact throughout than in the preceding computation because e' is here so much smaller. The greatest effect produced by all terms from the fourth to the seventh orders inclusive is but $0''.000001$, and it is evident that the terms of the eighth and higher orders are wholly inappreciable.

11. THE FINAL VALUES OF THE PERTURBATIONS.

Combining the results of the preceding pages, we now obtain the values of the perturbations stated in the following tables. For comparison with these, the results obtained by LEVERRIER[7] and NEWCOMB[15] are added, all of the results being reduced to the values of the masses here adopted and stated in Article 6.

SECULAR PERTURBATIONS OF MERCURY.

(Epoch 1850.0, G. M. T.)

Action of—	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{di}{dt}\right]_{00}$
	"		
Venus	+0.027739414	+2.7763615	−0.14811133
Earth	+0.011476557	+0.91448833	−0.014040890
Mars	−0.000607428	+0.02486334	−0.000301945
Jupiter	+0.00319413	+1.5400720	−0.049056191
Saturn	+0.000531095	+0.07312263	−0.004212776
Uranus	+0.000009638	+0.00142135	−0.000024450
Neptune	+0.000003320	+0.00041901	−0.000020027
	−0.04234673	+5.3307482	−0.21576761

Action of—	$\left[\frac{d\Omega}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
Venus	−1.9420214	+2.7618772	−3.2505323
Earth	−1.0037245	+0.90700208	−1.1935233
Mars	−0.01926435	+0.02471966	−0.03293324
Jupiter	−1.4795642	+1.5290366	−2.2066350
Saturn	−0.06979662	+0.07260205	−0.10657405
Uranus	−0.00134987	+0.00141128	−0.00201139
Neptune	−0.00044314	+0.00041570	−0.00060031
	−4.5161641	+5.2970646	−6.7928096

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.04246	+0.04234	+0.0423467
$e[d\pi/dt]_{00}$	+1.08946	+1.09601	+1.0891018†
$[di/dt]_{00}$	−0.21586	−0.21570	−0.2157676
$\sin i\,[d\Omega/dt]_{00}$	−0.55017	−0.55041	−0.5505495
$[dL/dt]_{00}$	−6.8190*		−6.79281

* Exclusive of the action of Uranus and Neptune.

† This unexpectedly large difference is a gradual accumulation from all of the computations Thus, the residuals, Newcomb–Gauss, are, in the several cases: 0″.00300, 0″.00151, 0″.00003, 0″.00227, and 0″.00010, the sum of which is the difference as found above.

SECULAR PERTURBATIONS OF VENUS.

(Epoch 1850.0, G. M. T.)

Action of—	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{di}{dt}\right]_{00}$
	"	"	"
Mercury	−0.013012279	−1.1893992	+0.0094965089
Earth	−0.04898290	−5.6289701	+0.000044940
Mars	−0.001963988	+0.74594759	+0.0013204280
Jupiter	−0.031162921	+6.5654682	−0.038659982
Saturn	−0.000675363	+0.07935156	−0.0052327048
Uranus	+0.000005263	+0.00278176	+0.0000018240
Neptune	−0.000000278	+0.00110440	−0.0000283988
	−0.09579247	+0.5762842	−0.033057385

Action of—	$\left[\frac{d\Omega}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
	"	"	"
Mercury	+ 0.0897732	−1.1892420	+ 0.74542525
Earth	− 7.293993	−5.6417558	− 5.4005288
Mars	− 0.0473504	+0.74586465	− 0.09940123
Jupiter	− 2.7242270	+6.5606924	− 5.5347410
Saturn	− 0.0824657	+0.07920700	− 0.26491624
Uranus	− 0.0028813	+0.00277671	− 0.00496096
Neptune	− 0.0007780	+0.00110304	− 0.00148569
	−10.061922	+0.5586460	−10.5606087

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	− 0.09558	−0.09576	− 0.0957925
$e[d\pi/dt]_{00}$	+ 0.00366	+0.00392	+ 0.0038229
$[di/dt]_{00}$	− 0.03318	−0.03306	− 0.0330574
$\sin i\,[d\Omega/dt]_{00}$	− 0.59530	−0.59551	− 0.5955192
$[dL/dt]_{00}$	−10.549		−10.5606087

SECULAR PERTURBATIONS OF THE EARTH.

(Epoch 1850.0, G. M. T.)

Action of—	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{dp}{dt}\right]_{00}$
	"	"	"
Mercury	−0.0011613570	− 0.10999815	+0.0025085775
Venus	+0.013483339	+ 3.4537341	+0.074457966
Mars	−0.015723904	+ 0.97519611	+0.0063443986
Jupiter	−0.081841849	+ 6.9652565	−0.025114405
Saturn	−0.000430571	+ 0.18725991	−0.0054235259
Uranus	+0.0000172788	+ 0.00566366	+0.0000236793
Neptune	−0.0000006006	+ 0.00179708	−0.0000364953
	−0.085660150	+11.4789092	+0.052760195

Action of—	$\left[\frac{dq}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
	"	"
Mercury	−0.002098681	+ 0.3930935
Venus	−0.28462399	+11.232473
Mars	−0.007195311	− 0.2342424
Jupiter	−0.16046446	− 9.1916336
Saturn	−0.013188086	− 0.4325140
Uranus	−0.0000784873	− 0.0080930
Neptune	−0.0000432488	− 0.0024199
	−0.46769226	+ 1.756664

Comparison with Other Results.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	−0.08569	−0.08563	−0.085660
$e[d\pi/dt]_{00}$	+0.19254	+0.19248	+0.192514
$[dp/dt]_{00}$	+0.05290	+0.05276	+0.052760
$[dq/dt]_{00}$	−0.46754	−0.46768	−0.467692
$[dL/dt]_{00}$	+1.7570*		+1.756664

The values of $[dp/dt]_{00}$ and $[dq/dt]_{00}$ obtained by Hill in the "*New Theory*" are given below. These were regarded as provisional results only, and were derived from the numerical values of the coefficients in the expansion of the perturbing function stated by Leverrier in the *Annales*, Vol. II.

It may also be of interest to add the results obtained by the first application ever made of the method of Gauss. This was a computation by Nicolai of the secular perturbations of the Earth, the final values only being published, in Bode's *Berliner Jahrbuch*, 1820, pages 224–226 (Aug. 30, 1817). These results are here reduced to the values of the masses stated in Article 6.

	Hill.	Nicolai.
		"
$[de/dt]_{00}$		−0.08606
$e[d\pi/dt]_{00}$	"	+0.19283
$[dp/dt]_{00}$	+0.0527225	+0.05182
$[dq/dt]_{00}$	−0.4676079	−0.46738

* Exclusive of the action of Neptune. If the value of this found above is included, we have $[dL/dt]_{00} = 1''.7546$; —a less exact agreement.

SECULAR PERTURBATIONS OF MARS.

(Epoch 1850.0, G. M. T.)

Action of—	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{di}{dt}\right]_{00}$
	"	"	"
Mercury	+0.000335670	+ 0.0061841	+0.000074482
Venus	+0.000795405	+ 0.4947286	−0.012829757
Earth	+0.021481158	+ 2.2915614	+0.000319911
Jupiter	+0.15813453	+12.476799	−0.25654077
Saturn	+0.006289141	+ 0.6679051	−0.024687281
Uranus	−0.000014965	+ 0.0120199	−0.000062454
Neptune	+0.000000608	+ 0.0034075	−0.000104366
	+0.18702155	+15.952606	−0.29383023

Action of—	$\left[\frac{d\Omega}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
	"	"	"
Mercury	+ 0.01479483	+ 0.0061918	+ 0.1940178
Venus	+ 0.30877426	+ 0.4948896	+ 4.1204933
Earth	− 2.2862242	+ 2.2903688	+ 6.6520970
Jupiter	− 8.3142000	+12.472464	−18.450874
Saturn	− 0.26298236	+ 0.6677678	− 0.8382821
Uranus	− 0.00741949	+ 0.0120160	− 0.0154576
Neptune	− 0.00302292	+ 0.0034059	− 0.0045985
	−10.5502799	+15.947104	− 8.342604

COMPARISON WITH OTHER RESULTS.

	Leverrier.	Newcomb.	Method of Gauss.
	"	"	"
$[de/dt]_{00}$	+0.18703	+0.18706	+0.187022
$e[d\pi/dt]_{00}$	+1.48645	+1.48787	+1.487355
$[di/dt]_{00}$	−0.29375	−0.29385	−0.293830
$\sin i\,[d\Omega/dt]_{00}$	−0.34099	−0.34066	−0.340709
$[dL/dt]_{00}$	−8.358*		−8.34260

12. COMPARISON WITH THE RESULTS OF OBSERVATION.

From a discussion of all the available observations of the planets and of the Sun, NEWCOMB has derived the most probable values of the preceding coefficients based upon observations alone. These will be found summarized in a convenient form on pages 107 and 108 of *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy* (Supplement to the American Ephemeris and Nautical Almanac, 1897).

* The value of $[dL/dt]_{00}$ arising from the action of Mercury was not stated by Leverrier. The value as found above has been added to his series of values in order to obtain this sum.

In order to compare the values here obtained with those given by NEWCOMB it is necessary to notice that the values of i and Ω stated by him are measured from the movable equator and equinox and that it is therefore necessary to free the values of $[di/dt]_{00}$ and $[d\Omega/dt]_{00}$ here given from the changes caused by the motion of the ecliptic itself. For this purpose we first compute ρ and L from the equations,

$$\rho \sin L = \left[\frac{dp}{dt}\right]_{00} \quad \text{and} \quad \rho \cos L = \left[\frac{dq}{dt}\right]_{00},$$

the secular variations being those which belong to the Earth's orbit, and then add the quantities $-\rho \cos (L - \Omega)$ to the several determinations of $[di/dt]_{00}$ and $-\rho \times \cos i \sin (L - \Omega)$ to those above given for $\sin i\ [d\Omega/dt]_{00}$. In this manner the values stated in the following tabulation are obtained.

In a similar way it might appear necessary to add the correction,

$$e \tan \tfrac{1}{2} i \left(\sin i \left[\frac{d\Omega}{dt}\right]_{00} + \rho \sin (L - \Omega)\right)$$

to the values obtained for $e\ [d\pi/dt]_{00}$, the first part arising from the change due to the plane of the orbit and the second from that produced by the motion of the ecliptic. And in the case of the Earth's perihelion, there is a secular motion due to the lack of sphericity of the Earth-moon system which is expressed by the equation,

$$e\left[\frac{d\pi}{dt}\right]_{00} = \tfrac{3}{2} en \cdot \frac{mm'}{(m + m')^2} \cdot \left(\frac{a'}{a}\right)^2,$$

the accented letters applying to the moon (*Annales de l'Observatoire de Paris*, Vol. IV, pages 42–46). Employing the values of a' and m' given in the *Astronomical Papers of the American Ephemeris*, Vol. IV, page 11, this correction is found to be $+0''.0157884$. But these last two corrections need not here be applied because the values of the variations obtained by NEWCOMB from observation have already been freed from their effects.

MERCURY.

	Newcomb.	Method of Gauss.	Observ.	δ_1	δ_2	ϵ
	"	"	"	"	"	"
$[de/dt]_{00}$	+0.0423	+0.0423	+0.0336	−0.0087	−0.0087	±0.0050
$e\ [d\pi/dt]_{00}$	+1.0960	+1.0891	+1.1824	+0.0864	+0.0933	±0.0040
$[di/dt]_{00}$	+0.0676	+0.0674	+0.0714	+0.0038	+0.0040	±0.0080
$\sin i\ [d\Omega/dt]_{00}$	−0.9250	−0.9234	−0.9189	+0.0061	+0.0045	±0.0045

VENUS.

	Newcomb.	Method of Gauss.	Observ.	δ_1	δ_2	ϵ
	"	"	"	"	"	"
$[de/dt]_{00}$	−0.0958	−0.0958	−0.0946	+0.0012	+0.0012	±0.0020
$e\ [d\pi/dt]_{00}$	+0.0039	+0.0038	+0.0029	−0.0010	−0.0009	±0.0020
$[di/dt]_{00}$	+0.0034	+0.0034	+0.0029	−0.0005	−0.0005	±0.0030
$\sin i\ [d\Omega/dt]_{00}$	−1.0600	−1.0603	−1.0540	+0.0060	+0.0063	±0.0012

EARTH.

	Newcomb.	Method of Gauss.	Observ.	δ_1	δ_2	ϵ
	"	"	"	"	"	"
$[de/dt]_{00}$	−0.0856	−0.0857	−0.0855	+0.0001	+0.0002	±0.0009
$e\ [d\pi/dt]_{00}$	+0.1925	+0.1925	+0.1948	+0.0023	+0.0023	±0.0012
$[d\epsilon/dt]_{00}$	−0.4677	−0.4677	−0.4711	−0.0034	−0.0034	±0.0023

MARS.

	Newcomb.	Method of Gauss.	Observ.	δ_1	δ_2	ϵ
	"	"	"	"	"	"
$[de/dt]_{00}$	+0.1871	+0.1870	+0.1900	+0.0029	+0.0030	±0.0027
$e\ [d\pi/dt]_{00}$	+1.4879	+1.4874	+1.4955	+0.0076	+0.0081	±0.0035
$[di/dt]_{00}$	−0.0225	−0.0229	−0.0226	−0.0001	+0.0003	±0.0020
$\sin i\ [d\Omega/dt]_{00}$	−0.7263	−0.7251	−0.7260	+0.0003	−0.0009	±0.0020

In the above tabulation the column headed δ_1 expresses the residuals from the computation of NEWCOMB and that headed δ_2 states the residuals from the results here obtained. It will be noticed that the differences are very minute throughout, the only appreciable improvement arising from the more accurate computation occurring in the case of the node of Mercury, where the residual is reduced by its fourth part.

The last column contains the mean errors of the observational results. If we multiply these by 0.6745 to reduce them to probable errors, we observe that in seven cases the residuals are less than the probable errors; in five cases they vary from one to three times as great but that in each of these cases where the divergence is greatest a slight change in the value of the masses will correct the disagreement, and that in the remaining three cases the difference is very much greater than can be ascribed to errors either in the adopted masses, the computation, or to errors in the observations themselves. These three cases are:

1. The motion of the perihelion of Mercury.
2. The motion in the node of Venus.
3. The motion of the perihelion of Mars.

The first of these is the well-known discordance. The second is well established, the discordance between observation and theory being nearly eight times the probable error, nor can the uncertainty remaining in the values of the masses account for more than a small part of the discrepancy. NEWCOMB estimates the mean error of the computed value arising from this uncertainty as not more than $\pm 0''.0012$, so that with this included the residual is nearly six times the probable error. The third discordance is the least of the three, but as the masses of Jupiter and Saturn, the principal disturbing planets for this case, are accurately known, the uncertainty of the computed results is almost negligible. NEWCOMB estimates the mean error of the result of computation arising from the uncertainties in the masses of all the planets as here but $\pm 0''.0004$, so that the residual remains between three and four times as large as the probable error.

13. COMPARISON WITH SEELIGER'S HYPOTHESIS ON THE CONSTITUTION OF THE ZODIACAL LIGHT.

Many hypotheses have been made for the purpose of explaining the discrepancies shown in the preceding article. In general, either the assumption is made that NEWTON'S Law of Gravitation is not strictly accurate* or else that certain additional matter in the solar system must be considered whose attraction has not hitherto been allowed for.† The most recent and the most plausible investigation of the second kind is that effected by SEELIGER[10], [11], [12] who seeks to account for all of the appreciable discrepancies by the perturbing effect of the cloud of particles known as the zodiacal light.

What the true form of this cloud is, and still more, what the law of the distribution of its density is, is very uncertain.‡ SEELIGER assumes that it can be roughly conceived as made up of two homogeneous ellipsoids of revolution whose semi axes have the values 0.24 and 1.20, respectively. Both the eccentricities of these ellipsoids and the position of the equator of the outer one can vary within wide limits without greatly altering the values of the perturbations which they produce; the distance from the focus to the center in each of them is arbitrarily chosen as equal in length to ten times the semi minor axis, and the equator of the outer one is assumed to be coincident with the plane of the equator of the sun. The respective densities and also the two constants which define the equatorial plane of the first ellipsoid remain as unknown quantities whose values are to be determined.

* See Tisserand's Mecanique Celeste, Vol. IV, Pages 494–542.

† See Newcomb's "Astronomical Constants. . . ."[17], Pages 110–120.

‡ See the article, "The Zodiacal Light" by Newcomb, in the Encyclopaedia Britannica, Vol. XXVIII.

From the known formulas which express the attraction exerted by an ellipsoid upon a point either wholly within or without its surface, the expression for the perturbing force in any case can readily be written, and from this the equations for the variations of the various elements are derived, each equation containing five unknown quantities whose values are to be so determined as to best account for the excess of the variations observed over those heretofore obtained from the theory. As the ellipsoids are assumed to be symmetrical with respect to their axes of rotation, however, they will cause no appreciable perturbation of any eccentricity. The variation of the obliquity of the Earth's orbit was also not considered by SEELIGER.

There remain therefore but ten discrepancies to be represented; namely, those of the four perihelia, those of the three nodes and those of the three inclinations. These ten discrepancies form the absolute terms of ten corresponding equations which contain five unknown quantities. It is to be noticed that in the "Astronomical Constants . . . " two tables of the theoretical variations are stated by NEWCOMB; the first, on page 109, are those computed from the values of the various masses assumed in Chapter V; the second, on page 185, are those computed from the definitively adopted masses. The latter values of the masses are in closer accordance with those assumed in the present paper than the former; the first values are, however, the ones adopted by SEELIGER in the computation.

The final results are as in the following table:

	Newcomb.	Method of Gauss.	Per. caused by Zod. L't.	Final Residuals.		Prob. Errors.
				Newcomb.	Meth. of Gauss.	
MERCURY.						
$ed\pi$	+8".64	+9".33	+8".49	+0".15	+0".84	±0".29
$\sin i d\Omega$	+0.61	+0.45	+0.62	+0.01	−0.17	±0.54
di	+0.38	+0.40	+0.49	−0.11	−0.09	±0.35
VENUS.						
$ed\pi$	−0.10	−0.09	+0.05	−0.15	−0.14	±0.17
$\sin i d\Omega$	+0.60	+0.63	+0.60	0.00	+0.03	±0.22
di	−0.05	−0.05	+0.20	−0.25	−0.25	±0.11
EARTH.						
$ed\pi$	+0.23	+0.23	+0.09	+0.14	+0.14	±0.09
MARS.						
$ed\pi$	+0.76	+0.81	+0.56	+0.20	+0.25	±0.24
$\sin i d\Omega$	+0.03	−0.09	+0.21	−0.18	−0.30	±0.14
di	−0.01	+0.03	−0.01	0.00	+0.04	±0.15

The first two columns of the table contain the residuals from the masses employed in the present paper; the third column states the perturbations caused by the zodiacal light when its elements are derived from the residuals of NEWCOMB's first tabulation. As the five elements were so determined as to represent NEWCOMB's first residuals as

accurately as possible, their agreement with these is naturally more exact than with the values here stated. Thus the first agreement for the motion of Mercury's perihelion is exact while here the discrepancy is considerable. On the other hand, the greatest discrepancy when the results are compared with the first tabulation, and which occurs in the motion of the node of Mars, is slightly lessened when the new masses are employed.

As the five elements were determined to represent the ten residuals of NEWCOMB's computation as accurately as possible, the numbers of the fourth column are, as might have been expected, generally smaller than those of the fifth. It may justly be inferred, however, that SEELIGER's hypothesis is capable of greatly reducing those discrepancies whose values are sufficiently large to establish their reality, without at the same time unduly increasing any of the smaller ones.

The last column contains NEWCOMB's estimate of the total probable errors arising both from the errors of observation and from the uncertainties in the values of the adopted masses.

The elements of the zodiacal light derived by SEELIGER are as follows:

Density of inner ellipsoid $= 2.52 \times 10^{-11}$ times the Sun's density.
Density of outer ellipsoid $= 0.0026 \times 10^{-11}$ times the Sun's density.
Total mass $= 35000 \times 10^{-11}$ times the Sun's mass.
Inclination of equator of I $= 6°.95$
Longitude of node of I $= 40°.03$.

The unit of time throughout this article is the Julian Century.

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PART 3

Tertiary Vertebrate Faunas of the North Coalinga Region of California.
A Contribution to the Study of Palæontologic Correlation in the Great Basin and Pacific Coast Provinces.

By JOHN C. MERRIAM,
Professor of Palæontology, University of California.

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ARTICLE III.

TERTIARY VERTEBRATE FAUNAS OF THE NORTH COALINGA REGION OF CALIFORNIA.

A Contribution to the Study of Palæontologic Correlation in the Great Basin and Pacific Coast Provinces.

By JOHN C. MERRIAM.

(Read April 24, 1915.)

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INTRODUCTION.

One of the most important problems in West-American palæontology and geology concerns the definite determination of time relations between the Pacific Coast marginal marine deposits and the sequence of continental formations of the Great Basin province. While similar time scales have been used in the regions east and west of the Sierra-Cascade Range, lines distinctly connecting the geological columns of these two regions are rare. Until recently, remains of the flora have been almost the only means of correlation, but we have known very few satisfactory occurrences of plants appearing in both areas. Marine invertebrates, found in a remarkably full faunal sequence in the Tertiary marginal marine province, and furnishing the principal materials for correlation between the areas or subdivisions of this province, are absolutely unrepresented in the Tertiary of the Great Basin. Fresh-water invertebrates are known sparingly in both regions, but the faunal sequence is far from satisfactory, while the characteristic simplicity of these forms reduces the possibilities of correlation. Land mammals are well represented in the Great Basin province and offer in their stages of evolution a satisfactory basis of correlation. They have been very rare, however, in the marginal marine areas.

Up to the present time, one of the most significant discoveries throwing light on the problem of correlation between the Pacific Coast and Great Basin provinces is the finding of a considerable number of remains representing land mammals in the marginal marine series of California. The collection was obtained by a party of advanced students from the University of California, in the North Coalinga region on the western border of the San Joaquin Valley, in December, 1913. The expedition visited the Coalinga field under the leadership of Dr. Bruce Clark for the purpose of

making a study of the sequence of invertebrate faunas, but the discovery of mammal remains seemed so significant that a large share of the work was devoted to examination of the vertebrate horizons.

For the material described in the following paper, the writer is indebted to all of the members of the University of California field party working in the Coalinga region in December, 1913. The first important specimens were obtained in the "Temblor" beds by C. L. Moody and J. M. Douglas. Later all of the members of the party collected at the locality visited by Moody and Douglas. A large part of the material secured at this locality was obtained by Neil C. Cornwall, who systematically excavated for mammalian remains in the soft sandstone and gravel. In the Jacalitos formation, the most important specimens were found by J. H. Ruckman, who worked very carefully over the basal portion of that formation. In the uppermost horizon, good material was obtained by all of the members of the party. The largest portion of this collection was secured by J. O. Nomland, who was making a special study of the Etchegoin. After the return of the December, 1913, expedition, Nomland made a second visit to the region and secured some of the most interesting material representing the latest fauna.

FAUNAL ZONES REPRESENTED.

The vertebrate material obtained in the North Coalinga region represents at least four horizons or faunal zones. Collections were obtained: (1) in the "Temblor" phase of the Monterey series; (2) in the Jacalitos formation; (3) in the lower portion of the Etchegoin formation as mapped in this region by the University ef California party in 1913; (4) a fauna later than that of the lower portion of the Etchegoin section, and obtained in the upper portion of the Etchegoin area.

The lowest horizon is characterized by abundance of horse teeth representing the genus *Merychippus*, and may be known as the Merychippus zone. From the Jacalitos horizon very few remains are known, and there does not seem to be sufficient characteristic material available to warrant final palæontologic designation. The lower Etchegoin horizon is characterized by the presence of *Pliohippus coalingensis*, and may be known as the Pliohippus coalingensis zone. The latest fauna is distinguished by a large specialized horse, with characters of *Pliohippus* and *Equus*, and by remains of a form near *Cervus*. It may be known as Cervus fauna.

The sequence of formations in the region examined is as follows:

Geological Periods.	Local Formations.	Vertebrate Faunas.
Pleistocene	Terraces	
Pliocene	Tulare	
	Etchegoin	Cervus or Odocoileus Pliohippus coalingensis Pliohippus?
	Jacalitos	Neohipparion
Miocene	"Santa Margarita"	
	"Big Blue"	Merychippus californicus
	Monterey ("Temblor")	Tetrabelodon? Desmostylus hesperus
Oligocene	Lillis formation[1]	
Eocene	Tejon	
Cretaceous	Chico	

The following generalized section of the region examined by the field party in December, 1913, was prepared for the writer by J. H. Ruckman.

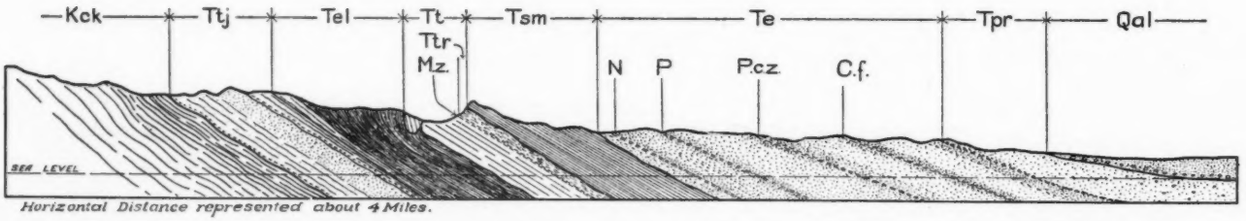


FIG. 1. Somewhat generalized east and west section across the North Coalinga horizons containing mammalian faunas Section constructed by J. H. Ruckman. *Kck*, Cretaceous; *Ttj*, Tejon Eocene; *Tel*, Lillis formation, Oligocene; *Tt.*, "Temblor" *Ttr*, "Rainbow beds" or "Big Blue"; *M.z.*, Merychippus zone; *Tsm*, Santa Margarita; *Te*, Jacalitos and Etchegoin; *N*; Neohipparion locality; *P*, Pliohippus? locality; *P.c.z.*, Pliohippus coalingensis zone; *C.f.*, Cervus fauna; *Tpr*, Tulare, formation; *Qal*, Pleistocene.

FAUNA OF MERYCHIPPUS ZONE.

OCCURRENCE.

The vertebrate collection representing the Merychippus fauna was obtained in a zone at the upper limit of the formation known as "Temblor"[2] or "Vaqueros" about twelve miles north of Coalinga. The best exposures are at locality 2124 in SW. ¼, Sec. 28, T. 18 S., R. 15 E., M. D. B. & M.

[1] From manuscript of J. H. Ruckman on North Coalinga section.

[2] "Temblor" of F. M. Anderson, *Proc. Calif. Acad. Sci.*, 4th ser., vol. 3, p. 18, 1908. Correlated with the "Vaqueros" of Hamlin by Arnold and Anderson, U. S. Geol. Surv. Bull. 389, p. 87, 1910.

The bones were found mainly in a zone of sandstone and conglomerate two or three feet thick immediately below the beds known as the "Big Blue," which overlies the characteristic "Temblor" beds. The zone containing bones consists in part of conglomerate, including pebbles ranging up at least to one inch in diameter. The upper portion of the zone is mainly fine sand. Fragments of bones and teeth have also been found scattered through the "Temblor" sandstone to a distance of at least one hundred feet below the conglomerate-sandstone zone in which the Merychippus fauna was obtained.

Question has naturally arisen as to the stratigraphic relation of the conglomerate and sandstone of the Merychippus zone to the "Temblor" beds below, and to the "Big Blue" above, as also regarding the relation of the "Big Blue" to the "Temblor."

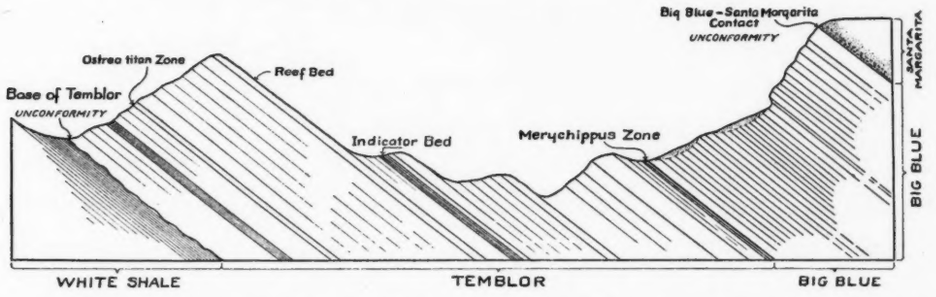


FIG. 2. Detailed section across the "Temblor" showing location of the Merychippus zone. Section prepared by C. L. Moody. Length of section approximately 1225 feet.

The work of the University party in December, 1913, seemed to show a gradual transition from the "Temblor" beds through the Merychippus zone up at least to the base of the "Big Blue." At some localities there is an irregular contact between the conglomerate and the underlying "Temblor" sandstone, but such relief as was observed is possibly not more than should be expected in an estuarine deposit in which coarse sediments were moved at varying rates at different times. The general consensus of opinion of those members of the University party who examined this section was that no important stratigraphic break exists between the characteristic "Temblor" portion of the sequence and the top of the Merychippus zone.

The beds north of Coalinga designated as the "Big Blue" were referred to by F. M. Anderson[1] as having the stratigraphic position of the Monterey series. Anderson stated that "their separation from the Temblor in the fields north of Coalinga is for convenience in logical treatment rather than for emphasis of their stratigraphic prominence."

Arnold and Anderson[2] mapped the "Big Blue" tentatively with the "Santa Mar-

[1] Anderson, F. M., *Proc. Calif. Acad. Sci.*, 4th ser., vol. 3, p. 21, 1908.

[2] Arnold, R., and Anderson, R., U. S. Geol. Survey, Bull. 398, pp. 76, 78, 82, 88, 89; 1910.

garita" but noted that it might really represent a part of the Monterey or a portion of the "Vaqueros" (p. 78). They called attention to its stratigraphic position corresponding to that of the Monterey (p. 76); to the probability that the line at its base may represent a great unconformity (p. 82); and to the overlap of the Tamiosoma zone upon it, indicating that it is a distinct unit (pp. 88 and 89).

In reporting on the Cantua-Panoche region north of Coalinga, after more careful study of the field than was possible in connection with the Coalinga report, Robert Anderson[1] states that in the area north of Coalinga mapped by Arnold and Anderson there is evidence indicating that the "Big Blue" belongs with the "Vaqueros." This evidence includes the presence in the "Big Blue" of marine fossils of early Miocene or "Vaqueros" type. Anderson suggests (p. 64) the possible equivalence of the "Vaqueros" of the Cantua-Panoche region to the lower portion of the Monterey shale in the region nearer the coast.

R. W. Pack, of the United States Geological Survey, who worked carefully over this region with Robert Anderson, states that the fauna of the marine beds below the "Big Blue" occurs also in sandy strata above the typical serpentinous shale, which forms the major part of the "Big Blue." This indicates that the time of deposition of the "Big Blue" belongs to the same biologic period as that of the "Temblor" or "Vaqueros" below, and that there was no great time interval between the deposition of the two.

COMPOSITION OF THE MERYCHIPPUS FAUNA.

The material obtained in the Merychippus zone consists of scattered teeth and skeletal elements. No connected parts of the skeletons, and no teeth so closely associated as to indicate their certain reference to the same individual, have been found. The specimens were all obtained from coarse sands and gravels, presumably representing the wash upon a beach near the mouth of a river. The number of specimens secured exceeds that obtained at any other locality of this nature known to the writer in the Pacific Coast region. The occurrence at this place presumably indicates that when these beds were being deposited the land mammals found here were abundant in this region. It also indicates an unusual combination of circumstances making possible the preservation of these remains.

Associated with the remains of land mammals in the Merychippus zone are several specimens representing marine types. At least three species of sharks appear in this collection. At a level about two feet above the layer in which *Merychippus* teeth were most abundant, Mr. Cornwall obtained a perfectly preserved tooth of the sirenian, *Desmostylus*. As the remains of *Desmostylus* are characteristic fossils of

[1] Anderson, R., U. S. Geol. Surv. Bull. 431–A, p. 65, 1910.

the "Temblor" in this region, question has naturally arisen whether this tooth may not have been derived from the "Temblor" and secondarily deposited in the conglomerate at the base of the "Big Blue." Remains of *Desmostylus* are by no means abundant at any horizon, and it seems improbable that the occurrence of this tooth above the *Merychippus* stratum is the result of secondary deposition. It is probable that this animal was living in the sea in the period in which *Merychippus* inhabited the adjacent land.

The fauna obtained from the Merychippus zone includes the following forms:

Merychippus californicus, n. sp.
Prosthennops?
Procamelus?, sp.
Tetrabelodon?, sp.
Desmostylus, near *hesperus* Marsh.
Isurus, sp.
Carcharodon, sp.

MERYCHIPPUS CALIFORNICUS, N. SP.

Type specimen, $M^{\frac{1}{}}$, no. 21247, locality 2124. From Merychippus zone between typical "Temblor" and "Big Blue."

Cheek-teeth of the *Merychippus* type, but tending to a more slender form than in *M. isonesus* of the Mascall Middle Miocene.

Remains representing Equidæ from locality 2124 comprise forty or fifty well-preserved upper and lower molars, several incisors, and limb elements including the calcaneum, astragalus, second phalanx, and a metapodial. The upper and lower molars seem all referable to a single species.

3a

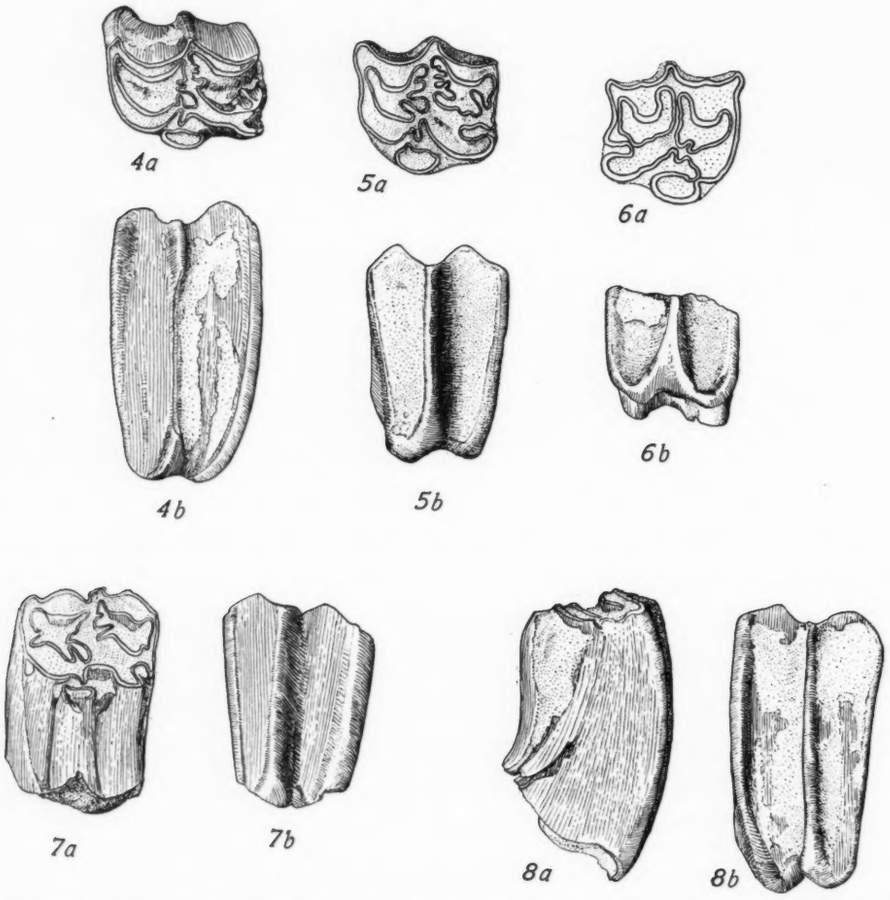
3b

3c

3d

FIGS. 3*a*–3*d*. *Merychippus californicus*, n. sp. $M^{\frac{1}{}}$, no. 21217, natural size, Merychippus zone, North Coalinga region, California. Fig. 3*a*, occlusal view; Fig. 3*b*, posterior view; Fig. 3*c*, inner view; Fig. 3*d*, outer view.

The characters of the upper molars (Figs. 3*a* to 8*b*) are in some respects similar to those of *Merychippus isonesus* of the Mascall Middle Miocene. The protocone is distinct from the protoconule and remains separate almost to the base of the crown.



FIGS. 4*a*–8*b*. *Merychippus californicus*, n. sp. Upper molars, natural size, Merychippus zone, North Coalinga region, California.

FIGS. 4*a* and 4*b*, P^4, no. 21280; Fig. 4*a*, occlusal view; Fig. 4*b*, external view.

FIGS. 5*a* and 5*b*, M^1, no. 21236; Fig. 5*a*, occlusal view; Fig. 5*b*, external view.

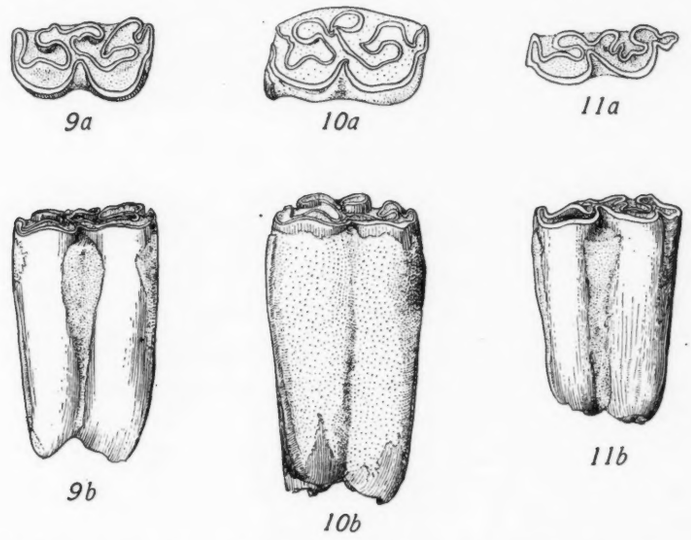
FIGS. 6*a* and 6*b*, M^1 ?, no. 21241; Fig. 6*a*, occlusal view; Fig. 6*b*, external view.

FIGS. 7*a* and 7*b*, P^4, no. 21230; Fig. 7*a*, occlusal and inner view; Fig. 7*b*, external view.

FIGS. 8*a* and 8*b*, M^1, no. 21238; Fig. 8*a*, anterior view; Fig. 8*b*, external view.

Only in very old, much worn teeth is there any suggestion of connection of the protocone and protoconule on the crushing surface. In cross-section, the protocone is round with a slight lateral compression. A ridge on the antero-external wall of the protocone reaches outward toward the protoconule, but does not unite with it until an advanced stage of wear is reached. The moderately complex folding of the enamel

enclosing the fossettes approximates that of *M. isonesus*. In form and strength of the outer styles the Coalinga form is not appreciably different from typical Mascall specimens. Compared with *M. isonesus*, the upper molars from Coalinga show a tendency to be a little smaller in cross-section, and a little larger in length of crown. The type of upper molar seen here is that of a representative of *Merychippus* in the division of the group leading to *Hipparion*. In this group *M. isonesus* of the Middle Miocene seems to be a less advanced stage and *M. calamarius sumani* of the Barstow Upper Miocene is more advanced.



FIGS. 9*a*–11*b*. *Merychippus californicus*, n. sp. Lower molars natural size, Merychippus zone, North Coalinga region, California.

FIGS. 9*a* and 9*b*, P_3, no. 21244; Fig. 9*a*, occlusal view; Fig. 9*b*, external view.

FIGS. 10*a* and 10*b*, P_4, no. 21508; Fig. 10*a*, occlusal view; Fig. 10*b*, external view.

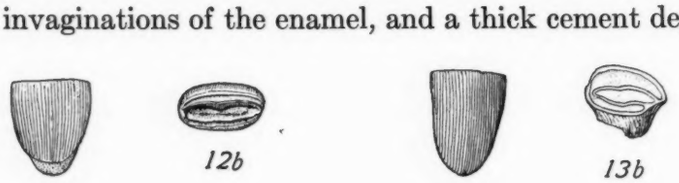
FIGS. 11*a* and 11*b*, M_3, no. 21249; Fig. 11*a*, occlusal view; Fig. 11*b*, external view.

The lower cheek-teeth (Figs. 9*a* to 11*b*) from Coalinga closely resemble *M. isonesus* in form and dimensions. The Coalinga specimens are possibly a little longer, but the difference is small. The metaconid-metastylid column is moderately flattened, and an antero-external ridge may be present on the protoconid.

Merychippus seversus, a form apparently widely distributed in the Mascall Miocene, is clearly separable from the Coalinga species by its much smaller size.

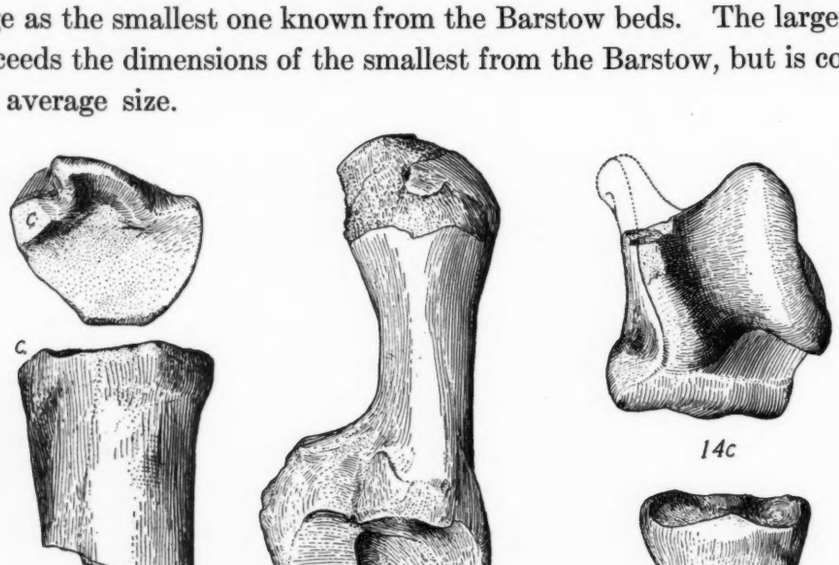
Merychippus calamarius, a type abundantly represented in the Barstow Upper Miocene of the Mohave Desert, is larger and more advanced. One form, *M. calamarius sumani*, occurring rather rarely in the Barstow fauna approaches the combination of characters seen in the Coalinga specimens, but is a more advanced type, and is specifically separable.

Two incisor teeth (Figs. 12*a* to 13*b*) found with the Coalinga *Merychippus* fauna show deep invaginations of the enamel, and a thick cement deposit in the pits.



FIGS. 12*a*–13*b*. *Merychippus californicus*, n. sp. Incisors, natural size. Merychippus zone, North Coalinga region, California. Figs. 12*a* and 12*b*, no. 21381; Fig. 12*a*, anterior view; Fig. 12*b*, occlusal view. Figs. 13*a* and 13*b*, no. 21374; Fig. 13*a*, anterior view; Fig. 13*b*, occlusal view.

Of the protohippine limb elements (Figs. 14*a* to 14*d*) from locality 2124, a single calcaneum is slightly larger than those referred to *Merychippus* from the Mascall and Virgin Valley. It is of approximately the size of this element in *Merychippus calamarius* from the Barstow beds. An astragalus has dimensions approximately the same as the average *Merychippus* specimens from the Mascall and Virgin Valley. Another much worn specimen is considerably smaller and corresponds in size to the smallest available astragalus from the Mascall stage. The smaller astragalus is not as large as the smallest one known from the Barstow beds. The larger specimen slightly exceeds the dimensions of the smallest from the Barstow, but is considerably below the average size.



FIGS. 14*a*–14*d*. *Merychippus californicus*, n. sp. Merychippus zone, North Coalinga region, California. All figures natural size. Fig. 14*a*. Proximal end of metatarsal III, no. 21509, anterior and proximal views; *c*, cuboid facet. Fig. 14*b*. Calcaneum, no. 21367, superior view. Fig. 14*c*. Astragalus, no. 21368, superior view. Fig. 14*d*. Second phalanx, no. 21366*a*, superior view.

A metatarsal III (Fig. 14*a*), no. 21509, from the Merychippus zone is of approximately the same size and form as the corresponding element in *Merychippus* specimens from the Mascall beds of Oregon. The Merychippus zone specimen is perhaps a little thicker anteroposteriorly. The facet for the cuboid is at least as large in no. 21509 as in the Mascall specimens, and is if anything a little nearer the plane of the ectocuneiform facet. The form of the proximal end of metatarsal III in no. 21509 can be matched almost exactly in *Merychippus* specimens from the Barstow fauna, but some of the Mohave specimens are larger and have a larger cuboid facet. The greatest transverse diameter of the proximal end is twenty-six millimeters,

The phalanges correspond in dimensions to those of the Mascall and Virgin Valley horses and are smaller than in the average *Merychippus* forms of the Barstow beds.

Measurements of Cheek-teeth.[1]

All measurements of anteroposterior and transverse diameter were taken about ten millimeters above the base of the crown.

Height of crown as given is the greatest height measured along the mesostyle of upper teeth or along the hypoconid of lower teeth.

	No. 21250 P^2	No. 21326 P^3	No. 21240 P^4	No. 21239 P^4	No. 21238 M^1	No. 21236 M^1	No. 21237 M^1	No. 21246 M^3
Anteroposterior diameter	21 mm.	20.3	17.8	19	17	18.3	19	15.2 with cement
Transverse diameter	18	*a*21	*a*20.5	20.8	21	20.2	19.9	17.7 with cement
Height of crown	22.5 worn	35.5	36	27 worn	37.4	25.9 worn	28.5 worn	29.5

	No. 21325 P_2	No. 21245 P_3	No. 21508 P_4	No. 21249 M_2	No. 21324 M_3	No. 21329 M_3
Anteroposterior diameter	20 mm.	19.3	21.2	16.7	20.5	21.5
Transverse diameter	9.9	10.2	10.4	9	7.5	*a*8
Height of crown	16.5 worn	33.7	38.8	28.5	21.5 worn	33.4

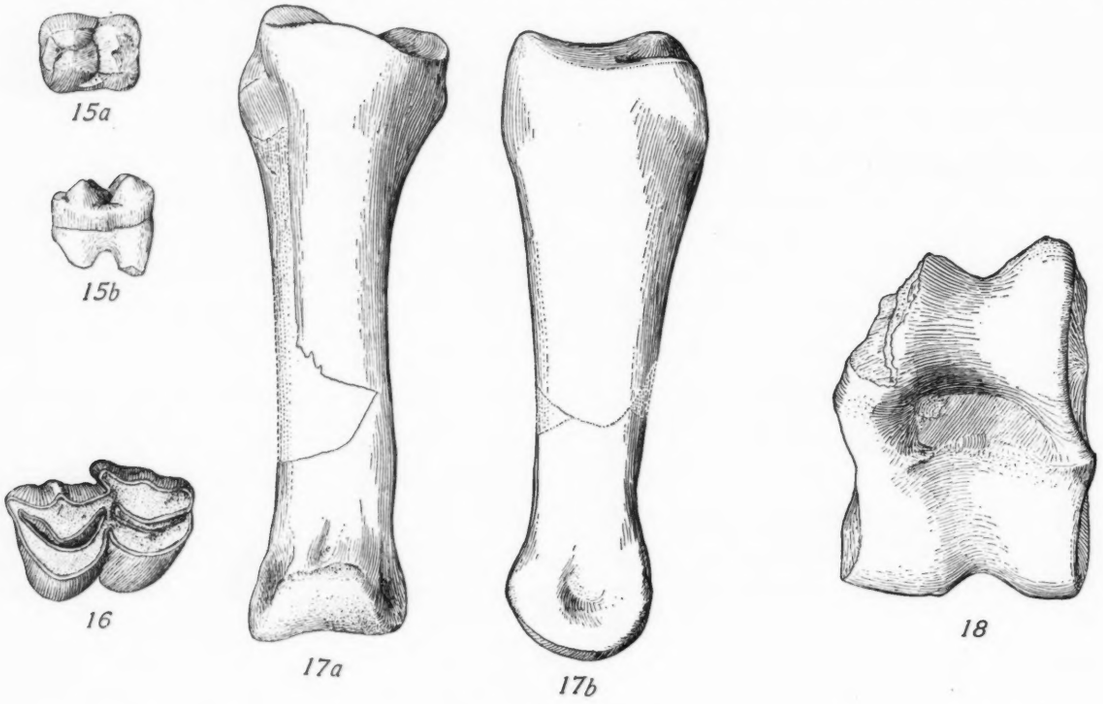
Prosthennops?, sp.

A single lower molar (no. 21511, Figs. 15*a* and 15*b*) from the Merychippus zone represents a peccary near *Prosthennops*. The tubercles are low and less acute than in *Bothriolabis* of the John Day.

[1] For method of measurement see Univ. Calif. Publ. Bull. Dept. Geol., vol. 7, p. 409, 1913.

MEASUREMENTS OF No. 21511.

	mm.
? $M_{\overline{2}}$, greatest anteroposterior diameter	14.5
? $M_{\overline{2}}$, greatest transverse diameter	11.4



FIGS. 15*a* and 15*b*. *Prosthennops* ?, sp. $M_{\overline{2}}$?, no. 21511, natural size. Merychippus zone, North Coalinga region, California.

FIGS. 16–18. *Procamelus* ?, sp. Merychippus zone, North Coalinga region, California. All figures natural size, Fig. 16, M^2, no. 21510, occlusal view. Figs. 17*a* and 17*b*, proximal phalanx, no. 21343; Fig. 17*a*, superior view; Fig. 17*b*, medial view. Fig. 18. Astragalus, no. 21344, superior view.

PROCAMELUS ?, sp.

The remains referred tentatively to *Procamelus* (Figs. 16 to 18) consist of several somewhat worn astragali, a portion of a calcaneum, a fragment of the distal end of a metapodial, a proximal phalanx and an upper molar. These specimens are near the size and dimensions of *Procamelus* forms from the late Miocene. The proximal phalanx (no. 21343) is much larger than that of the Recent llama. It is like that of *Procamelus*, but is larger than in some forms in the later Miocene. The astragali are a little larger than in the Recent llama, and smaller than in many specimens of *Procamelus*. An astragalus from locality no. 2028, in the Cedar Mountain region of Nevada, is similar in form, but a trifle larger. The astragali (no. 21344 especially) are almost identical in form with certain small cameloid astragali from the Upper Miocene of the Mohave Desert which are tentatively referred to *Procamelus*.

A single upper molar tooth, no. 21510, constitutes the only representation of the dentition.

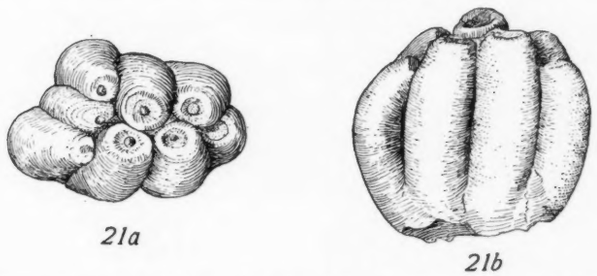
MEASUREMENTS OF UPPER MOLAR NO. 21510.

	mm.
? M^2, greatest anteroposterior diameter	26.2
? M^2, greatest transverse diameter	15.9

FIGS. 19*a*–20*b*. *Tetrabelodon* ?, sp. Cheek-teeth, × ½. Merychippus zone, North Coalinga region, California. Figs. 19*a* and 19*b*, occlusal and end view of fragment of cheek-tooth no. 21366. Fig. 20*a*, occlusal view of milk molar no. 21377; Fig. 20*b*, lateral view of specimen 21377.

TETRABELODON?, sp.

A considerable number of fragments represent a proboscidean near *Tetrabelodon* (Figs. 19*a* to 20*b*). The only complete tooth is a milk molar. Though certain generic determination does not seem possible, the absence of all proboscidean remains from beds older than Middle Miocene in America makes these specimens of importance in determining the epoch that this fauna could be presumed to represent.



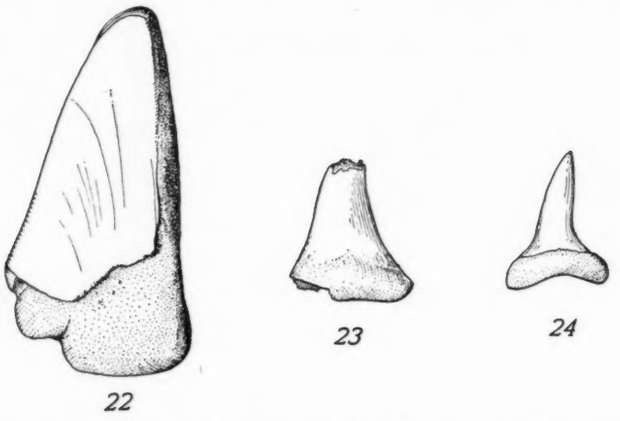
FIGS. 21*a* and 21*b*. *Demostylus*, near *hesperus* Marsh. M^1, no. 21375 × ½. Merychippus zone, North Coalinga region, California. Fig. 21*a*, occlusal view; Fig. 21*b*, lateral view.

DESMOSTYLUS, NEAR HESPERUS MARSH.

Two well-preserved cheek-teeth, an upper molar (Figs. 21*a* and 21*b*), and a lower molar, represent *Desmostylus*, the peculiar sirenian now known to have ranged widely around the North Pacific in Middle Tertiary time. In an earlier article[1] the writer has assembled all available information relating to *Desmostylus*, with the result that the best known occurrences seem to fall in the zone of the "Temblor" or "Vaqueros" of California. Other occurrences are apparently near that horizon. If the vertical range of this form is not unusually long, the wide geographic range

[1] Merriam, J. C., Univ. Calif. Publ. Bull. Dept. Geol., vol. 6, pp. 403–412, 1911.

through marine deposits of the North Pacific gives us an almost unprecedented opportunity for correlation, as the horses with which *Desmostylus* occurs in these beds have a wide geographic range over the land.



FIGS. 22–24. Selachian teeth from the Merychippus zone, North Coalinga region, California. All figures natural size.

FIG. 22. *Carcharodon*, sp. no. 21563.

FIG. 23. *Isurus*, sp. no. 21565.

FIG. 24. *Isurus?*, sp. no. 21564.

SELACHIAN TEETH.

Three types of shark teeth from the beds at locality 2124 represent forms corresponding approximately to species occurring in strata referred to the "Temblor" horizon of middle and southern California. One specimen (Fig. 22) is a fragment of a *Carcharodon* tooth from which the denticles have been removed by wear in shifting about in the sand before final burial. A second worn specimen (Fig. 23) is evidently a form of *Isurus;* a third (Fig. 24) is a worn *Lamna* or an *Isurus*.

STRATIGRAPHIC AND PALÆONTOLOGIC CORRELATION OF THE MERYCHIPPUS ZONE.

The appearance of a *Merychippus* fauna in the essentially marine section of the California area is of significance with respect to the broad problem of time relations of the West-American Tertiary. In considering the meaning of this occurrence, it is desirable: *first,* to determine as certainly as possible the position of the Merychippus zone in the marine section of the California area and to ascertain the basis for determination of age of this particular member of the California marine series; *second,* to determine the relation of the Merychippus fauna to those in the mammalian sequence of America, and the time relations of the American mammalian series to the succession in typical sections of the Old World.

Position of Merychippus Zone in the California Marine Series.—As has been

shown in the general discussion of occurrence of the Merychippus fauna, the evidence indicates that this zone is included within a portion of a series designated as "Temblor" by F. M. Anderson and referred to as "Vaqueros" in all recent accounts of this region published by members of the United States Geological Survey. The strata referred to the "Temblor" or "Vaqueros" of the North Coalinga field are evidently a part of a wide-spread series of deposits generally recognized as representing one great period of sedimentation and designated as the Monterey series.[1]

In the earliest attempt at separation of faunal zones in the section between the Tejon Eocene and the San Pablo Miocene the writer[2] contrasted the fauna of the lowest portion of the beds referred to the Monterey series in the Contra Costa Hills with that of higher members of that section, exclusive of the contrast between sandstone and shale members due to varying conditions of deposition. The faunal zone of the lowest beds above the Tejon Eocene of the Contra Costa Hills was designated the Agasoma gravidum zone, and was considered as Lower Miocene. This fauna was compared with that of beds presumed to be of Lower Miocene age in the southern part of the state. Two faunal assemblages from the southern area were referred to the Lower Miocene. One assemblage, characterized by the presence of *Turritella ocoyana* and a number of *Agasoma* species, was referred to as the Turritella ocoyana zone. The other fauna, characterized by *Turritella inezana* and an *Agasoma*, was distinguished as the Turritella (hoffmani) inezana[3] zone. It was suggested that these zones did not represent the same horizon, but that they were not widely separated. The ocoyana zone was shown to have a fauna of more recent aspect than that of the inezana zone. The *Agasoma* forms of the ocoyana zone were described as in some respects intermediate between *Agasoma gravidum* of the Contra Costa Hills, and *Agasoma sinuatum*, a species from the upper portion of the Contra Costa Hills section.

Following shortly after the faunal separation of the Agasoma gravidum zone, Hamlin[4] described as the "Vaqueros formation," a section containing the fauna of the Turritella inezana zone. Later, from the southern part of the state, F. M. Anderson[5] described as the "Temblor formation" an important series of strata representing in part at least the Turritella ocoyana zone.

F. M. Anderson and Arnold and Anderson have considered the "Vaqueros"

[1] Lawson, A. C., Univ. Calif. Publ. Bull. Dept. Geol., vol. 1, pp. 1–59, 1893. See also Louderback, G. D., Univ. Calif. Publ. Bull. Dept. Geol., vol. 7, pp. 177–241, 1913.

[2] Merriam, J. C., Univ. Calif. Publ. Bull. Dept. Geol., vol. 3, pp. 377–381, March, 1904.

[3] *Turritella hoffmani* Gabb has been shown by Arnold to be identical with the previously described *T. inezana* of Conrad.

[4] Hamlin, H., U. S. Geol. Surv. Water Supply Paper No. 89, distributed June 15, 1904. Hamlin's use of name preceded by H. W. Fairbanks, U. S. Geol. Surv. Folio 101, distributed June 10, 1904.

[5] Anderson, F. M., *Proc. Calif. Acad. Sci.*, 3d ser., Geol., vol. 2, p. 170, 1905.

of Hamlin and the "Temblor" of F. M. Anderson as practically representing the same stratigraphic and faunal unit. Arnold and Anderson, by virtue of their decision as to the stratigraphic identity of the deposits containing these two faunas, have used the name "Vaqueros"[1] as having precedence in use over "Temblor." F. M. Anderson,[2] also holding that the two zones represent the same stratigraphic unit, considers the description of the "Vaqueros" presented by Hamlin as insufficient. In reality the description by Fairbanks, based on a typical section, precedes that of Hamlin and establishes the name.

The most recent investigations of the stratigraphic, geographic, and faunal relations of the Turritella inezana and Turritella ocoyana zones seem to the writer still to suggest that these zones may represent recognizable stages in the evolution of the Tertiary marine faunas of the California area, and that there is a possibility of stratigraphic separation of the two. Under the circumstances, the writer is inclined to continue his use of the faunal designations of these horizons suggested in 1904. If the faunal separation is continued, and if the stratigraphic relations are not clear, it seems desirable to use for geologic reference to the strata containing these faunas terms which suggest their geographic occurrence. The writer is therefore inclined for the present to use the name "Vaqueros" for the series of strata containing the *Turritella inezana* fauna. The name "Temblor" is tentatively used for those sections in the southern part of the state containing the *Turritella ocoyana* fauna, on the presumption that this part of the series may be stratigraphically distinct from the "Vaqueros." It is probable that the "Temblor" as thus designated is really a part of the Monterey series as now recognized.

In a recent discussion of the geologic range of Miocene faunas of California, Professor James Perrin Smith[3] has taken the view that the "Temblor" fauna was synchronous with that of the Monterey, and that the Turritella inezana zone represents a horizon below the typical Monterey. The name "Vaqueros" is used by Professor Smith for the beds containing the *Turritella inezana* or "Vaqueros fauna."

The available evidence indicates that the Merychippus zone occupies a position within the "Temblor," and presumably some distance below the upper limit of the Monterey series. If the beds of this zone were considered unconformable upon the "Temblor," they would presumably correspond in time equivalence to some portion of the upper Monterey beds as represented in middle California. The "Big Blue" resting above the Merychippus zone is separated by marked unconformity from the "Santa Margarita" above it. The "Santa Margarita" of the

[1] Arnold, R., and Anderson, R., U. S. Geol. Survey Bull. 398, p. 87, 1910.

[2] Anderson, F. M., *Proc. Calif. Acad. Sci.*, 4th ser., vol. 3, pp. 38 and 39, 1908.

[3] Smith, J. P., *Proc. Calif. Acad. Sci.*, 4th ser., vol. 3, pp. 164, 165 and 169, 1912.

Coalinga region, according to Dr. Bruce Clark,[1] contains a fauna closely similar to that of the middle San Pablo. The unconformity between the "Santa Margarita" and "Big Blue" apparently marks a long period, which probably represents a time sufficiently long to equal the period of deposition of the earlier portion of the San Pablo.

Age of the Monterey.—During several decades past the Monterey series has been considered as Miocene and strata containing a faunal assemblage of the stage seen in the "Temblor" of the Coalinga region have been generally recognized as Lower Miocene. In actual practice the determinations of age of strata referred to the Miocene have never been based on very extensive or exact comparisons.

In describing a collection of fossils from Astoria in 1849 Conrad[2] expressed his view as to age determination of fossiliferous deposits as follows:

". . . the forms are decidedly approximate to those of the Miocene period which occur in Great Britain and the United States. *Nucula divaricata*, for instance, closely resembles *N. Cobboldiæ* (Sowerby) of the English Miocene, and *Lucina acutilineata* can scarcely be distinguished from *L. contracta* (Say), a recent species of the Atlantic Coast and fossil in the Miocene beds of Virginia. *Natica heros*, a shell of similar range, is quite as nearly related to the *N. saxea*. A similar number of species might be obtained from some of the Miocene localities of Maryland or Virginia and yet no recent species be observed among them. In the Eocene, and also in the Miocene strata, there are peculiar forms which obtain in Europe and America, and although the species differ, yet they are so nearly allied that this character alone, independent of the percentage of extinct forms, is quite a safe guide to the relative ages of remote fossiliferous rocks. On this foundation, I speak with confidence, when I assign the fossils of the Columbia River to the era of the Miocene."

In one of the first references to the Miocene of California, Conrad,[3] in speaking of beds at Santa Barbara, called Miocene, expresses the following view as to the age of the strata:

"The *Mercenaria* and *Pecten* are closely related to species of the Virginia Miocene, and indeed there is an extraordinary analogy in all of the above mentioned shells to species of the Atlantic Miocene deposits. . . ."

Discussing the age of fossil-bearing deposits in California in 1857 Conrad[4] makes the following statement:

"Like the Miocene of Virginia, the Estrella group is characterized by large and even comparatively gigantic species of Pectinidæ, so unlike any living on the coast of California or the Atlantic

[1] Clark, B., Unpublished manuscript on the Fauna of the San Pablo Miocene.

[2] Conrad, T. A., U. S. Exploring Expedition, 1849, p. 659 (see original reference, *Am. Jour. Sc.*).

[3] Conrad, T. A., *Proc. Acad. Nat. Sc. Philad.*, vol. 7, p. 441, 1834.

[4] Conrad, T. A., Pacific Railroad Survey, vol. 7, p. 189, 1857.

states. It would seem that this family had then reached their maximum of development and the genus *Pallium* was first introduced, and of far larger size than any which now exists. It is worthy of remark that the generic character is developed on a far grander scale than appears in subsequent epochs, the prominent teeth and thick hinge reminding us of the genus *Spondylus*.

"Every new collection of Miocene fossils shows more clearly the connection between some of the Tertiary strata of California and those of Virginia. The species in the present collection are far more interesting than in others of the same formation on the Pacific slope which I have yet seen. It does not appear that this group of fossils has any living representative in the present fauna of the Pacific Coast, but several of them approximate to extinct Virginia species; and I am not sure that the large *Pecten magnolia*, herein described, is not identical with the Virginia species *P. jeffersonius*.

"I think it might safely be assumed that the San Rafael Hills, Santa Inez Mountains and Estrella Valley contain strata which are parallel to Miocene sands and clays of the James and York rivers in Virginia."

Later writers have generally followed the lead of Conrad in fixing the age of middle Tertiary strata, including the Monterey. In an early descripion of the type locality of the Monterey, Blake[1] simply refers to it as Tertiary. In the description of these beds by Lawson[2] the statement is made that characteristic Miocene fossils have been found in the Monterey series at various parts of the coast by former observers, and in particular at the town of Monterey. A list of species obtained from a locality near Carmelo Bay, and determined by W. H. Dall, is published in Professor Lawson's paper.

In recent years, the results of work on the Tertiary of California have brought to light previously unknown thicknesses of strata, new and important unconformities indicating large gaps in the record, and new faunal zones intercalated between those previously known. The addition of these factors, which lengthen the geologic record, has placed in the division recognized as Miocene time an unexpectedly long series of events, and has very naturally raised a question whether the lowest beds referred to the Miocene do not represent Oligocene. In a discussion of the geologic range of *Desmostylus*,[3] a sea-cow, in 1911, the writer described this form as most common in beds called Lower Miocene of the southern part of the state, and suggested that the beds marking its downward limit of geologic range might correspond to Oligocene.

Recently Arnold[4] and Hannibal, in a classification of the Tertiary formations of the Pacific Coast region, have stated that the "Monterey . . . as far as our present knowledge goes, might be placed equally well in the latest Oligocene or the earliest Miocene on the basis of the general faunal facies."

[1] Blake, W. P., Pacific R. R. Report, vol. 5, chapter 13, 1856.

[2] Lawson, A. C., Univ. Calif. Publ. Bull. Dept. Geol., vol. 1, p. 27, 1893.

[3] Merriam, J. C., Univ. Calif. Publ. Bull. Dept. Geol., vol. 6, p. 407, 1911.

[4] Arnold and Hannibal, H., *Proc. Amer. Phil. Soc.*, vol. 52, p. 575, 1913.

Though practically all determinations of the age of marine Tertiary formations in California have been based upon palæontologic correlation, the true basis of comparison of these horizons has never been clearly expressed. The determinations were made either by very general correlation with determined faunas of eastern United States and Europe, or by use of the Lyellian percentage method.

The comparisons of West Coast faunas of post-Eocene age with those of the Atlantic area, upon which the original age determinations of Conrad and Gabb were based, were made at a time when the faunas of the Atlantic area were not as well known as at present, and the study of the Pacific Coast faunas was hardly begun. The results of a thorough comparison with the Atlantic faunas are not available to us as the basis for a fully satisfactory judgment from this point of view.

In connection with a recent study of the marine Tertiary of the Washington-Oregon area, Arnold and Hannibal[1] express the opinion that "a direct correlation between the Pacific Coast marine Tertiary and the deposits of Europe and bordering the Gulf of Mexico is impossible owing to almost total absence of identical species except in the Eocene." In consideration of possible correlation of strata on the Pacific Coast region with those of the Oligocene of Europe, attention is called to the fact that there do not appear to be any marine forms that will serve as a basis for direct comparison. The suggestion is, however, made that an Oligocene facies is indicated by absence of all Recent molluscan species from the older strata considered as post-Eocene, and the gradual addition of a small percentage of Recent species in the Monterey. The presence of certain Eocene-Oligocene genera as *Crassitellites*, *Aturia*, etc., also suggests Oligocene age.

A number of age determinations of Pacific Coast Tertiary formations have been based upon use of the percentage method, and while fairly consistent results were secured, it must be borne in mind that determinations obtained by this means may be subject to modification by several factors. It is, for example, evident that since the time of Lyell views as to the limits of species have greatly changed, and the percentages used for the species of Lyell's day would not apply with the same result upon the same faunas with present-day limitations of species. Species as now defined are much more closely restricted in range of characters. This limitation of the characters in specific groups has naturally restricted also the geographic and geologic range. Studies in the general accuracy of percentage method age-determinations recently made by Bruce Martin at the University of California have shown surprisingly large errors from various sources in ordinary computations, unless a very full representation of the fauna is available.

[1] Arnold, R., and Hannibal, H., *Proc. Amer. Philos. Soc.*, vol. 52, p. 574, November–December, 1913.

Aside from purely palæontologic determinations of the age of Tertiary formations in the California area, we have practically nothing as a basis for definition of historic stage in comparison with the later portion of geologic history in other parts of the world. Though every one admits the exceeding desirability of some method of comparison connected solely with a study of crustal movements, scarcely any attempt has been made to apply such a means of determination for stratigraphic units of this region older than the close of the Pliocene. Presuming that we have in the Pleistocene a satisfactory medium of comparison through crustal movement, without reference to climatic and faunal characters which are commonly the means of correlation employed, there seems no satisfactory method of comparison for systems older than Pleistocene other than by approximate matching of unconformities.

The California Tertiary geologic sequence is notoriously complicated. Crustal movements have been frequent; unconformities are common; and it is difficult to see how any one working in California geology up to the present time could have selected the particular unconformities or determined the movements needed to give us the key to comparison of our Tertiary history with that of eastern United States or of Europe. At any rate, this has not been done with any assurance of accuracy. At best there could be nothing more than a rough approximation based on the assumption that some of the diastrophic events here have a definite relation to those that have occurred elsewhere.

A general consideration of the present situation with reference to determination of age of the members of the California Tertiary sequence makes it evident that our present determinations are not based on thoroughly satisfactory studies in any of the several possible fields of endeavor, and that much work may profitably be given to an examination of this problem.

Position of the Merychippus Fauna in the Mammalian Sequence of America.—In terms of the vertebrate series of western North America, the fauna of the Merychippus zone in the North Coalinga region is clearly later than Lower Miocene, and not later than Upper Miocene. Proboscidean remains are not certainly known in America at a horizon lower than Middle Miocene. The wide-spread genus *Merychippus* is not known earlier than Middle Miocene, and the characteristic forms of that genus in the Middle Miocene of America are not more advanced than the species of the Merychippus zone in the Coalinga region. The number of horse remains found gives ample opportunity to determine the stage of evolution of the cheek-tooth dentition, and it seems scarcely possible that there could be any misunderstanding as to the stage in the history of this group represented here. The available material representing camels does not suggest an age determination different from that based on the representatives of the horse and mastodon groups.

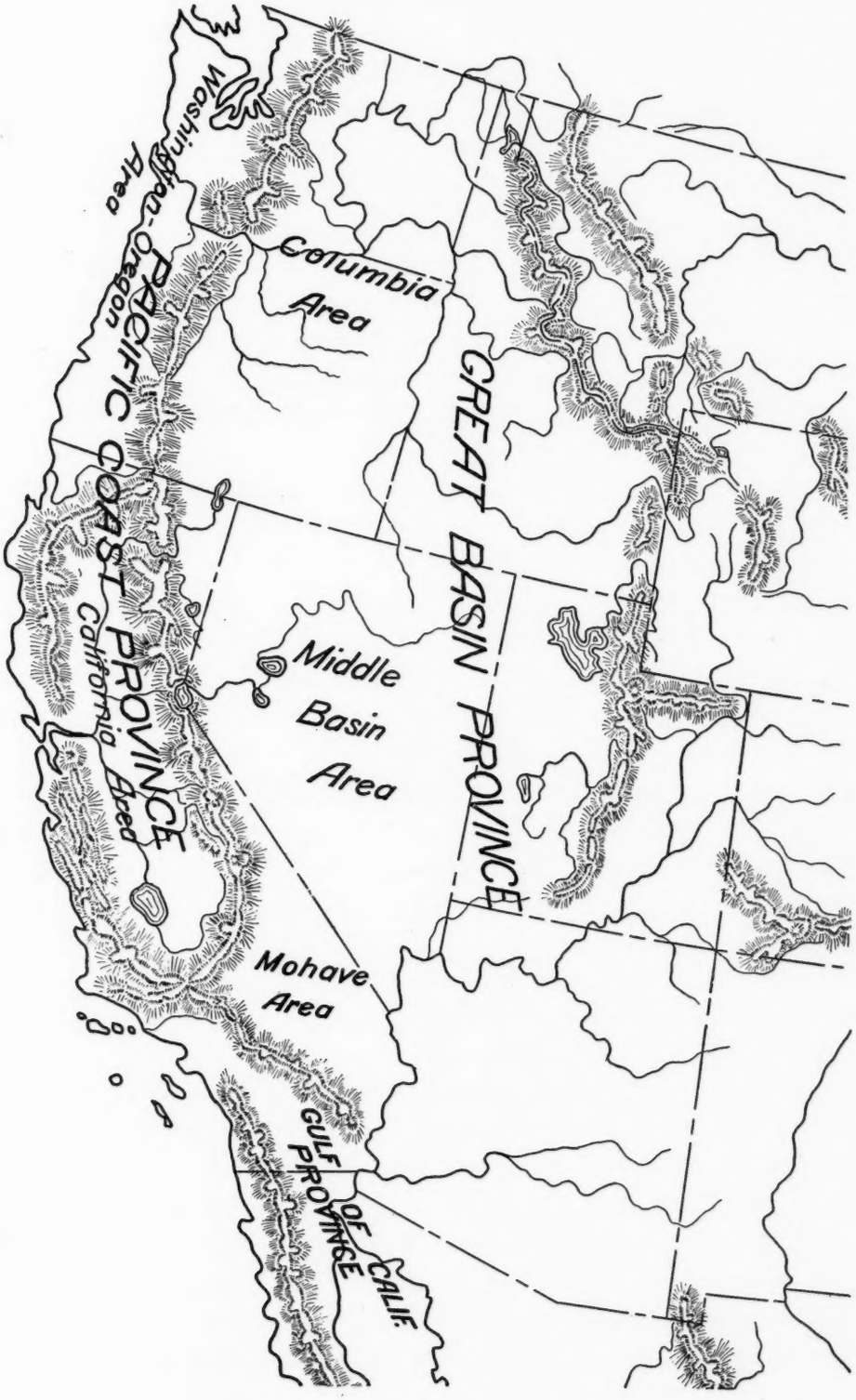


FIG. 25. Sketch map of Pacific Coast and Basin regions showing geographic divisions based on distribution of Tertiary faunas.

Whatever may be the relation of the divisions of the American continental Miocene fauna or formation sequence to the sequence of the Old World based upon the succession of mammalian forms, it seems clear that the Merychippus zone of the Coalinga region is near the stage of the Great Basin Middle Miocene fauna represented in the Mascall of eastern Oregon and the Virgin Valley of the northern Nevada region. Through the medium of comparison with the Great Basin sequence, there is justification for correlating the stage of the Merychippus zone approximately with that of the Pawnee Creek beds of northeastern Colorado and other occurrences of beds referred to Middle or late Miocene in Western North America.

The only means of escape from the conclusions stated above as to relative age of the Coalinga *Merychippus* fauna would presumably be reached through consideration of the possibility that the fauna of the California Merychippus zone originated west of the Wasatch, or that it first appeared in California or possibly in the Great Basin and remained in the extreme west for some time before migrations populated the Great Plains area.

The presence of a form of the mastodon type in the Coalinga Merychippus zone may suggest the possibility that this fauna represents the earliest appearance in America of Old World immigrants not known until considerably later in the region to the east. While it is possible that Asiatic forms colonizing America followed the west coast of the continent south to California, it is unsafe to consider this as necessarily the most inviting path for migration. Other routes leading into the Great Plains region may have been easily accessible. Aside from the California specimens under consideration here, the oldest American representatives of the mastodon group are found in formations situated to the east of the eastern boundary of the Great Basin.

The *Merychippus* group evidently originated in America. The Equidæ were largely represented here in late Oligocene time, and were unrepresented at that stage in the Old World so far as known. Unfortunately for consideration of the possibility of western origin of *Merychippus* types, we have, as yet, no representation of the mammalian fauna of the Great Basin province between the close of the John Day Oligocene and the beginning of the Middle Miocene. As nearly as can be judged, this interval represents a long period. During the earlier portion of this time the John Day beds were subjected to erosion. In the latter part of the interval the Columbia Lava accumulated in the northern portion of the province, and great rhyolite outbursts seem to have occurred in the Middle Basin area. It seems that the Great Basin province may have been a region with somewhat varied relief in which no extensive accumulation of deposits occurred during the first part of the

Lower Miocene, and was widely covered with lava flows in the second part of this epoch.

As *Merychippus* is an animal with tooth and foot structure better adapted to grazing and to plains-living than the known American Lower Miocene horses, it possibly developed in the Great Basin on wide stretches of plains in early Lower Miocene, or on the great lava plains of late Lower Miocene. So far as environmental conditions have influence in the evolution of specializing types, one might expect to find such a form as *Merychippus* arising in response to the stimulus of environment on the considerably greater areas of the Great Plains country farther to the east, rather than in the Great Basin.

The further possibility is available that *Merychippus* originated in Lower Miocene time on the extreme western border of the continent, in California.

According to the view that the greatest number of new progressive forms will arise on the largest continental masses, one almost unavoidably considers the middle of the continent as the locus of evolutionary or creative activity. This is true mainly in the sense in which the center of gravity of any mass is the controlling point. It may be that temporarily isolated marginal areas bordering large land masses furnish at least as satisfactory points of origin or nurseries of new forms as could be provided by any set of conditions on other portions of a large land mass, and possibly the California region has been favorably conditioned for inducing the development of new types of mammals.

The California area, as it appears today, offers on the plains of the Great Valley an unusual combination of conditions which might well be favorable to production of a new type of horse. In the time of deposition of the Merychippus zone of Coalinga, a large part of the Coast Range region and the southern end of the Great Valley were under water, and the degree of isolation conditioned by the Sierra boundary was probably less marked than at the present day. For a considerable time preceding the deposition of the Merychippus zone, conditions were not materially different from those during the deposition of the beds at that horizon. Somewhat earlier the land area was larger. The conditions do not, however, seem to have been as favorable for the development of the *Merychippus* group in this area as they probably were in many other regions. At least, there is no special warrant for supporting the hypothesis that California was the point of origin of this group.

On the whole, the evidence indicates that the fauna of the Coalinga Merychippus zone is at least as advanced as that of the Mascall and Virgin Valley formations of the Great Basin; and that there is no reason for assuming that California was the place of origin of this group. It also appears that the Mascall and Virgin Valley

faunas are not younger than the stage of evolution of assemblages considered to represent the Middle Miocene of the areas to the east, and that there was probably close faunal connection with the region to the east. The Middle Miocene fauna of the Great Basin evidently represents approximately the same period as that of other areas farther to the east. There is no reason for believing that the Great Basin fauna in general or the *Merychippus* group in particular necessarily originated in this region. A portion of the fauna derived from Asiatic immigration may have appeared here before it reached the Great Plains area, but the chances are against its having been largely represented here before it was known elsewhere on the continent. The possible Californian or Great Basin origin of Miocene groups should not be overlooked in further consideration of the Pacific Coast faunas, but it is improbable that the geographic provinces were so sharply separated in early Miocene time that a group of the *Merychippus* type could become widely spread in the Great Basin and Pacific Coast provinces and remain unrepresented east of these regions.

Relation of American Miocene Mammal Horizons to Those of the Old World.—In correlating marine Middle Tertiary faunas of western North America with those of Europe, it is difficult to make direct comparisons, as the North Pacific Ocean was connected with the marine area bordering Europe only by circuitous routes, or by paths leading through zones of widely varying temperatures. For this reason, one might expect to find relatively little correspondence, excepting in similarity of stage of evolution of forms surviving from faunas common to both regions at some earlier period. There is little known in the comparative history of marine life suggesting migration of faunas between these areas in middle Cenozoic time to such an extent as to leave a distinct impression.

In a comparison of the history of the mammalian fauna of North America with that of the Old World, one might expect to find that the grouping of continents around the North Pole has permitted moderate crustal movements to bring about occasional union of those land masses, and that this connection of the areas would invite faunal mingling.

In the critical regions where connection and separation of the continental areas might be expected, especially the Alaskan-Siberian region, there is evidence of varying position of land and water, suggesting alternations of union and separation of the land areas. Whatever inhibitive influence might be exerted by an arctic climate, such as that of the present day, was largely eliminated throughout the greater part of Tertiary time, as the earth climate was somewhat warmer than at present, and the polar regions did not present an insuperable temperature barrier.

In the Tertiary history of mammalian faunas in the Northern Hemisphere, we

note certain epochs marked by similarity of life over nearly the whole region. These epochs alternate with times of development of provincial faunas. Without calling in the suggestion of independent origin of similar types in different regions too frequently to satisfy the law of probability, it is difficult to account for these recurrences of similar faunas unless we accept the view that they have been induced by crustal movements uniting and separating land areas, and also closely related to or concurrent with considerable climatic variation. The opportunity for intermigration of faunas on the land areas concerned appears on the whole better than in the case of marine life in the bordering seas, as the path of migration was shorter and involved less range of latitude. Other factors being approximately even, in a study of the Tertiary of the Northern Hemisphere, one might expect more satisfactory results in correlation based on comparative studies in the history of land mammals than in comparison of marine invertebrates.

Through the combined work of many investigators, and especially by the comparatively recent assembling of all results in this field by Osborn, Matthew, Scott, Schlosser, and others, it is evident that the portion of Tertiary time, including Oligocene, Miocene, and Pliocene, is actually marked by important intermigrations of land mammals between America and Eurasia. Within this time, the early Oligocene is characterized by appearance of similar forms in North America and in the Old World. In late Oligocene, intermigration seems less marked. In Middle Miocene the movement is particularly noticeable again, especially in the first certain appearance in America of the proboscideans, the teleocerine rhinoceroses, and probably several carnivore groups. In later Miocene and Pliocene time, the number of common forms increases considerably.

What is known as the Middle Miocene mammalian fauna of America shows some resemblance to that of the Lower Miocene of Europe, and question has been raised as to the time relations of the lower and middle divisions. In many respects there seems, however, closer correspondence of the American Middle Miocene, as represented by the Mascall and Pawnee Creek faunas, with the Middle Miocene of Europe than there is between these American faunas and that of any European Lower Miocene division. There is general agreement in the succeeding stages of the two areas, which indicates that the mammalian sequences of North America and Eurasia are brought into approximately the same relation throughout the later Tertiary series. If any difference exists, or if further adjustment occurs in the scale, it is doubtful whether this modification will do more than to bring to the same level the upper portion of the Lower Miocene of Europe and the lower portion of the Middle Miocene of America.

SIGNIFICANCE OF THE MERYCHIPPUS ZONE OF COALINGA WITH RELATION TO THE PROBLEM OF CONSTRUCTING THE WEST AMERICAN TERTIARY GEOLOGICAL SCALE.

A summation of results obtained in a study of the position and relationships of the zone containing a *Merychippus* fauna in the North Coalinga region of California may be stated as follows:

1. The Merychippus zone is evidently included within the limits of the Monterey series of California as now defined.

2. The Merychippus zone evidently represents the faunal stage of the Turritella ocoyana zone, and the stratigraphic stage of the "Temblor" beds of F. M. Anderson.

3. The fauna of the Merychippus zone is not older than the stage of the Middle Miocene as represented by the Mascall and Virgin Valley faunas of the Great Basin region, and by the Pawnee Creek stage of the Great Plains area.

4. It seems inconceivable that the fauna of the Merychippus zone can represent any stage of the Oligocene of the mammal-bearing beds of western North America. The possibility that it represents any portion of the mammal-bearing Lower Miocene of this continent carries with it the supposition of wide divergence of faunas in different areas of America in Lower Miocene time.

5. The Mascall and Virgin Valley formations of the Great Basin region are of approximately the same age or older than the "Temblor."

6. The lack of adjustment between the time scale of the California area of the Pacific Coast Marginal Marine province and that of the Great Basin province suggests that correlation dependent on use of percentages of marine molluscan species as commonly defined has tended to place time divisions relatively too far from the present, in comparison with the original time scale of Europe. The lack of adjustment suggests further that the Middle Miocene mammal-bearing beds of North America should possibly occupy a lower position in the scale, in comparison with the scale of Europe, than that to which they have generally been assigned.

JACALITOS FAUNA.

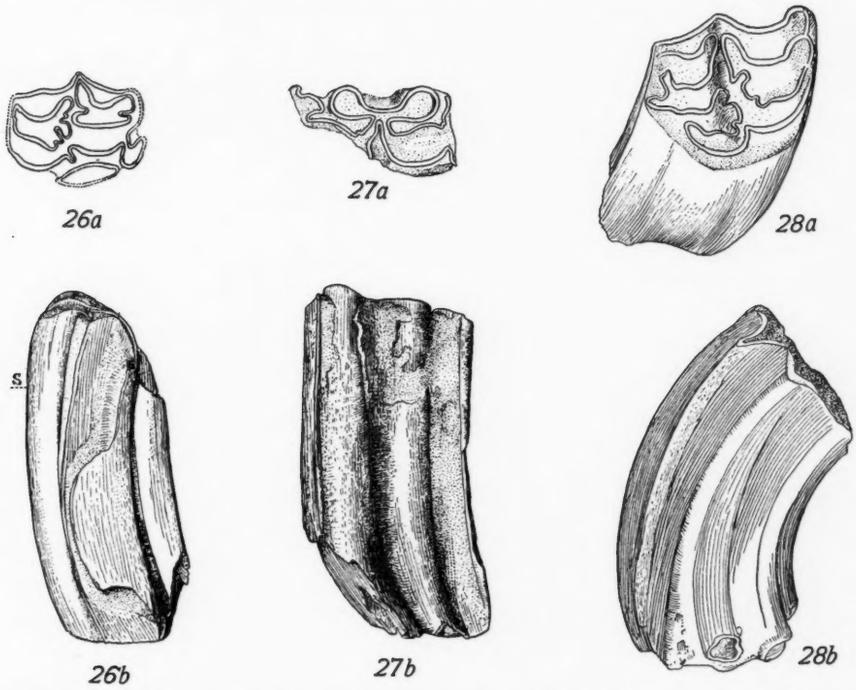
Within the limits of the beds mapped as Jacalitos a few vertebrate remains representing *Pliohippus?* were reported by Arnold and Anderson.[1] Additional material of this horse and specimens representing a *Neohipparion* species have been found by investigators since that time.

[1] Ralph Arnold and Robert Anderson, U. S. Geological Survey Bull. 398, p. 300, and pl. 33, figs. 3 and 3a, 1910.

Pliohippus or Protohippus.

One specimen figured by Arnold and Anderson was determined by J. W. Gidley as *Pliohippus*. It represents a species of medium size with somewhat flattened protocone, and wide fossettes with slightly crinkled enamel walls. (See Figs. 28*a* and 28*b*.) Two teeth of a type similar to that from the Jacalitos figured by Arnold and Anderson were presented to the University by F. M. Anderson. They are two upper molars rather strongly curved, and with the protocone connected with protoconule. They were obtained in Sec. 35, T. 19 S., R. 15 E., M. D. B. and M. Section 35 is largely covered by the upper portion of the Jacalitos, with Etchegoin on the east side. The chances favor the occurrence of these teeth in the upper portion of the Jacalitos.

Fragments of horse teeth found by the University party in December, 1913, at three additional localities in the area mapped as Jacalitos represent advanced types of horses. All of these localities are at the base of the Jacalitos.



Figs. 26*a* and 26*b*. *Neohipparion molle* Merriam $M^{\frac{3}{3}}$?, no. 21370, natural size. Lower Jacalitos, North Coalinga region California. Fig. 26*a*, section of tooth at point indicated by *s* on Fig. 26*b*; Fig. 26*b*, inner view.

Figs. 27*a* and 27*b*. *Neohipparion*, sp. $M_{\frac{3}{3}}$?, no. 21369, natural size. Lower Jacalitos, North Coalinga region, California. Fig. 27*a*, occlusal view; Fig. 27*b*, outer side.

Figs. 28*a* and 28*b*. *Pliohippus*?, sp. $M^{\frac{3}{3}}$, no. 165665, U. S. National Museum, natural size. Jacalitos formation, North Coalinga region, California. Adapted from Arnold and Anderson. Fig. 28*a*, occlusal view; Fig. 28*b*, posterior ? view.

NEOHIPPARION.

An interesting upper molar three (Figs. 26*a* and 26*b*, no. 21370, locality 2076) found by J. H. Ruckman at the base of the Jacalitos, but below the line as mapped by Arnold and Anderson, represents a species of *Neohipparion*. The much flattened protocone of this tooth is widely separated from the protoconule. The fossettes are rather narrow, and their walls are only moderately plicated.

Compared with a number of practically perfect specimens of M^3 in the *Hipparion* collection from the Ricardo fauna of the Mohave area, this tooth is somewhat longer, the walls of the fossettes show less plication, the protocone is absolutely wider anteroposteriorly and much narrower transversely. The species represented by specimen 21370 is evidently distinct from the Ricardo forms, and from all other described *Hipparion* species of the Pacific Coast and Basin provinces. Whether it is a more or a less advanced species than the Mohave form is not entirely clear. The slightly greater length of crown, and the large, much-flattened protocone may indicate a more advanced stage in the Coalinga species. The *Neohipparion* species represented by no. 21370 is described as *Neohipparion molle*. This species is characterized by length and narrowness of upper molar crown, simplicity of enamel borders of the narrow fossettes, and unusually large anteroposterior diameter of the laterally compressed protocone.

A fragmentary long-crowned lower molar (no. 21369, Figs. 27*a* and 27*b*) from locality 2126 is very narrow transversely. The metaconid-metastylid column is very long anteroposteriorly, while the inner groove on this column is wide and deep. The anteroexternal region shows the sharp angle commonly seen in *Hipparion*. The fold separating the protoconid and hypoconid inserts its inner end between the folds separating protoconid and metaconid and hypoconid and metastylid. This tooth corresponds in many characters to DM_3 of *Equus occidentalis*, but is much too long to represent a temporary tooth. The characters of this tooth correspond in general to those of *Hipparion*, but it does not resemble the forms from the Ricardo fauna. In the Coalinga specimen the metaconid-metastylid column is wider anteroposteriorly, the groove between the metaconid and metastylid is wider, and the characters in general are suggestive of a more specialized form than those of Ricardo.

The Coalinga form seen in no. 21369 most nearly resembles a *Hipparion* or *Neohipparion* type quite certainly derived from the Rattlesnake Pliocene of the John Day region in eastern Oregon. Specimen 554 in the University collections from the John Day Valley was found in a basin containing both Rattlesnake Pliocene and Mascall Miocene, but the matrix covering the specimen is like the reddish-brown Rattlesnake beds, and very different from the white or gray Mascall beds of this

area. The $M_{\overline{2}}$ of this Rattlesnake specimen has dimensions near those of the Coalinga specimen. The metaconid-metastylid column shows a somewhat flatter outer groove in no. 21369, but this may be due to difference in wear. The Rattlesnake specimen shows a very prominent fold on the anterior side of the hypoconid. The corresponding region of the Coalinga specimen is broken away, but there is a sudden bend in the small portion of the wall remaining at this point suggesting the presence of this fold.

A *Hipparion* or *Neohipparion* form from the Thousand Creek beds of northern Nevada shows characters near those of the Rattlesnake form, and suggests the structure in no. 21369 from the Coalinga region, but these resemblances do not necessarily mean specific identity, and the Thousand Creek form may represent a distinct type.

It is perhaps significant that both the upper and the lower teeth referred to the genus *Hipparion* or *Neohipparion* suggest a type different from that of the well-known Ricardo species, and possibly more advanced. The Jacalitos species seems most nearly related to types of the Rattlesnake beds.

MEASUREMENTS.

	No. 21370 Coalinga.	No. 21311 Ricardo Unworn.	No. 21369 Coalinga.	No. 19847 Ricardo.	No. 554 Rattlesnake.	No. 19414 Thousand Creek.
$M^{\overline{3}}$, anteroposterior diameter	19.3 mm.[1]	21				
$M^{\overline{3}}$, transverse diameter	15.8[1]	17.5				
$M^{\overline{3}}$, length of crown	48	38				
$M^{\overline{3}}$, anteroposterior diameter of protocone at middle height of crown	9.4	7.3				
$M_{\overline{2}}$, greatest anteroposterior diameter			23.4	24.5	25.9	29
$M_{\overline{2}}$, transverse diameter			11.4	9	10.5	11
$M_{\overline{2}}$, length of crown			48 worn	48.9	57.5 worn	66
$M_{\overline{2}}$, anteroposterior diameter of metaconid-metastylid column			15	11.1	14.4	14.6

AGE OF JACALITOS VERTEBRATE FAUNA.

The Jacalitos fauna as now known is characterized by the presence of *Neohipparion* occurring only in the lowest beds, and by *Pliohippus* or *Protohippus* apparently occurring a little higher than the *Neohipparion* specimens in the basal portion of the section. Specimens resembling the *Pliohippus?* of this formation are presumed to come also from the upper portion of the section.

[1] Approximate.

The time relations of the Jacalitos vertebrates to the faunas of the Great Basin Province are not entirely clear, but the closest relationships seem to be with the Lower Pliocene.

FAUNAS OF THE ETCHEGOIN AREA.

Vertebrate remains were found at numerous localities in the area mapped as Etchegoin. The forms obtained fall naturally into two groups. One fauna was found at a series of localities near the base of the section, the localities being in or above the zone of the pelecypod *Glycimeris coalingensis*. The other localities are scattered over an area underlain by the upper portion of the Etchegoin of this section.

PLIOHIPPUS COALINGENSIS ZONE.

At several localities near the base of the Etchegoin section in this region, a fauna is found including remains of horses of a considerably more advanced type than those of the Jacalitos. The most characteristic form is *Pliohippus coalingensis*. This fauna is best known from localities 2073, 2074, 2078, and 2090. It comprises the following forms:

Pliohippus coalingensis (Merriam).
Pliohippus?, sp. small.
Procamelus?, sp.
Platygonus?, sp.

The faunal zone near the base of the Etchegoin in the North Coalinga region represents a distinctly more advanced stage than that of the Jacalitos.

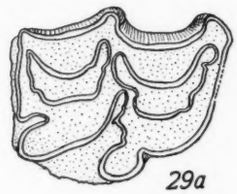
PLIOHIPPUS COALINGENSIS (Merriam, J. C.).

Protohippus coalingensis Merriam, J. C., *Science*, N. S., vol. 40, p. 645, August, 1914.

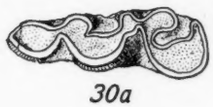
Type specimen no. 21341 from locality 2073. Lower Etchegoin ten miles north of Coalinga, California. A species of medium size, approaching the dimension of *Pliohippus supremus*. The type specimen (Figs. 29*a* and 29*b*) differs from the type specimen of *P. supremus* in the heavier mesostyle, relative narrowness of the crown even toward the base, narrower fossettes, much less complicated enamel walls of the fossettes, and much smaller more nearly circular protocone.

The nearest approach to this form among the Equidae known west of the Wasatch is *Pliohippus fairbanksi*, a species imperfectly represented in the Ricardo beds of the Mohave Desert. The lower Etchegoin form resembles the Ricardo species approximately in size, in simplicity of the fossettes, in the very small, round protocone, and weak connection of the protoloph and metaloph. The crown of the Etchegoin form

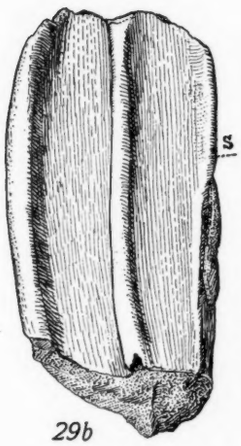
is, however, less curved and is much narrower. The narrowness of the Coalinga form is apparently a normal character of worn teeth. It may be that later collections will show that these two forms are specifically identical, or at most not removed



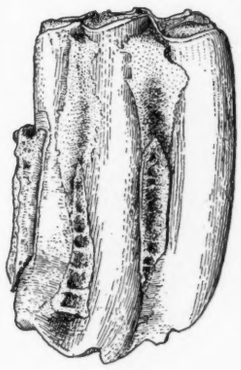
29a



30a



29b



30b

FIGS. 29*a* and 29*b*. *Pliohippus coalingensis* (Merriam). P^4, type specimen, no. 21341, natural size. Etchegoin formation, North Coalinga region, California. Fig. 29*a*, cross-section of tooth at point indicated by *S* on Fig. 29*b*; Fig. 29*b*, outer view.

FIGS. 30*a* and 30*b*. *Pliohippus?*, sp. $M_{\overline{3}}$, no. 21362, natural size. Etchegoin formation, North Coalinga region, California. Fig. 30*a*, occlusal view; Fig. 30*b*, outer view.

farther than by subspecific characters. The imperfectly known *Pliohippus* species of the Thousand Creek beds seems to have a somewhat different form and differently constructed fossettes.

Pliohippus coalingensis is somewhat larger and more advanced than the *Pliohippus* or *Protohippus* species of the Jacalitos.

MEASUREMENTS.

	No. 21341 Coalinga.	No. 19789 Ricardo.	P. supremus Type.
P^4, transverse diameter	24.4 mm.	26	
P^4, anteroposterior diameter	27.2	25	
P^4, height of crown	53	55	
M^1, transverse diameter			25
M^1, anteroposterior diameter			24

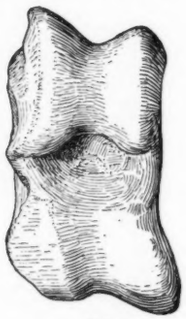
Protohippus or Pliohippus, sp.

Associated with *P. coalingensis* at locality 2074 are fragmentary remains of teeth evidently representing a species near the *Pliohippus?* of the Jacalitos. The material available is not sufficient to permit definite determination of the affinities of the form. At locality 2090 a lower molar, no. 21362 (Figs. 30*a* and 30*b*), represents a species that may be identical with the smaller protohippine form at locality 2074.

Camelidæ.

A single fragment of a metapodial (Fig. 31) from locality 2090 represents a camel approximating in size the large species of *Procamelus*.

31



32*a*

32*b*

Fig. 31. Camelid. Portion of distal end of metapodial. No. 21363, × ½. Etchegoin formation, North Coalinga region, California.

Figs. 32*a* and 32*b*. Astragalus of *Platygonus*-like form. No. 21371, natural size. Etchegoin? formation, North Coalinga region, California. Fig. 32*a*, superior view; Fig. 32*b*, medial view.

Suidæ.

A single astragalus (no. 21371, Figs 32*a* and 32*b*) from locality 2123 in the lower Etchegoin zone represents a peccary-like form. In dimensions and form it is near *Platygonus*. It is possible that this specimen belongs with the latest fauna from this region.

LATER FAUNA FOUND ON UPPER PORTION OF ETCHEGOIN AREA.

Vertebrate remains were found in considerable quantity on exposures representing the upper portion of the Etchegoin section in the North Coalinga region. The members of the University party all obtained specimens in this area, but the largest part of the collection was secured by J. O. Nomland, who was engaged in making a special study of the Etchegoin and its relation to adjacent formations. The collections from this area were obtained from a considerable number of localities situated mainly in a zone ranging from one-half mile to a little more than a mile east of the Glycimeris zone at the base of the Etchegoin. The only exception to the rule

is the possible representation of the fauna by a peccary astragalus at locality 2123 near the Glycimeris zone. The localities range in elevation from near 600 feet to a little more than 900 feet above sea level. At a number of localities this fauna is associated with invertebrates of several horizons representing the upper portion of this Etchegoin section. The vertebrate specimens were not found in place, but appeared on exposures which seemed to consist solely of Etchegoin material.

As the later fauna in general resembles that of the Pleistocene as closely as it does any known Pliocene of America, the writer made special inquiry as to the possibility of derivation of the specimens from Pleistocene accumulations. Terrace deposits bordering the Great Valley are reported in the region to the north of Coalinga at several levels, some of which are considerably above that at which the collections representing the later fauna were made. The bones were all obtained on hills seemingly consisting entirely of Etchegoin, and with no suggestion of terrace deposits. However, it is known that important terraces in the region to the north may have an exceeding small amount of material left upon them, so that the rarity of loose material may not indicate absence of terraces. Members of the University party who collected vertebrate remains in this area state that there are terraces at levels both above and below the localities at which bones were collected, and that the hills upon which the vertebrate material was obtained seem to have been formed by erosion since the upper terraces were completed, while the lower terraces do not reach up to the level of these hills.

It does not appear to the writer that final evidence as to the occurrence of this fauna is now before us. Since the question is of importance it is desirable to continue the examination of this area with a view to obtaining material in place.

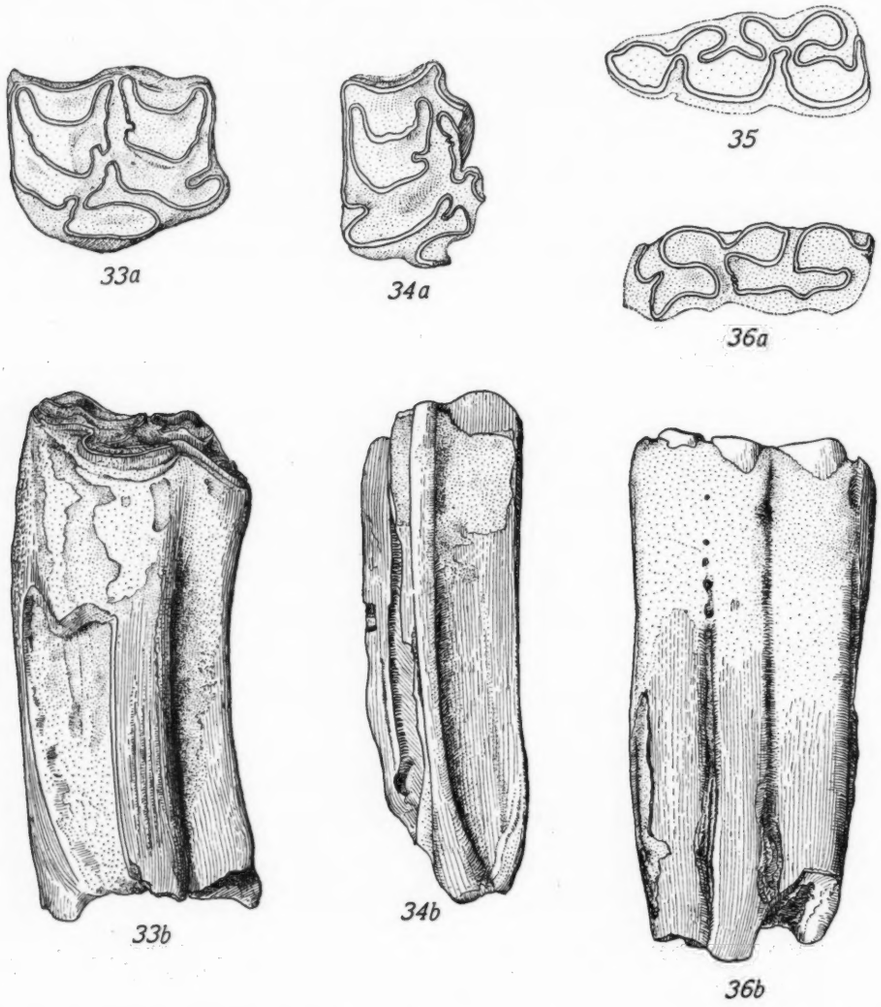
The fauna found in the upper Etchegoin area includes the following forms:

Equus or *Pliohippus*, sp.
Camelops or *Pliauchenia*, sp.
Procamelus?, sp.
Cervus or *Odocoileus*, sp.
Tayassu or *Mylohyus?*, sp.
Mastodon.
Testudo?, sp.
Fish vertebræ, and scattered bulbous fish bones.

Equus or Pliohippus, sp.

The equid remains known from the upper Etchegoin area include upper and lower molars, the astragalus, and all of the phalanges. The molars are very large, exceeding

slightly the dimensions of the largest specimens of *Pliohippus* available for comparison. The form of the upper cheek-teeth (Figs. 33*a* to 34*b*) strongly resembles



FIGS. 33*a* and 33*b*. *Equus* or *Pliohippus*, sp. M^1, no. 21330, natural size. From upper Etchegoin area, North Coalinga region, California. Fig. 33*a*, occlusal view, outer enamel wall absent; Fig. 33*b*, inner view.

FIGS. 34*a* and 34*b*. *Equus* or *Pliohippus*, sp. P^4?, no. 21331, natural size. From upper Etchegoin area, North Coalinga region, California. Fig. 34*a*, occlusal view; Fig. 34*b*, outer view.

FIG. 35. *Equus* or *Pliohippus*, sp. $M_{\overline{3}}$, occlusal view. No. 21333, natural size. From upper Etchegoin area, North Coalinga region, California.

FIGS. 36*a* and 36*b*. *Equus* or *Pliohippus*, sp. M_1?, no. 21332, natural size. From upper Etchegoin area, North Coalinga region, California. Fig. 36*a*, occlusal view; Fig. 36*b*, inner view.

that of *Equus*. The protocone differs from that of all *Pliohippus* species known to the writer in its relatively great anteroposterior diameter. Suggestions of *Pliohippus* are seen in the simplicity and unusual width of the fossettes, in the somewhat more

slender mesostyle, and in the convex inner face of the protocone. The protocone is also short anterior to its union with the protoconule. The general form of the protocone may, however, be approximated in *Equus*.

MEASUREMENTS.

	mm.
$P^{\frac{4}{4}}$?, transverse diameter (no. 21331)	29
M^{1}, anteroposterior diameter (no. 21330)	30
M^{1}, anteroposterior diameter of protocone (no. 21330)	8.8

The lower cheek-teeth (Figs. 35 to 36*b*) are large, long, and heavily cemented. The dimensions approximate those of the largest specimens of *Equus occidentalis* from the Pleistocene of California. The crowns are larger and relatively narrower than in any *Pliohippus* available to the writer for comparison. The metaconid-metastylid column is divided internally by a deep, wide groove, which approaches more nearly the normal form in *Equus* than to that in typical *Pliohippus*.

MEASUREMENTS.

	mm.
No. 21332.	
Lower cheek-tooth, anteroposterior diameter	34.1
No. 21333.	
$M_{\overline{1}}$, anteroposterior diameter	27.5
$M_{\overline{3}}$, anteroposterior diameter	36.2
$M_{\overline{3}}$, transverse diameter	13.8

The limb elements (Fig. 37), including the astragalus, metapodials, and the phalanges, are somewhat smaller than in average specimens of *Equus caballus*, or in the Pleistocene *Equus occidentalis*. The form of the limb elements is much as in *E. caballus*. If it differs appreciably from modern species, the difference is in the more slender form of the Coalinga specimens. The proximal end of a metacarpal shows the facets as in *Equus*. The distal keels of several metapodials are large and prominent. In one specimen representing an ungual phalanx the lateral wing of the hoof is somewhat smaller than in *Equus caballus*.

A summation of the characters of the equid form from the upper Etchegoin area shows that it very closely approaches the *Equus* group. If, as is suggested by certain characters of the upper molars, there is close affinity with *Pliohippus*, this would seem the most advanced species thus far known in that group, and one presumably furnishing a transition to *Equus*.

Proboscidean Remains.

A considerable number of fragments representing a mastodon-like form have been found at several localities in the upper Etchegoin area. The greater part of a tooth (no. 21339, Fig. 38) was found at locality 2119, and another tooth of different

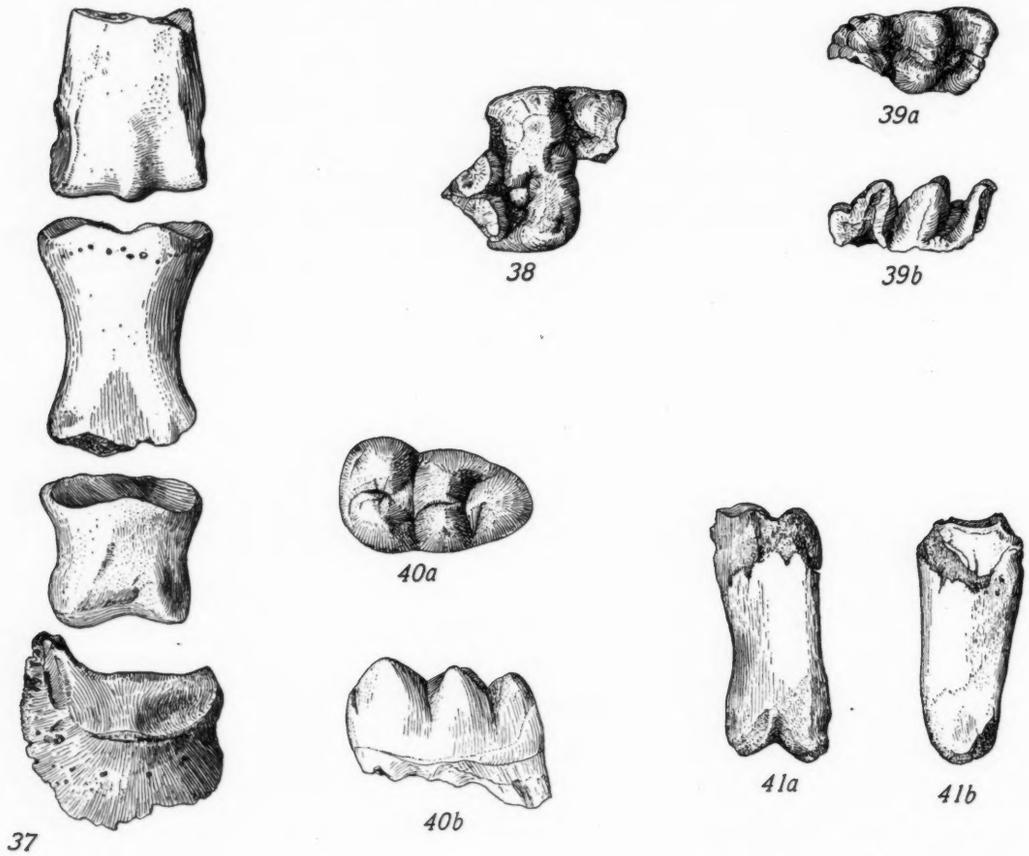


Fig. 37. *Equus* or *Pliohippus*, sp. Anterior view of distal end of third metapodial, and phalanges of third digit. No. 21380 (not all elements known to represent one individual) × ½. From upper Etchegoin area, North Coalinga region, California.

Fig. 38. Portion of a mastodon cheek-tooth seen from above. No. 21339 × ¼. From upper Etchegoin area, North Coalinga region, California.

Figs. 39*a* and 39*b*. Portion of a mastodon cheek-tooth. No. 21340 × ½. From the upper Etchegoin area, North Coalinga region, California. Fig. 39*a*, occlusal view; Fig. 39*b*, lateral view.

Figs. 40*a* and 40*b*. M_3 of *Mylohyus*-like form. No. 21360, natural size. From upper Etchegoin area, North Coalinga region, California. Fig. 40*a*, occlusal view; Fig. 40*b*, side view.

Figs. 41*a* and 41*b*. Proximal phalanx of *Platygonus* or *Mylohyus*-like form. No. 21338, natural size. From upper Etchegoin area, North Coalinga region, California. Fig. 41*a*, superior view; Fig. 41*b*, side view.

form (no. 21340, Figs. 39*a* and 39*b*) at locality 2079. The first specimen represents a very large mastodon, which is, so far as determinable, not far from the stage of development of the Pleistocene species. The second specimen apparently represents

a smaller form with narrower teeth. Neither form is necessarily excluded from the types of mastodon occurring in the Pliocene.

SUIDÆ.

Two large third lower molars represent a peccary-like type not previously known in the Pacific Coast region. The tubercles are low, blunt-conical as in *Tayassu* and *Mylohyus*, and are not distinctly connected by transverse crests as in *Platygonus*. Intermediate tubercles do not appear as well developed as in the *Tayassu* and *Mylohyus* specimens examined by the writer. In size the larger specimen (Figs. 40*a* and 40*b*) corresponds approximately to *Mylohyus*, sp. *a* from Conard Fissure, as figured by Brown.[1]

This form is near *Tayassu* or *Mylohyus*. If other specimens show somewhat advanced development of the secondary tubercles, it may represent *Mylohyus*. The species is probably new.

MEASUREMENTS.

	No. 21360, mm.	No. 21359.
$M_{\overline{3}}$, greatest anteroposterior diameter	27.2	24.3
$M_{\overline{3}}$, transverse diameter	15.9	13.4

A single proximal phalanx (no. 21338, Figs. 41*a* and 41*b*) from locality 2079 represents a form near *Platygonus*, but the element is a little less enlarged inferiorly than that genus. This difference is possibly due in part to wear on the fossil specimen.

CAMELIDÆ.

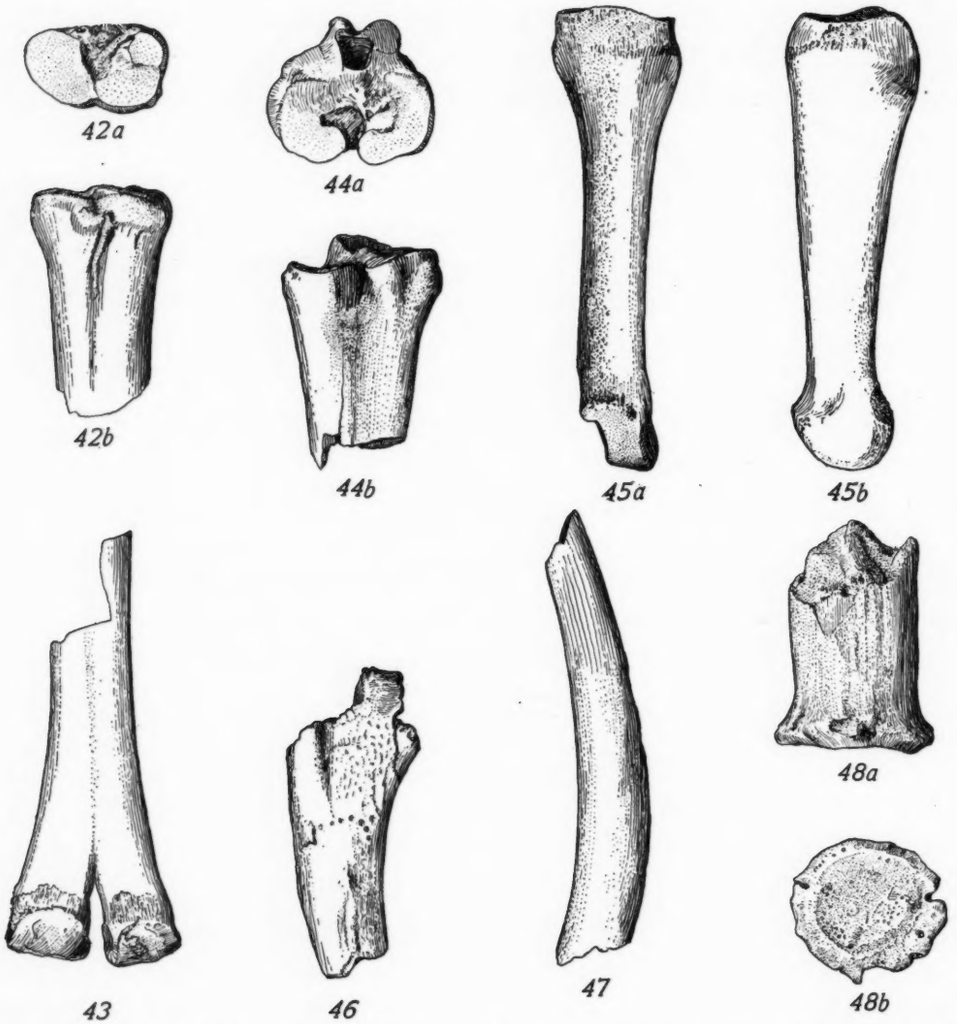
Remains representing two groups of camels are found in the upper Etchegoin area. One contains forms near *Camelops* or *Pliauchenia*. The other type is not more than half as large as the first, and is possibly a representative of a large species of *Procamelus*. The two species were found at locality 2119.

The first form is represented by large metapodial fragments (nos. 21334 and 21379, Figs. 42*a* to 43). A proximal phalanx and several fragmentary bones possibly belong also to this type. This animal was near the size of *Camelops hesternus* of the California Pleistocene. The metapodial fragments (no. 21334 and 21379) of the large camel represent the proximal end of an anterior cannon-bone, and the distal portion of an element which is possibly from the hind limb. The proximal end of the anterior cannon-bone is closely similar to that of *Camelops*. The main difference between this specimen and that of *Camelops hesternus* from Rancho La Brea suggested is in the slightly fuller bridge of bone uniting metacarpals three and four posteriorly

[1] Brown, B., *Mem. Amer. Mus. Nat. Hist.*, vol. 9, pl. 24, 1908.

at the proximal end. If the fragment of the distal end of a cannon-bone represents an anterior limb, this element is more slender than in *Camelops hesternus*.

A proximal phalanx (Figs. 45*a* and 45*b*, no. 21356) from locality 2370 is longer and much more slender than the phalanges of the *Camelops* species from Rancho La



Figs. 42*a*–43. Metapodials of large *Camelops*-like form. From upper Etchegoin area, North Coalinga region, California. Figs. 42*a* and 42*b*. Proximal end of anterior cannon-bone. No. 21334 × ¼. Fig. 42*a*, proximal view; Fig. 42*b*, anterior view. Fig. 43. Anterior view of distal end of metapodial, possibly of posterior limb. No. 21379 × ¼.

Figs. 44*a* and 44*b*. *Procamelus*?, sp. Proximal end of cannon-bone from posterior limb. No. 21335 × ½. From upper Etchegoin area, North Coalinga region, California. Fig. 44*a*. Proximal view; Fig. 44*b*, anterior view.

Figs. 45*a* and 45*b*. *Pliauchenia*?. Proximal phalanx. No. 21356 × ½. From upper Etchegoin area, North Coalinga region, California. Fig. 45*a*, superior view; Fig. 45*b*, side view.

Figs. 46–48*b*. *Cervus* or *Odocoileus*, sp. From upper Etchegoin area, North Coalinga region, California. Fig. 46. Portion of flattened forked antler. No. 21336 × ½. Fig. 47. Portion of slender tine with round cross-section. No. 21337 × ½. Figs. 48*a* and 48*b*. Base of antler with burr. No. 21357 × ½. Fig. 48*a*, lateral view; Fig. 48*b*, basal view.

Brea, or than in the large camel of the Pleistocene Manix beds in the Mohave area. It evidently represents a large species quite different from the Pleistocene *Camelops* forms, and from all other camels known in the Pacific Coast Pleistocene. It possibly corresponds to *Pliauchenia* or to some genus other than *Camelops*.

The second camelid form known from the upper Etchegoin area is represented by the proximal end of a posterior cannon-bone (no. 21335) of an adult individual somewhat larger than the Recent llama. This animal was less than half the bulk of *Camelops hesternus* or of the *Camelops*-like form represented by specimen no. 21334. The form of the proximal end of the cannon-bone (Figs. 44*a* and 44*b*) is entirely different from that of *Camelops* and *Camelus*. It is much narrower transversely, and the posterior facet for the cuboid is much larger than in *Camelops*. The form is near that of *Procamelus*, though the transverse diameter is a little larger than in any *Procamelus* species available for comparison. The facets for the cuboid and mesocuneiform are lunate instead of oval as in *Procamelus*.

Cervidæ.

At several localities (no. 2119, 2079, 2370 and 2374) in the upper Etchegoin area fragments of antlers have been found representing forms of a cervid type. Two specimens consist of the basal portion with the burr of large antlers that had been shed. They evidently represent a very advanced form of the true cervid type. The larger specimen (Figs. 48*a* and 48*b*, no. 21357) is from an individual with antlers approximating in size those of large specimens of the Recent *Cervus* or *Odocoileus*.

One specimen (no. 21336, Fig. 46) consists of a flattened beam apparently dividing above into two branches of different dimensions. The shape of the beam is like that in *Cervus*, and resembles a form that may occur in *Merycodus*. It is larger than in specimens of the nearest forms of *Merycodus* known to the writer, and the branches above are more uneven in size than in *M. necatus*, the nearest merycodont. Another specimen (no. 21337, Fig. 47) is a portion of a long, slender, gently curved tine or branch with nearly circular cross-section. The convex side of the tine shows a double row of very small, irregular tubercles separated by a faint groove. This tine is larger than that of any merycodont examined by the writer.

The evidence before us indicates that the specimens from this fauna represent a true cervid of the *Odocoileus* or *Cervus* type.

Testudinate Remains.

A fragment of a carapace representing a very large tortoise-like form (no. 21378, Figs. 49*a* and 49*b*) was obtained with the collection found in the upper Etchegoin area. The species is unlike any form known in this region at the present time.

Age of Later Fauna from Upper Etchegoin Area.

The aspect of the later fauna from the Etchegoin area resembles in some features that of the Pleistocene. The horse in this assemblage seems more advanced than any American Pliocene form known to the writer. *Equus simplicidens* Cope from the Blanco Pliocene of Texas is considered by Gidley[1] to represent *Pliohippus*. The protocone of the Coalinga species is very large, long anteroposteriorly, compressed laterally, and nearly flat on the inner side. In Cope's figure of *E. simplicidens* the protocone is incomplete, but seems smaller than in the Coalinga form, and is restored as smaller by Gidley.

The metapodial of the larger camel in this fauna approaches that of a Pleistocene *Camelops*, but the large phalanges, apparently representing this fauna, do not correspond to any of the Pleistocene camel species known in the Pacific Coast region. The smaller camel seems more nearly to represent a Pliocene *Procamelus* or some allied

49a 49b

Figs. 49*a* and 49*b*. Portion of carapace of a large tortoise. No. 21378 × ½. From the upper Etchegoin area, North Coalinga region, California. Fig. 49*b*, superior view of marginal? plate, lateral margin to left of figure; Fig. 49*a*, transverse section at *s*, at lower end of Fig. 49*b*.

group. The cervid remains represent deer of a distinctly modern type. Nothing of this character has been known in the Pliocene of America so far as the writer is aware. The peccary is of an advanced type and is near the Pleistocene stage of development. The mastodon remains are fragmentary, and little weight has been placed on them in distinction between Pliocene and Pleistocene. This is true also of the tortoise, and of the fish remains.

In attempting to fix the age of the fauna it is first necessary to determine whether the collection available comes from one source or whether it is derived from two or more sets or deposits. It is possible that one part comes from Pleistocene terraces, and that other elements, such as the *Procamelus*, are from the Etchegoin formation. The fact that both the small camel, and the large species show difference from the known Pleistocene forms naturally suggests that a really new faunal assemblage is present here. However, it will be difficult to give satisfactory evidence that this is a faunal unit until some of the forms with a distinctly Pleistocene aspect have

[1] Gidley, J. W., *Bull. Amer. Mus. Nat. Hist.*, vol. 14, p. 124, 1901.

been found in place in the Etchegoin, or until some of those representing an older type, as the *Procamelus*-like form, have been discovered in terrace deposits.

If the fauna is a unit and occurs in place in the Etchegoin, it indicates either that the Etchegoin is younger than the Miocene or Pliocene stage which it has been presumed to represent, or that a modern type of fauna has appeared relatively early in California.

The Etchegoin, upon the outcrops of which the odd Cervus fauna is found, is a formation of considerable thickness. It is covered by the Tulare, which is generally considered to be represented by much more than 2,000 feet of strata. Since the close of Tulare time important events referred to the Pleistocene have taken place in this region. From purely physical studies there seems good reason for believing that the upper portion of the Etchegoin represents an epoch separated from Recent time by a considerable period.

The invertebrate fauna of the upper portion of the Etchegoin area from which the vertebrate remains were obtained consists, according to Mr. Nomland, of about forty-nine species of which twenty are extinct. Arnold and Anderson[1] considered the Etchegoin as of Miocene age. The total fauna was noted to contain 65 per cent. of extinct species. Arnold and Anderson were in agreement with F. M. Anderson that the Etchegoin is to be correlated with the San Pablo formation. Arnold and Anderson also considered the upper Etchegoin as the equivalent of the lower Purisima. The most recent studies of Bruce Clark, Bruce Martin, and J. O. Nomland, show that the Etchegoin fauna is distinctly more recent than that of the San Pablo. The suggestion that the Etchegoin was contemporaneous with the San Pablo, but of a different geographic phase, seems not to be adequate in view of the fact that the San Pablo fauna closely matches that of the "Santa Margarita" occurring in the second formation below the Etchegoin, and separated from it by the stage referred to the Jacalitos in the North Coalinga region.

From the lower portion of the Tulare formation above the Etchegoin, Arnold and Anderson[2] list thirteen species, of which all but three are considered as extinct forms. It was suggested by these authors that the basal portion of the Tulare represented the lower Pliocene, but that the upper portion extended into the Pleistocene.

It appears to the writer improbable that the Etchegoin can be included in the Miocene. The assemblage of evidence from a study of the invertebrate faunas, interpreted in terms of the Lyellian percentage method as it was understood when that method was proposed, suggests Pliocene age. This was the determination given by F. M. Anderson in the original description of the Etchegoin.

[1] Arnold, R., and Anderson, R., U. S. Geol. Surv. Bull. 398, p. 139, 1910.

[2] Arnold, R., and Anderson, R., U. S. Geol. Surv. Bull. 398, p. 154, 1910.

If the vertebrates obtained from the upper portion of the Etchegoin area in the North Coalinga region were actually derived from Etchegoin strata, we apparently have a relatively advanced fauna in beds which according to evidence from geology and invertebrate palæontology suggest Pliocene age. This relation of the evidence from vertebrate and invertebrate palæontology is in a manner similar to that in the lower portion of the Coalinga geologic section, where the "Temblor" invertebrates are determined as older than the vertebrate Merychippus fauna. The fact that the relation is the same in the Etchegoin area as in the "Temblor" area may suggest that the vertebrate fauna actually occurs in the Etchegoin strata. If this is true, the balance of evidence would seem to the writer to indicate that the strata in question are not older than middle Pliocene.

Alternative possibilities of explanation that remain open are the suggestion that this peculiar fauna is derived from vestiges of terraces that reached only across the outer or upper portion of the Etchegoin area, or that it is composed in part of Pleistocene terrace material and in part of Pliocene forms derived from strata of the Etchegoin formation. Although the field work already done has been carefully conducted, the final settlement of this important problem seems to require still more intensive study.

TABLE REPRESENTING POSITION OF FORMATIONS CONTAINING VERTEBRATE FAUNAS IN NORTH COALINGA REGION.

The following table gives approximate position of beds containing land vertebrate faunas in the North Coalinga region in comparison with the situation of the principal divisions of the geological scale in the Great Basin and California regions.

Geological Periods.		Local Formations, California Area.	Local Formations, Great Basin Province.	Faunal Zones.	
				Vertebrate Faunas.	Invertebrate Faunas.
Pleistocene		Rancho La Brea	Manix	Smilodon, Equus, Camelops, Elephas } Stages not defined	Echinarachnius excentricus Turritella jewetti
Pliocene		Tulare Etchegoin Jacalitos	Thousand Creek Rattlesnake Ricardo	Hyaenognathus Ilingoceras Pliohippus Hipparion	Echinarachnius gibbsii
Miocene	Upper	San Pablo ("Santa Margarita")	Barstow Beds	Merychippus calamarius Merycodus necatus	Astrodapsis
	Middle	Monterey ("Temblor")	Mascall and Virgin Valley	Merychippus isonesus Parahippus Dromomeryx	Turritella ocoyana
	Lower	?"Vaqueros"	Columbia Lava	No known fauna	Turritella inezana
Oligocene	Upper Middle Lower	San Lorenzo	Upper John Day Middle John Day ?Lower John Day	Promerycochoerus Eporeodon Imperfectly known fauna	Turritella diversilineata
Eocene	Upper	Tejon	Upper Clarno	No fauna (Flora—Sequoia heeri, Betula, Alnus, Quercus, Acer, Ficus)	Turritella merriami Turritella uvasana
	Lower	Martinez	Lower Clarno	No fauna (Flora—Lygodium, Asplenium, Equisetum, Juglans, Magnolia)	Turritella pachecoensis Turritella infragranulata

LIST OF IMPORTANT LOCALITIES AT WHICH VERTEBRATE SPECIMENS WERE COLLECTED IN THE NORTH COALINGA REGION.

All Townships and Ranges referred to Mount Diablo Base and Meridian.

MERYCHIPPUS ZONE—

Locality 2124: SW. ¼ sec. 28, T. 18 S., R. 15 E.

JACALITOS—

Locality 2126: NE. ¼ of NE. ¼ of sec. 15, T. 19 S., R. 15 E.

Locality 2076: SW. ¼ of sec. 34, T. 18 S., R. 15 E.

Locality 2077: NE. ¼ sec. 15, T. 19 S., R. 15 E.

ETCHEGOIN, PLIOHIPPUS COALINGENSIS ZONE—

Locality 2073: SW. ¼ sec. 13, T. 19 S., R. 15 E.

Locality 2074: SW. ¼ sec. 12, T. 19 S., R. 15 E.

Locality 2078: NW. ¼ of SW. ¼ sec. 13, T. 19 S., R. 15 E.
Locality 2090: NW. ¼ sec. 13, T. 19 S., R. 15 E.

FAUNA FROM UPPER ETCHEGOIN AREA:

Locality 2079: the NW. ¼ of NW. ¼ sec. 19, T. 19 S., R. 16 E.
Locality 2080: NW. ¼ of SW. ¼ sec. 6, T. 19 S., R. 16 E.
Locality 2082: SW. ¼ of NE. ¼ sec. 1, T. 19 S., R. 15 E.
Locality 2084: north line of SE. ¼ of sec. 12, T. 19 S., R. 15 E.
Locality 2086: NW. ¼ of SW. ¼ of sec. 6, T. 19 S., R. 16 E.
Locality 2089: NW. ¼ of sec. 19, T. 19 S., R. 16 E.
Locality 2119: SE. ¼ of sec. 12, T. 19 S., R. 15 E.
Locality 2368: NW. ¼ of the SW. ¼ of sec. 19, T. 19 S., R. 15 E.
Locality 2370: NE. ¼ of sec. 25, T. 19 S., R. 15 E.
Locality 2372: NE. ¼ of the SE. ¼ of sec. 25, T. 19 S., R. 15 E.
Locality 2374: W. ½ of the NW. ¼ of sec. 25, T. 19 S., R. 15 E.

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TRANSACTIONS

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PART 4

The Feeding Habits of Pseudomyrmine and Other Ants.

By W. M. Wheeler and I. W. Bailey.

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1920

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ARTICLE 4.

THE FEEDING HABITS OF PSEUDOMYRMINE AND OTHER ANTS.[1]

(Plates I–V.)

By W. M. WHEELER and I. W. BAILEY.[2]

(Read April 24, 1920.)

INTRODUCTION.

All observers of ants have been impressed with their greed and the peculiarities of their feeding habits, and although much has been written on the nature of their food, our knowledge is largely confined to generalities. We know that some species are predominantly insectivorous, some granivorous, some fungivorous and that many others feed to a considerable extent on the saccharine excreta of Aphids, Coccids and Membracids. We also know that certain ants have such specialized appetites that they confine themselves to one or a very few food-substances. Thus some (*e.g.*, *Megaponera fœtens* Fabr., *Carebara*, *Aëromyrma*, *Pœdalgus*, *Liomyrmex*, etc.) feed mainly or exclusively on termites, others on land isopods, or slaters (*e.g.*, *Lobopelta* species according to Wheeler (1904), Arnold (1915) and Williams (1919)), others, like the Attii on particular species of fungi, while still others, intimately associated with some of the higher plants (*Cecropia*, certain neotropical acacias), have long been supposed to eat only particular plant structures or exudates (Beltian and Müllerian bodies, nectar). We are, nevertheless, far from possessing a knowledge of the precise nature, variety and quantity of the food substances ingested by any species of ant, and our knowledge of the substances which these insects feed to their larvæ is even more unsatisfactory. Such a knowledge, however, is urgently needed on account of its bearing on many matters of both theoretical and economic importance.

The feeding habits of animals are so fundamental and have such profound and far-reaching effects not only on their bodily structure but also on their general

[1] Contributions from the Entomological Laboratory of the Bussey Institution, Harvard University. No. 170.

[2] There are peculiar rows of excavations in the hollow stems of a number of Ethiopian myrmecophytes that are closely correlated in distribution with the phyllotaxy of the plants. The junior author was able to demonstrate by means of a detailed study of the minute anatomy of the myrmecodomatia that these excavations contain zoöcecidia, which in certain cases are as highly differentiated histologically as many of the complex gall-structures induced by Cynipidæ. In endeavoring to prove that the ants actually feed upon the "nutritive layers" of these zoöcecidia, he had occasion to analyze the castings or pellets from the infrabuccal pockets of a number of the ants. The contents of these pellets were so significant from both the botanical and zoölogical points of view, that it seemed desirable for an entomologist and a botanist to collaborate in studying the feeding habits of various groups of Formicidæ.

physiology and behavior that naturalists have always taken pains to ascertain and record the methods of feeding and the character of the food. On the other hand we know that the injuries or benefits accruing to the human race from insects are the direct outcome of their feeding habits, so that the economic is even more interested than the theoretical biologist in a knowledge of these peculiarities. One reason why the economic importance of many common ants remains so dubious or ambiguous is the lack of precise information in regard to the quality and quantity of their food. Moreover, ants are of economic importance not only because they actually feed on certain substances valuable to man, but also because they have a habit of collecting and carrying certain living organisms for considerable distances, either purposely or incidentally. Sernander (1906), in a beautiful paper, has shown how many of the common European ants—and the same is undoubtedly true of many North American species—are an important factor in the distribution of the herbaceous flora, owing to their habit of collecting and transporting seeds, on the arilli of which they feed. And, as will be shown in the sequel, the extraordinary number and variety of fungus spores and Bacteria which they carry on their bodies or in their mouths, may have a great but hitherto only vaguely apprehended importance for the student of plant, animal and human pathology.

There are at least ten sources that may yield indications of the nature of the food eaten by a particular species of ant. These are:

1. The prey or booty (living or dead insects, seeds, nectar) carried into the nest.

2. The materials placed on the refuse-heaps, or kitchen-middens, either in the superficial chambers of the nest or on the surface of the soil around the nest entrance.

3. The substances produced by myrmecophiles, larval or adult ants (exudates or glandular secretions) within the nest or by trophobionts (Aphids, Coccids, Psyllids, Membracids, Lycaenid larvæ, etc.) outside the nest.

4. The contents of the pellet contained in the infrabuccal cavity of the adult ants.

5. The contents of the pellet placed in the trophothylax of the larvæ of certain ants (Pseudomyrminæ).

6. The contents of the larval stomach.

7. The meconial pellet of the adult larva.

8. The fungi that may grow on the walls of the nest-chambers or on various substances in the nest (carton, vegetable débris, insect excrement, etc.).

9. Ectoparasitic mites of the ants themselves may be collected and perhaps partially eaten, to judge from the remains of these organisms sometimes found in infrabuccal pockets.

10. The eggs, larvæ, pupæ or even the adult ants of a colony may be devoured under certain usually abnormal conditions, as, *e.g.*, when other food is unobtainable.

All of these sources present problems of considerable difficulty and may easily lead to fallacious interpretations. Thus ants often carry various solids (seeds, buds, flowers, leaves, bird, mammal and insect excreta, etc.) into the nest for no apparent reason, except to avoid returning home with empty mandibles. Such materials may be later thrown on the kitchen-middens and create the erroneous impression that they have been partially eaten. The exudates and secretions of myrmecophiles and of larval ants are notoriously difficult of observation in particular instances. As will be shown in the sequel, the pellets contained in the infrabuccal pocket and the trophothylax consist in large part of extraneous substances derived from cleaning the surfaces of the body or from excavation in friable soil or wood, and the contents of the larval stomach and meconial pellets are in most cases so finely triturated and have been so much altered by the action of the digestive juices as to be undeterminable. Furthermore, the meconial pellet can be recovered, as a rule, only in the subfamilies of ants which make cocoons (Ponerinæ and Formicinæ). The cropping by ants of mycelia growing in their nests is by no means proof that the fungus filaments are actually eaten. Cannibalism, though not infrequent, is, of course, an exceptional or abnormal source of food.

On the other hand, since the pellets of the infrabuccal cavity and trophothylax and the materials in the stomach of the larva often contain recognizable fragments of food as well as substances excavated or manipulated by the ants in making their nests, careful analysis, especially of the pellets, often throws considerable light not only on the nature of the food but also on that of the environment in which the ants nest. For this reason we have dissected out and examined several hundred of the pellets in a considerable number of species, representing well-known genera in all the subfamilies (except the Cerapachyinæ) and have devoted considerable space to an account of the substances found. Particular attention was devoted to the Pseudomyrminæ, because it is the only subfamily in which it is possible in the same species to study both the pellets of the infrabuccal pocket of the adult and those of the larval trophothylax.

The whole subject is thus seen to be one of considerable scope and complexity. Our results have been obtained exclusively from a study of preserved material and must therefore be regarded as merely furnishing a foundation for future experimental researches on the living ants. Such researches will be most advantageously pursued in tropical or subtropical regions where the family Formicidæ is represented by the greatest number of genera and species of morphological and ethological interest. Though the scope of our work is circumscribed, it yields, nevertheless, some indications of the magnitude and importance of a study of the trophic activities of ants and may furnish some useful hints to future investigators.

I. THE FEEDING HABITS OF ADULT ANTS.

A. Historical.

An adequate account of feeding in adult, or imaginal ants would require a description of the structure and functions of the entire alimentary tract. This would be a work of supererogation, because these subjects have been set forth in sufficient detail in several treatises, such as those of Donisthorpe (1915), Escherich (1906) and the senior author (1910). There is one structure, however, the infrabuccal pocket, which may be considered more exhaustively, as well as certain peculiarities of feeding in such specialized ants as the harvesters and fungus-growers, since in these cases it seems to be most difficult to determine the precise composition of the substances actually taken into the crop and stomach and assimilated by the insects.

The Dutch entomologist Brants (1841), while studying the mouth-parts of various Hymenoptera, discovered in all three phases of wasps (*Vespa crabro, vulgaris* and *holsatica* and in *Odynerus*), but not in bees, Ichneumonids and Crabronids, a peculiar subspherical pouch just behind the tongue and below the buccal orifice. He discussed the structure at length and supposed at first that it might be used either as a receptacle in which the materials for the paper nest could be prepared, or as a crop for the feeding of the young. He accepted the former as the more probable function and therefore called the structure the "lijmholte." On examining it under the microscope he found it to contain small fragments of wood .1 to .5 mm. in length, in which he clearly recognized the cells with their pitted walls. He also found algæ, but supposed that they had been growing on the wood, and though he detected some animal substances, such as a piece of spider's skin, he believed them to be accidental and that the pouch was properly a mortar in which minute wood-particles could be mixed with saliva and thus made into paper.

The same pocket was discovered in ants by Meinert (1860), but called the "Mundsak." He regarded it as being morphologically an infolding of the hypopharynx and says that he often found it filled with a "dark brown, formless mass." The walls were described as clear and transparent and devoid of musculature. He dissented from Brants' opinion in regard to its function, pointing out that it is present in male and female as well as worker ants, that it is well-developed in at least the majority of European species, none of which, except *Lasius fuliginosus*, makes a carton nest, and that in this species the organ shows no enlargement or peculiarities of structure. He was inclined to believe that it may function, at least to some extent, as a kind of crop for feeding the larvæ or adult ants. He therefore returns to an hypothesis which Brants suggested but rejected.

Lubbock in 1877 gave a very similar account of the organ, with figures illustrating its appearance in median sagittal sections of the head of a female *Lasius niger* and a worker *Myrmica ruginodis*. Concerning the structure, for which he adopts Meinert's term "mouth sac," he makes the following statements: "Its membranous walls appear to be firm and elastic. I have not found any muscles attached to it, and presume that it is kept open by the elasticity of the walls. The orifice of the sac can be closed at will by a small flap (Pl. CLXXXIX, Fig. 3 *fl.*) which is supplied with several muscular fascicles (Plate CLXXXIX, Fig. 3 *M M*). The cavity generally contains a brown, spongy mass, in which I once (in a specimen of *Formica rufa*) found a small hematoid [*recte* Nematode] worm." Lubbock also discusses the mechanism employed in emptying the sac. He finds Brants' account of the process obscure and concludes that in ants "the general action of the muscles which open the pharynx would tend to empty the mouth sac; those which draw down the lower wall of the pharynx, by directly constricting the mouth sac, while even those attached to the upper wall of the pharynx would tend to empty the mouth sac by sucking out its contents." His figures and his description of the pharynx are also of interest. "The walls of the pharynx are more or less chitinous, and the lower portion immediately above the entrance to the mouth sac is covered with small teeth, which point downwards. This arrangement is unusual, since in most animals the palatal teeth point backwards, so as to prevent any food which has once entered from returning; and, on the contrary, to promote its passage down the throat. I presume, however, that in ants, which feed principally on animal and vegetable juices, it is advantageous to prevent the entrance of solid particles."

Adlerz (1886) considers the "mouth sac" rather briefly, referring to the results of Brants and Meinert and differing from the latter in regard to its function. He believes that the crop (ingluvies) and not the "mouth sac" serves to carry the food to the larvæ, and adds that in *Camponotus, Lasius fuliginosus* and other ants which commonly nest in decayed wood, "the mouth sac is often found filled with wood-fragments and earth particles," in *L. fuliginosus* "mixed with a dark brown humor," which arises, not from the cells of the organ itself, but from some of the glands in its neighborhood. Adlerz makes no statement as to what he considers the true function of the mouth sac to be.

Janet was the first accurately to describe the method of feeding in adult ants and social wasps. His results were presented in several papers (1895*a*, 1895*b*, 1897, 1900, 1905). He found the sac described by Brants, Meinert, Lubbock and Adlerz just behind the upper surface of the labium and beneath the buccal orifice, as a small subspherical pouch and called it the "cavité supralabial," "poche prébuccal,"

"cavité de moulage des boulettes," or "poche gnathale" (Text-fig. 1 *h*). At various times the senior author has referred to it as the "infrabuccal cavity, or chamber" or "hypopharyngeal cavity." Donisthorpe (1915) has adopted the term "infrabuccal

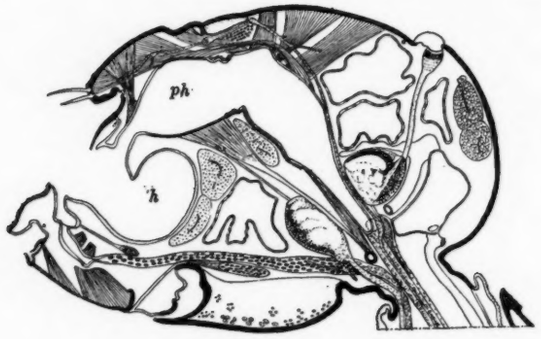

FIG. 1. Sagittal Section of head of femur *Lasius niger* L. (After C. Janet) *ph*, pharynx, *h*. infrabuccal pocket.

cavity" in his monograph of the British ants, Escherich (1906) employs the term "Infrabuccaltasche" and Stitz (1914) "Intrabuccaltasche," possibly a typographical error for "Infrabuccaltasche." We prefer to designate it as the "infrabuccal pocket" throughout our discussion.

Janet showed that whereas liquid food is directly imbibed by ants from the surface of the labium and carried into the pharynx and thence through the tenuous œsophagus into the crop, particles of solid food, obtained by licking with the tongue or rasped off by the maxillæ, are carried into the infrabuccal pocket where they are moulded in the form of a pellet, which he calls a "corpuscle enroulé," or "corpuscle" or "boulette de nettoyage." He demonstrated that this pellet, which as we have seen, had been noticed by Meinert, Lubbock and Adlerz, consists not only of bits of food but also in large part or even exclusively of particles of foreign matter which have been cleaned off from the surfaces of the body by the tongue or the strigils, the comb-like organs—really modified spurs—at the tips of the fore tibiæ (Plate I, Fig. 2). The strigils, used primarily and most beautifully adapted for cleaning the antennæ, are frequently drawn over the maxillæ and labium, thus depositing the particles that adhere to the teeth of their combs. The particles are probably kept together by the secretion of a gland which Janet found to open by many minute pores at the bases of the strigillar teeth. But ants also use the tongue in cleansing one another and their brood and hence many particles of foreign matter may be carried more directly into the infrabuccal pocket and agglutinated or mixed with the food particles of the pellet. The solid portions when no longer of nutritive value, are cast out as a small pellet of refuse.

Janet studied the pellets of wasps and of several species of ants and by powdering the insects with various substances demonstrated the fact that the pellets contain particles from the surface of the body. He says of the conditions in the ants: "The composition of the 'boulettes de nettoyage' shows that they arise from the food residue and the dust and detritus which the ants remove by means of their toilet organs from their own bodies, from their progeny and from their companions. Sometimes two ants may be seen facing each other, in a position that might lead one to suppose that one of them, with its mandibles wide open, is regurgitating liquid food to the other, whose mandibles are closed. Actually, however, they may be engaged in a different occupation, the worker with the closed mandibles being absorbed in carefully licking and cleaning the open mandibles of its companion. This may be easily observed in large species such as *Camponotus ligniperdus*. It is mainly the mandibles which are subjected to such thorough cleaning, but it extends also to the other buccal appendages and neighboring parts of the head, parts which the animal possessing them doubtless cannot cleanse as easily and completely as can one of its companions." Concerning the conditions in the hornets Janet makes this statement 1895*b*): "The hornets, like the other wasps and the ants, not infrequently cast out little rolled-up corpuscles from the supralabial cavity. These corpuscles are usually 1.2 to 1.5 mm. long and 1 to 1.2 mm. broad. All powdery substances accidentally deposited on the hornets' bodies and the solid residue of their meals are found thus molded in the form of rolled-up corpuscles. The chitinous fragments of malaxated insects and the wood-paste that serves in the construction of the nest, commonly figure among the elements of these corpuscles, which in shape are in all respects comparable with the corpuscles I have already described in the ants."

That the analysis of the pellet in the infrabuccal cavity is capable of yielding very interesting and important results was demonstrated by H. von Ihering (1898) and Huber (1905) in their studies on the fungus-growing ants of the neotropical tribe Attiini. These authors showed that the peculiar fungus, which the large Brazilian *Atta sexdens* L. cultivates on a substratum of triturated vegetable substances such as pieces of leaves or caterpillar excrement, is transmitted generation after generation, from the maternal to the daughter colonies, in the form of the food-pellet of the recently fecundated *Atta* queen. Before leaving the maternal nest for her marriage flight she fills her infrabuccal pocket with a mass of hyphæ from the fungus gardens. After excavating her own small cell in the ground and closing the aperture, so that she is entirely shut off from the outside world, she spits out the pellet. The hyphæ at once begin to grow in the moist atmosphere, deriving their nutriment from the detritus, or cleanings contained in the pellet, and later the queen adds her own feces

and crushed eggs as manure, thus keeping the diminutive garden alive and flourishing till her first batch of small workers have hatched. They then break through the soil and collect and bring in the vegetable substances required as a nutrient substratum for the continuous growth of the fungus.

Recently Bruch, in a preliminary note (1919), has given a similar account of colony formation by an allied fungus-growing ant, *Acromyrmex lundi* Guérin of Argentina. He says, however: "The female ant, before leaving the formicary, takes into her buccal cavity a pellet consisting of vegetable substances containing fungus spores. After the nuptial flight she loses her wings, enters the soil and makes a cavity, the initial chamber of the future fungus-garden. Twenty-four hours later she deposits the pellet, the spores of which germinate in the humid environment, producing slender filaments, the fungus hyphæ." It thus appears that, in this species at least, some of the vegetable substratum of the fungus-gardens of the parental nest is carried over in the infrabuccal pocket, but since true spores are not produced by the fungus *Rhozites gongylophora*, according to Moeller, it is rather difficult to understand how the gardens of the daughter colony can arise in the manner described by Bruch. His completed account, when published, will no doubt clear up this difficulty.

The feeding habits of the harvesting ants have been rather carefully investigated by Moggridge (1873), McCook (1879), Neger (1910) and Emery (1899, 1912). Moggridge studied *Messor barbarus* and its subspecies *structor* at Mentone, and confirmed many statements of the ancient Greek and Roman authors who had seen these ants harvesting, storing and eating the seeds of various plants. He ascertained that the seeds may be kept for a long time in a sound condition in the chambers ("granaries") of the nests, without showing signs of germination. When germination does occur in the nests—and Moggridge believed that the ants usually soften the seeds and make them sprout—"before they are consumed by the ants, it is very curious to see how the growth is checked in its earliest stage, and how, after the radicle or fibril—the first growing root of dicotyledonous and monocotyledonous seeds—has been gnawed off, they are brought out from the nest and placed in the sun to dry, and then, after a sufficient exposure, carried below into the nest." And he adds: "The seeds are thus in effect malted, the starch being changed into sugar, and I have myself witnessed the avidity with which the contents of seeds thus treated are devoured by the ants." On Plate 6 he shows how the ants mutilate the germinating seeds by cutting off the radicle, thus confirming statements of Plutarch and Pliny. Moggridge does not, however, go into the actual details of feeding in *Messor*.

McCook in his voluminous study of the Texan harvester, *Pogonomyrmex barbatus* subsp. *molefaciens*, gives a much more detailed account. The seeds are broken open

or squeezed by the mandibles and "the juices or oils of the seed, and the fine, starchy powder of the grains rasped off by the tongue. . . . The process of eating," he says, "is a steady licking of the surface of the seed." Although inclined to believe that *Pogonomyrmex* prefers sound seeds as food, he nevertheless cites Mrs. Mary Treat as having "formed the opinion from her observations of the Florida harvesters [*P. badius*], that they eat the grains only after or during sprouting, and that their appetite especially affects the saccharine substance, which is only manifest at fermentation." After discussing the salivary glands of ants he cites Lubbock's observations on the infrabuccal pocket and surmises that the "pappy mass of flour" licked by the ants from the seed, may have first found lodgment within this sac," but at the time he was studying *P. barbatus* he was not aware of the English naturalist's observations.

In 1899 Emery published a very brief but important paper on the feeding of the Italian *Messor barbarus* subsp. *structor* in artificial nests of the Janet pattern. He noticed the ants cropping the mycelium which grew on the walls of the nest and inferred—perhaps erroneously—that the insects were feeding on the fungi. They also fed on green seeds and buds, boiled, dried beef, hulled rice and other ripe grain, whole or in fragments, bread and the wheat paste (semolina) employed by the Italians in making macaroni, spaggetti, etc. Crude starch, however, was constantly rejected. The wheat paste was the principal food used in the experiments. "This paste was given to the ants in small round grains. The ants held these grains between their mandibles during whole days till they swelled and softened to form a ductile dough which could be kneaded. Then the residue was abandoned in the dry, illuminated chamber of the nest. With this aliment alone the ants have reared young larvæ to the perfect state." He therefore believes that the insects extract from this food not only sugar, but also and mainly the proteids. From the fact that they also utilize boiled, dried beef containing very little soluble material he infers that "the saliva of ants seems to be able to peptonize albuminoid substances, as Plateau has proved for *Periplaneta orientalis*. The fact that these ants reject crude starch leads to the belief that they are unable to dissolve it. Be this as it may, we are confronted with a digestion that occurs or at least begins in the mouth, though it may be completed in the crop under the influence of the saliva."

Neger studied the habits of *Messor barbarus* subsp. *meridionalis* on the island of Arbe, off the Dalmatian coast. He found that the ants chew up and insalivate the soft part of the seeds to form small pasty masses which he calls "ant-bread crumbs" ("Ameisenbrodkrümmel"), which are carried out and placed on the surface of the soil to dry. Although he never saw them being removed he supposes that the ants eventually carry them into the nest. These crumbs, he finds, frequently contain

an amylolytic and proteolytic fungus (*Aspergillus niger*) which renders them more nutritious so that they can be fed to the young as "larva-bread" ("Larvenbrod").

Emery in 1912 regards Neger's conclusions as "addirittura fantastiche"—downright fantastic, and confirms his own former observations on *Messor structor* by an experimental study of the Italian *Messor barbarus* subsp. *minor* kept in artificial nests. The ants were given their choice between the embryonic and starchy portions of wheat grains and were found to prefer the former. He concludes that this is why the ants attack the germinal or radicle end of the seed and not because they wish to prevent its growth. The cutting off of the radicle, he believes, is due to similar causes though the reasons for this behavior "are, perhaps, more complicated." He agrees with Neger that the germination of the seed is merely useful to the ants in facilitating the removal of the often very hard glumes and coatings. As Neger observed, the process of germination is not sufficiently advanced to produce an abundant transformation of the starch into maltose. Emery also fed his ants on the wheat-paste (semolina) made up in the form of small rings. When given to the insects these were malaxated like the softer parts of the seed and converted into contorted masses corresponding to Neger's ant-bread crumbs. With the assistance of Prof. L. Pesce, Emery determined the weight and starch content of 50 rings before and after their malaxation. The reduction in starch-content in the rejected crumbs varied from 7.3 to 15.51 per cent., but other substances, in all probability proteids, had also been extracted by the ants. He infers that the workers of *M. barbarus minor* either digested or fed to their larvæ at least 7.3 per cent. of the starch in the paste, that they consumed also an unknown quantity of nonamylaceous substances, probably proteids, and that the latter are a much more important aliment than the starch. He concludes with a statement which precisely confirms the observations of the senior author on the various species of *Pogonomyrmex* in the Southwestern States: "The granivorous ants are derived from insectivorous ants and represent an adaptation to the climatic conditions of dry prairies, steppes and deserts. When, owing to the summer droughts, insects become scarce and are no longer sufficiently numerous to satisfy the needs of the ants, the granivorous species substitute the living but dried seeds of plants, but at least the species I have observed, will not refuse any insects that may be obtainable. The seeds, however, have the very great advantage that they can be stored for a long time, that they can be accumulated in granaries and that they contain abundant provisions, not exactly for the winter alone, as the ancients maintained, but for any periods of scarcity."

B. Observations.

We find that the infrabuccal pocket in the numerous Formicidæ we have dissected agrees closely with Janet's account and figures. Among the latter we select his sagittal section of the head of a queen *Lasius niger* L. for reproduction as Text-fig. 1, as it shows the relations of the pocket (*h*), the pharynx (*ph*), buccal orifice, tongue, musculature, and nerve ganglia (brain) of the head very clearly. The tongue is

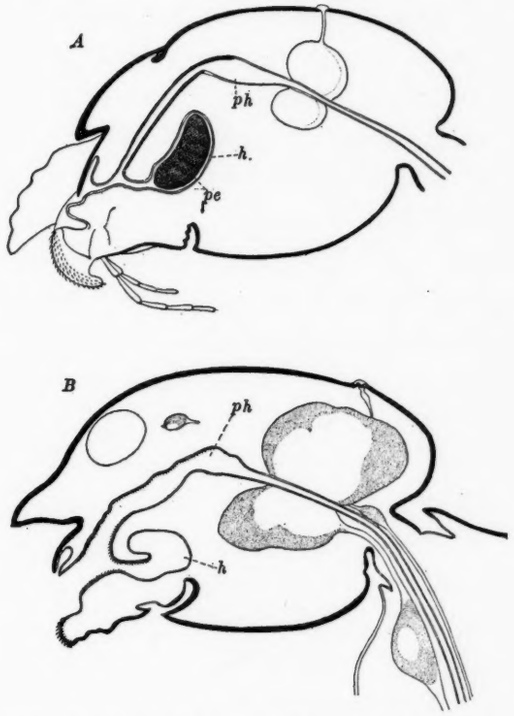

Fig. 2. *A*, Sagittal section of head of female *Camponotus* (*Myrmoturba*) *brutus* Forel. *B*, Sagittal section of head of mature worker pupa of *Viticicola tessmanni* Stitz. *ph*, pharynx; *h*, infrabuccal jacket; *pe*, pellet.

represented as depressed, so that the mouth is wide open. In Text-fig. 2 sagittal sections of the heads of two other ants are represented. *A* is a drawing of the median sagittal surface of the right half of the head of a female *Camponotus* (*Myrmoturba*) *brutus* Forel, with the mouth in the usual closed condition and the pellet (*pe*) *in situ* in the infrabuccal pocket. This view was obtained by simply bisecting the insect's head in the median line with a Gillette razor blade. The pellet stands our very conspicuously among the white surrounding tissues as a blackish, slightly curved, sublenticular body, filling the infrabuccal pocket, which determines its form, and showing a distinct stratification, each layer of which evidently represents the compacted strigil-sweepings of a single toilet operation.

Text-fig. *B* is a sagittal section of the head of a mature worker pupa of *Viticicola tessmanni* Stitz and is introduced, because it shows the very fine rows of chitinous spinules on the tongue, buccal and pharyngeal walls, originally described and figured by Lubbock. It will be noticed that these spinules, as he observed, point forwards in the anterior portion of the pharynx and backwards about the opening of the infrabuccal pocket, so that particles of solid matter would naturally tend to be kept out of the alimentary tract proper and to be directed into the pocket.

Although previous observers have called attention to the thin wall of the pocket and to the complete absence of musculature, no attention seems to have been devoted to the structure of the chitinous lining. We find that the latter has a beautiful pattern of polygonal areas, evidently the expression of the underlying, very flat hypodermal cells and that this pattern appears to vary so much in different species and genera that it might furnish characters of some taxonomic value, at least in certain groups of Formicidæ. We have not, however, deemed it advisable to devote much attention to such details in the structure of the infrabuccal pocket because their bearing on the investigation in hand is too remote.

The pellets, with which we are here concerned, were rather easily dissected out, each was mounted in a drop of glycerine on a slide and then crushed or spread out by pressure on the cover glass. The following table is a very condensed enumeration of the contents of the pellets, 736 in number, taken from species belonging to all the subfamilies, except the Cerapachyinæ and Pseudomyrminæ. No material of the Cerapachyinæ was available, and the Pseudomyrminæ may be reserved for special consideration (pp. 256 and 260).

The following analysis of the infrabuccal pellets of 38 different ants, of very different habits and from widely separated localities reveals several striking results. It will be seen that remains of insect food are by no means as abundant as would be expected. Owing to the nature of such food, however, softer particles can be identified only with difficulty, so that in many instances we have had to place a query in the column in which they are recorded. On the other hand, spores and particles of plant tissue occur in the pellets with surprising constancy, and are often numerous or very abundant, quite irrespective of the nesting habits.

The subfamily Pseudomyrminæ embraces only four genera: *Tetraponera* F. Smith (= *Sima* Roger), *Pachysima* Emery, *Viticicola* Wheeler and *Pseudomyrma* Lund. *Tetraponera* comprises a large number of species which are distributed over Ethiopian Africa, Madagascar, Indomalaya, Papua and Northern Australia. *Pachysima* includes only two species, *aethiops* Fabr. and *latifrons* Emery, and is confined to western equatorial Africa. *Viticicola* is monotypic, the single species, *tessmanni*

PONERINÆ.[1]

Ant.	Locality.	Nesting in	Food.			
			Insect Tissue.	Plant Tissue.	Fungi.	No. Pellets.
Myrmecia tarsata Sm.	N. S. W.	Ground	S	A	F–VN	14
M. gulosa Fabr.	N. S. W.	Ground	S–A	A	N–VN	8
Paraponera clavata Fabr.	Panama	Ground	S	A	VN	10
Diacamma australe Fabr.	Queensland	Ground	S	A	VN	16
Odontomachus hæmatoda L.	Costa Rica	Ground	S	A	N–VN	10
DORYLINÆ.						
Dorylus (*Anomma*) *nigricans* Illiger var.	Congo	Ground	A	?	F	18
Eciton burchelli Westwood	British Guiana	Ground	A	A	F–N	10
MYRMICINÆ.						
Leptothorax curvispinosus Mayr	N. Y.	Dead twigs	?	A	N	41
Aphænogaster tennesseensis Mayr	Mo.	Rotten stumps and logs	?	A	N	30
Veramessor pergandei Mayr	Ariz.	Ground (H)	?	A	F	18
V. andrei Mayr	Calif.	Ground (H)	?	A	F	12
Pogonomyrmex barbatus Sm. var. *marfensis* Wheeler	Texas	Ground (H)	S	A	F–N	46
P. barbatus Sm. subsp. *rugosus* Em.	Ariz.	Ground (H)	S	A	F–N	42
Solenopsis geminata Fabr. var. *rufa* Jerdon	Philippines	Ground (H)	?	A	N	6
S. geminata var. *diabola* Wheeler	Texas	Ground (H)	?	A	F–N	18
Myrmicaria eumenoides Gerst.	Congo	Ground	?	A	F–VN	12
Atta cephalotes L.	(3) Costa Rica	Ground	?	A	F–VN	22
Trachymyrmex septentrionalis McCook var. *obscurior* Wheeler	Florida	Ground	?	A	F–N	8
Cryptocerus multispinus Em.	Guatemala	Dead branches	S	A	F–VN	8
DOLICHODERINÆ.						
Liometopum apiculatum Mayr	Ariz.	Carton nests in hollow live oak	?	A	F–N	10
L. occidentale Em.	Calif.	Carton nests in hollow live oak	S–A	A	A–VN	18
Leptomyrmex nigriventris Guérin	N. S. W.	Hollow logs	S	A	A–VN	14
Dolichoderus (*Hypoclinea*) *doriæ* Em.	N. S. W.	Ground	S–A	A	F–VN	19
Azteca instabilis Sm.	Guatemala	Carton nest	S	A	VN	24

[1] S = fragments of tissue scanty
A = " " " abundant
F = spores or fragments of hyphæ infrequent
N = " " " " " numerous
VN = " " " " " forming considerable fraction of total volume of pellet
H = seed storing ants

FORMICINÆ.

Ant.	Locality.	Nesting in	Food.			
			Insect Tissue.	Plant Tissue.	Fungi.	No. Pellets.
Œcophylla longinoda Fabr....	Congo	Silk and leaf nests on trees	S	A	N	16
Lasius (*Dendrolasius*) *fuliginosus* Latr.....	Switzerland	Carton nests in hollow trees	?	S	VN	33
L. (*Acanthomyops*) *interjectus* Mayr.....	Illinois and Colorado	Ground	?	A	F–N	12
Formica rufa L. var. *obscuripes* Forel.....	B. C.	Mound nests	?	A	F–VN	17
Myrmecocystus melliger Llave	N. Mex.	Ground	?	A	F	12
C. herculeanus L. subsp. *ligniperda* Latr. var. *noveboracensis* Fitch (2)	N. Y.	Decaying stumps	S	A	N	41
Camponotus (*Myrmoturba*) *maculatus* Fabr. subsp. *maccooki* Forel var. *sansabeanus* Buck.....	Texas	Ground	A	S	F–N	11
C. (*Myrmoturba*) *nigriceps* Sm.....	Queensland	Ground	A	A	F–VN	18
C. (*Myrmoturba*) *acutirostris* Wheeler.....	Ariz.	Ground	S–A	A	F	7
C. (*Myrmoturba*) *brutus* Forel (3)	Congo	Rotten wood	S	A	VN	50
C. (*Myrmothrix*) *abdominalis* Fabr. var. *costaricensis* Forel.....	Costa Rica	Dead trees	S	A	A–VN	17
C. Myrmepomis sericeiventris Guérin.....	Guatemala	Rotten tree trunks	?	A	VN	35
Polyrhachis (*Myrmhopla*) *dives* Sm.....	Philippines	Silk and detritus nests on trees	A	A	F–VN	10
P. (*Hagiomyrma*) *ammon* Fab.	N. S. W.	Ground	S–A	A	F–VN	23

Stitz having apparently an even narrower range in the Kamerun and Congo Basin. *Pseudomyrma* is exclusively neotropical, extending from Argentina to Texas, Southern California and Florida and represented by a great number of species, especially in Central and northern South America. Our taxonomic knowledge of this genus is unsatisfactory, because Frederick Smith published very poor descriptions of many of the species and because Forel, who more carefully described many others, very rarely figured his types. The difficulties are increased by the fact that many of the species are highly variable and are often represented by inadequate series of specimens in our collections.

All the species of Pseudomyrminæ nest in the cavities of plants, except *Pseudomyrma elegans* Smith, which nests in the ground or in the walls of termitaria (*vide infra*). The very long, slender body of these ants and of their brood is evidently an adaptation to living in narrow cavities and is most pronounced in the female of *Ps. filiformis* Fabr. (See Wheeler 1919). It is apparent also in other ants which live

under similar conditions (*Cylindromyrmex*, *Simopone*, *Metapone*, *Podomyrma*, *Leptothorax*, some *Camponotus*, etc.).

Although the senior author's collection contains a large number of Pseudomyrminæ, we have selected for the following study only the species which were accompanied by well-preserved larvæ. The reasons for this will be evident from the sequel. The list of forms is as follows:

1. *Tetraponera allaborans* Walker (Text-fig. 4*D*). Los Baños, Philippines (F. X. Williams), from hollow twigs.
2. *Viticicola tessmanni* Stitz (Text-figs. 4*E* and 6). Medje, Belgian Congo (Lang, Chapin and Bequaert), nesting in the hollow stems of a Verbenaceous liana, *Vitex staudtii*. For an account of the larva see Wheeler, 1918, p. 303, Fig. 6.
3. *Pachysima aethiops* F. Smith (Text-fig. 4*B*). Several localities in the Belgian Congo (Lang and Chapin), nesting in the hollow twigs of a Rubiaceous myrmecophyte, *Barteria fistulosa*. For an account and figs. of the larvæ of all stages see Wheeler, 1918, p. 305, Figs. 7 and 8.
4. *Pachysima latifrons* Emery (Text-fig. 4*C*). Niangara, Belgian Congo (Lang and Chapin), nesting in the hollow twigs of *Barteria fistulosa*. See Wheeler 1918, p. 308, Figs. 9, 10, for a description and figures of the various larval stages.
5. *Pseudomyrma gracilis* Fabr. (typical) Text-figs. 3, 4*A* and 5). Corozal and Las Sabanas, Panama (Wheeler), nesting in hollow twigs; Quirigua, Guatemala (Wheeler), living in hollow thorns of *Acacia*.
6. *Ps. gracilis* var. *mexicana* Roger. Cuernavaca, Mexico; San Jose and Cartago, Costa Rica; Lake Atitlan and Guatemala City, Guatemala (Wheeler), nesting in hollow twigs. This variety occurs as far north as Brownsville and Victoria, Texas.
7. *Ps. gracilis* var. *dimidiata* Roger. Patulul and Escuintla, Guatemala (Wheeler), in hollow twigs.
8. *Ps. gracilis* var. nov. Escuintla, Guatemala (Wheeler), nesting in *Acacia* thorns.
9. *Ps. rufomedia* F. Smith. Antigua and Guatemala City, Guatemala (Wheeler), in hollow twigs. The specimens agree with Smith's very poor description and are from the same locality as the types. The species has not been recognized by recent authors.
10. *Ps. elegans* F. Smith. Kartabo, British Guinana (Alfred Emerson), nesting in a termitarium. According to Forel this is the only *Pseudomyrma* known to nest in the ground.
11. *Ps. belti* Emery. Escuintla, Guatemala (Wheeler), nesting only in the thorns of acacias (*A. cornigera* L.). This and the two following forms feed on the Beltian bodies and nectar of their host-trees.
12. *Ps. belti* var. *fulvescens* Emery. Quirigua and Zacapa, Guatemala (Wheeler), like the type of the species, nesting only in *Acacia* thorns (*A. hindsi* Benth.).
13. *Ps. spinicola* Emery. Las Sabanas, Panama (Wheeler), nesting in the thorns of *Acacia sphaerocephala* S. & C.
14. *Ps. sericea* Mayr var. *fortis* Forel. Escuintla, Guatemala (Wheeler), nesting in the hollow internodes of *Triplaris macombii* (D. Smith).
15. *Ps. championi* Forel var. Lake Atitlan, Guatemala (Wheeler), nesting in hollow twigs.

16. *Ps. filiformis* Fabr. Patulul and Zacapa, Guatemala (Wheeler), nesting in dead branches. The three phases and the habits of this species are described in a recent paper by the senior author (1919).
17. *Ps. decipiens* Forel. Cartago, Costa Rica and Antigua, Guatemala (Wheeler), nesting in hollow twigs.
18. *Ps. caroli* Forel. Guatemala (Wheeler) nesting in hollow twigs.
19. *Ps. elongata* Mayr. Bahamas and Cuba (Wheeler), nesting in hollow twigs.
20. *Ps. flavidula* F. Smith. Andros and New Providence Islands, Bahamas (Wheeler), nesting in hollow twigs and the culms of grasses.
21. *Ps. flavidula* var. *delicatula* Forel. Guadalajara, Mexico (McClendon), Austin, Texas and San Jose, Costa Rica (Wheeler), nesting in hollow twigs.
22. *Ps. arboris-sanctæ* Emery. Frijoles, Panama (Wheeler), in hollow trunks and branches of *Triplaris cummingiana.*
23. *Pseudomyrma* sp. Patulul, Guatemala (Wheeler); in hollow twigs.
24. *Pseudomyrma* sp. Antigua, Guatemala (Wheeler), in hollow twigs.
25. *Pseudomyrma* sp. Escuintla, Guatemala (Wheeler), in hollow twigs.
26. *Pseudomyrma* sp. Cartago, Costa Rica (Wheeler), in hollow twigs.

The pellets were dissected out of several workers of a number of the Pseudomyrminæ in the foregoing list, but as they had precisely the same composition as the pellets taken from the trophothylaces of the cospecific larvæ, apart from a somewhat smaller number of particles of insect tissue, a detailed account of the components is here omitted and the reader is referred to the description of the larval pellets on p. 261.

II. FEEDING HABITS OF LARVAL ANTS.

A. Historical.

In 1918 the senior author rather hastily reviewed what was known concerning the feeding of various ant-larvæ. It was shown that the most general method was with liquid food regurgitated by the worker nurses, a method probably almost universal in many species of certain subfamilies (Myrmicinæ, Dolichoderinæ and Formicinæ) for the larvæ throughout their development. In this connection attention may be again called to the following observation by Miss Fielde (1901) on the young larvæ of the Myrmicine *Aphaenogaster fulva* Roger: "The feeding of the larva, which is bent nearly double in the egg, with regurgitated food begins as soon as it straightens itself and protrudes its mouth. When the larvæ begin to appear in the egg-packet, the workers lift the packet and hold it free and still, while one of their number holds a transparent white globule of regurgitated food to the larval mouth projecting from the surface of the egg-packet. I have repeatedly seen the workers thus feeding the very young larvæ, a single globule of regurgitated food serving for a meal of which four or five larvæ successively partook."

Newell (1909) studied the feeding of the larva in a Dolichoderine, the Argentine ant, *Iridomyrmex humilis* Mayr, and describes the process as follows: "The larva ordinarily lies upon its side or back. The attending worker approaches from any convenient direction, usually from one side or from the direction in which the head of the larva lies and, spreading her mandibles, places them over the mouth-parts of the larva which are slightly extruded. The tongue of the worker is also in contact with the larval mouth. While the worker holds body and mandibles stationary a drop of light-colored, almost transparent fluid appears upon her tongue. This fluid disappears within the mouth of the larva, but it cannot be ascertained to what extent the larval mouth-parts are moved during the operation, owing to their being obscured from view by the mandibles and head of the attending worker. Slight constrictions of the larval abdomen during feeding are sometimes noticeable, at other times not. The time required for feeding a single larva varies from 3 to 30 seconds, depending doubtless on the hunger of the "baby." The workers proffer food to, or at least inspect, each larva, for the worker doing the feeding will place her mandibles to the mouth of one larva after another, feeding those which seem to require it."

Undoubtedly many Ponerinæ and even some Myrmicinæ and Formicinæ feed their larvæ directly with whole insects or pieces of insects. The facts relating to this method of feeding, especially in the Ponerinæ were also reviewed by the senior author in 1918. Janet has seen the larvæ of the European *Lasius flavus* DeG. feeding in this manner, and the senior author has made similar observations on the American species of *Lasius* belonging to the subgenus *Acanthomyops*, on *Aphaenogaster fulva* and *Pogonomyrmex imberbiculus* Wheeler. Workers of *P. imberbiculus* were given house-flies, and the senior author, writing in 1902, stated that "these were not only eaten with avidity by the adult *Pogonomyrmex*, but cut into pieces and fed to the larvæ in the same manner as I have described for the Ponerinæ and Myrmicinæ (1900). On one occasion nearly every larva in the nest could be seen munching a small piece of house-fly."

Certain agricultural ants, which feed on seeds (*Pogonomyrmex, Messor*) also nourish their young with the same food. Thus the workers of *P. imberbiculus*, referred to above, were seen to bring seeds from their granaries, crack them open and "after consuming some of the softer portions themselves, to distribute the remainder among their larvæ. The latter could be seen under the lens, cutting away with their mandibles and devouring the soft, starchy portions of the seeds." Emery (1915, p. 185) has observed the same method of feeding the larvæ in *Messor barbarus* L. He says: "The granivorous ants also nourish their larvæ with seeds, either in the form of the regurgitated contents of the crop, or in the form of seed-fragments satur-

ated with saliva and directly administered. In an artificial nest I have also seen a worker of the court-yard ant carrying in her mouth one or two larvæ attached to a piece of masticated seed."

Still another type of feeding prevails among the fungus-growing Myrmicinæ of the tribe Attiini. Tanner in 1892 observed *Atta cephalotes* L. in Trinidad and reported as follows: The eggs become enveloped in a "pearly white fluffy growth," evidently masses of fungus hyphæ. The larvæ, on hatching from these eggs "are usually placed on top of the nest and are constantly attended by the smallest workers—the nurses—who separate them into divisions according to size. At first it seemed a mystery, how these minute grubs could be fed so systematically, knowing that each individual larva was only one among so many, yet certain it was, that all were equally attended to. Further observations showed that nature had provided most efficiently for them to ask for food when they required it. This the larvæ do by pouting their lips; at this notification of their requirement the first nurse who happens to be passing stops and feeds them. The nurses are continually moving about among them with pieces of fungus in their mouths ready for a call for food. The nurses feed the minute larvæ by merely brushing the fungus across their lips showing that the spores alone are sufficient for its food at that period of its life. But it is not so when the larvæ have increased so much in size, that the pout can be seen without a glass, for then the whole piece after having been manipulated by the nurse's mandibles into a ball, in the same manner as the leaves are served, when they are first brought into the nest, is placed in its throat and if that is not sufficient the pout continues when the next one and even the next passing proceeds with the feeding, till the pout is withdrawn, showing that it is satisfied. No further notice is then taken of it by the feeders, until it again asks for a meal by pouting later on in the day."

Tanner's observations were made on adult colonies. J. Huber (1905), however, shows that the conditions in incipient colonies of the closely allied *Atta sexdens* L. are different. As soon as the first larvæ hatch, they are fed directly with eggs placed in their mouths by the queen. To quote part of his description, "after the mother ant has laid an egg she first palpates it for several seconds and then turns to a larva, which she tickles with her antennæ till it begins to move its jaws, whereupon she thrusts the egg, usually with some force, end foremost between the jaws, which continue to move against it. The egg sometimes stands off perpendicularly from the body of the larva, sometimes and more frequently it lies more or less closely applied to the larva's ventral surface. In the latter case the mother ant often presses the egg down with her foot. If the larva is still small, the egg is usually taken away after a short time and given to another larva; a large larva, however, is able to suck out an egg com-

pletely in the course of three to five minutes, so that only the collapsed shell remains, which is later licked away by the mother. . . . I believe that eggs, at least till the appearance of the first adult workers, are the exclusive food of both the mother and her brood. I have never seen *Atta* females giving their larvæ the mycelium or kohlrabi of the *Rozites*. And, unlike von Ihering, I have never seen the mother ant devouring kohlrabi." The best proof that the larvæ of the first brood do not receive fungus food is that the *Atta* queen is occasionally able to bring them up without a fungus garden. Later, according to Huber, after the first workers have matured the larvæ are fed by the latter with the peculiarly modified hyphæ called "kohlrabi" by Moeller and Huber.[1]

According to Bruch (1919), the queen of the Argentine fungus-grower, *Acromyrmex lundi*, also nourishes the larvæ of her first worker brood with some of her eggs.

We may now turn to a consideration of the Pseudomyrminæ, the larvæ of which were first described and figured by Emery in 1899. His remarks, referring to *Pseudomyrma* and *Sima* (*Tetraponera*) are here translated *in extenso:* "These two genera present larvæ of a very peculiar type. I have examined those of *Pseudoponera flavidula* F. Sm. (Cayenne, collected by Pillaut) and of certain species of *Sima*, particularly *S. natalensis* F. Sm. and *S. clypeata* Emery (Cape Colony, collected by Dr. Brauns).

"The larvæ are subcylindrical anteriorly and somewhat narrowed behind; the first postcephalic segments are more developed dorsally, shortened ventrally, so that in profile they seem to have a fan-shaped arrangement, their dorsal outlines forming together a curve or convexity, which constitutes the apparent anterior extremity of the larva, while the head, or morphological anterior end is found to be situated on the ventral surface of the body. Hence these larvæ may be called *hypocephalic*, in contradistinction to those belonging to the great majority of ants, which I would call *orthocephalic*.

"In *Sima* (Fig. 7) the head is depressed and scarcely projects from the ventral surface of the larva, its buccal extremity being, when in a state of repose, embedded in a cavity of the third and fourth segments of the trunk on which it rests. In *Pseudomyrma* (Fig. 8), the head is rounded and distinctly projecting and the third and fourth segments of the trunk are not hollowed out to receive it. The maxillæ

[1] A recent study by the senior author of various Attiine larvæ, belonging to the genera *Atta*, *Trachymyrmex*, *Cyphomyrmex*, *Sericomyrmex*, and *Apterostigma*, shows that their mouthparts are beautifully adapted for the methods of feeding described by Tanner and Huber. The mandibles are short, stout and acute, and except in *Apterostigma*, covered with numerous sharp spines, so that the delicate egg-shell or the thin walls of hyphal filaments or of kohlrabi spherules can be held and firmly squeezed and at the same time perforated with many small openings, thus allowing the liquid contents to be rapidly expressed and trickle into the mouth.

have no conical projections, there being in their places groups of small tubercles. Two similar groups are seen on the labium, where the cones are also lacking. The mandibles are small, slightly projecting, but robust and bidentate at the apex. A character quite peculiar to the larvæ of *Sima* and *Pseudomyrma* is the presence on the head of a pair of small appendages which I regard as rudiments of antennæ. Each of them consists of two small unequal conical or subcylindrical projections terminating in a very minute, obtuse hair (olfactory hair?). The hairs of the body are short and simple, but one observes, distributed along the body, four double series of long setæ, uncinate at the tip and regularly arranged on the single segments, as shown in Fig. 7*a*." The last remark, on the hooked hairs, refers to *Tetraponera*, as Emery neither figures nor describes the homologous structures in *Pseudomyrma*.

In 1912 Emery gave a brief account and a figure of the full-grown larva of *Pachysima aethiops*. He saw some of the peculiar exudatoria in the final stage of their development but erroneously interpreted them as the "beginnings of legs."

In 1918 the senior author figured and described the various stages of the larvæ of *P. aethiops* and *latifrons* and the larva of *Viticicola tessmanni*, but at that time his attention was mainly concentrated on the development and structure of the exudatoria. Nevertheless the structure of the trophothylax and enclosed food-pellet was figured and briefly described.

The only other published account of immature stages of Pseudomyrminæ is a study of the embryology of an unidentified species of *Pseudomyrma* by Strindberg (1917), treating only of the early stages. The type of cleavage, which occurs in two periods, is unique or unusual, not only in the Formicidæ but among insects in general. The serosa is absent, a peculiarity which the Swedish embryologist has also observed in *Leptothorax* and *Tetramorium*.

B. Observations.

The stomach contents of the great majority of ant-larvæ are so finely comminuted and have been submitted to such protracted digestion that it is impossible to determine their precise nature. It will be shown below how the comminution is probably brought about. Occasionally a minute fragment of chitin or a few hairs may indicate that parts of insects have been ingested. Still there are a few genera that show very clearly that the larvæ are nourished with pellets of insect flesh or with rather coarse fragments of insects. We have found peculiar conditions in the Dorylinæ. Examination of a number of half- and nearly full-grown worker larvæ of *Eciton burchelli* Westwood received from Mr. Wm. Beebe, shows that the stomach is unlike that of other known ant-larvæ in being very long and slender and in having unusually thick,

muscular walls. The larva is fed, apparently at considerable intervals, with rather large pellets consisting of the rolled up soft-parts of insects. These pellets are so compact that they retain their form in the narrow lumen of the stomach, where they lie in an irregular longitudinal series. Occasionally minute fragments of chitin or a few fungus spores are present, but owing to the feeble development of the larval mouth-parts so characteristic of the Dorylinæ, it is evident that the worker must prepare these pellets by carefully trimming away the hard, chitinous portions of their insect prey and rolling up the denser, muscular portions of the flesh. The worker probably consumes much of the exuding juices while engaged in this operation and before stuffing the pellets into the gullets of the larvæ.

The other genera in which the larval stomach was found to contain evidence of the precise nature of the food are *Cataulacus*, *Cryptocerus* and *Leptothorax*. We have examined the larvæ of *Cryptocerus egenus* Santschi, *Cryptocerus minutus* Klug, *C. multispinus* Emery, *C.* (*Cyathocephalus*) *varians* F. Smith and *wheeleri* Forel and some eight species of *Leptothorax* of the sugbenera *Leptothorax* sens. str., *Mychothorax* and *Goniothorax* from such widely different localities as the Congo, Central America, the Northern United States and Southern Europe. In all the forms cited the larval stomach is voluminous and closely packed with coarse chitinous fragments of small insects (Plate I, Fig. 8), in some cases interspersed with numerous fungus spores (Plate I., Fig. 6). In some species of *Leptothorax* the whole contents consist of entire or nearly entire legs of small insects. The mandibles of the larvæ of these three genera are short, broad and stout and therefore well-adapted to crushing, so that the coarse fragments may have been bitten off by the larvæ from larger pieces or whole insects proffered by their worker nurses. The pieces may, however, have been cut up to a considerable extent by the workers. The fungus spores may have come from their infrabuccal pouches, but some of the coarse insect materials at least, could hardly have such a provenience, especially in such small ants as the species of *Leptothorax*.

An examination of the meconial pellets, taken from the cocoons of the Ponerinæ and Formicinæ, furnishes no satisfactory information in regard to the larval food, apart from the fact that they contain black or dark brown, very finely comminuted solid matter. A biochemical investigation, which we are not competent to undertake, would probably reveal the presence of significant decomposition products.[1] The

[1] The workers and soldiers of highly carnivorous ants, such as the Dorylinæ and certain species of *Pheidole*, *e.g.*, *Ph. ecitonodora* Wheeler and *fallax* Mayr, *Megaponera foetens* Fabr. and *Paltothyreus tarsatus* Fabr., have a very powerful odor like that of the species of *Chrysopa* and therefore very similar to indol. Melander and Brues (1906) regarded the substance as probably being a leucin. Though this substance seems to be normally present in the feces of insects, there is some doubt as to whether in *Eciton* it is derived from the alimentary tract or from certain dermal glands, such as those in the epinotum.

Ponerinæ, with very few exceptions, are so patently and exclusively insectivorous that it is unnecessary to resort to the meconium for information concerning their diet. In the Pseudomyrminæ interesting information is obtained from a study of the pellets

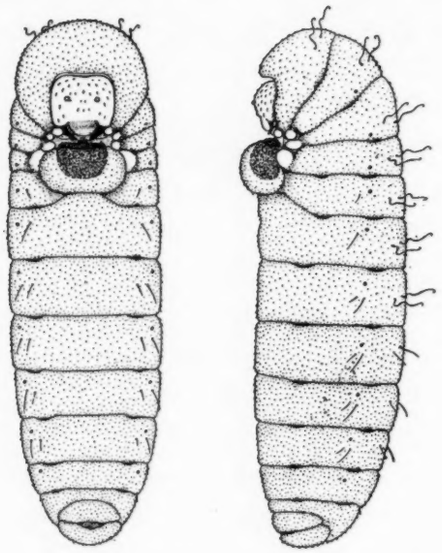

FIG. 3. Ventral and lateral view of larva of *Pseudomyrma gracilis* Fabr.

found in the trophothylaces. Before entering on this study, however, it will be advisable to introduce a more comprehensive account of the peculiar larvæ of this subfamily.

The adult larvæ of all four genera of Pseudomyrminæ are much alike (Text-figs. 3 and 4). The body is long, straight and cylindrical, not broader posteriorly as in nearly all other ant-larvæ. The anterior and posterior extremities are blunt and rounded and the segments are all sharply defined. The integument is uniformly thin and perfectly transparent, though tough, only the mandibles, as a rule, being strongly chitinized and the lining of the buccal cavity somewhat pigmented. The prothoracic segment is large and hood-shaped, and in certain species can be drawn down over the head; the meso- and metathoracic segments are narrowed ventrally, the head is large, somewhat flattened, usually subrectangular, about as broad as long and embedded in the ventral portions of the thoracic segments. The antennal rudiments are always distinct as small, rounded papillæ, each bearing three sensillæ. The mandibles are small, stout and bidentate, sometimes with a vestige of a third tooth, their upper surfaces covered with regular rows of subimbricate papillæ. The maxillæ are large, swollen and rounded, lobuliform, the labium short and broad, with the transverse, slit-shaped opening of the salivary duct in the middle. The sensory organs which

in many other ants have the form of papillæ or pegs on the maxillæ and labium are in the Pseudomyrminæ usually reduced to small areas or feeble eminences, bearing the groups of sensillæ. The anterior maxillary organ has five, the posterior two and each labial organ has five of these sensillæ. The buccal cavity is broad and transverse, its dorsal and ventral walls being in contact and both furnished with fine, regular

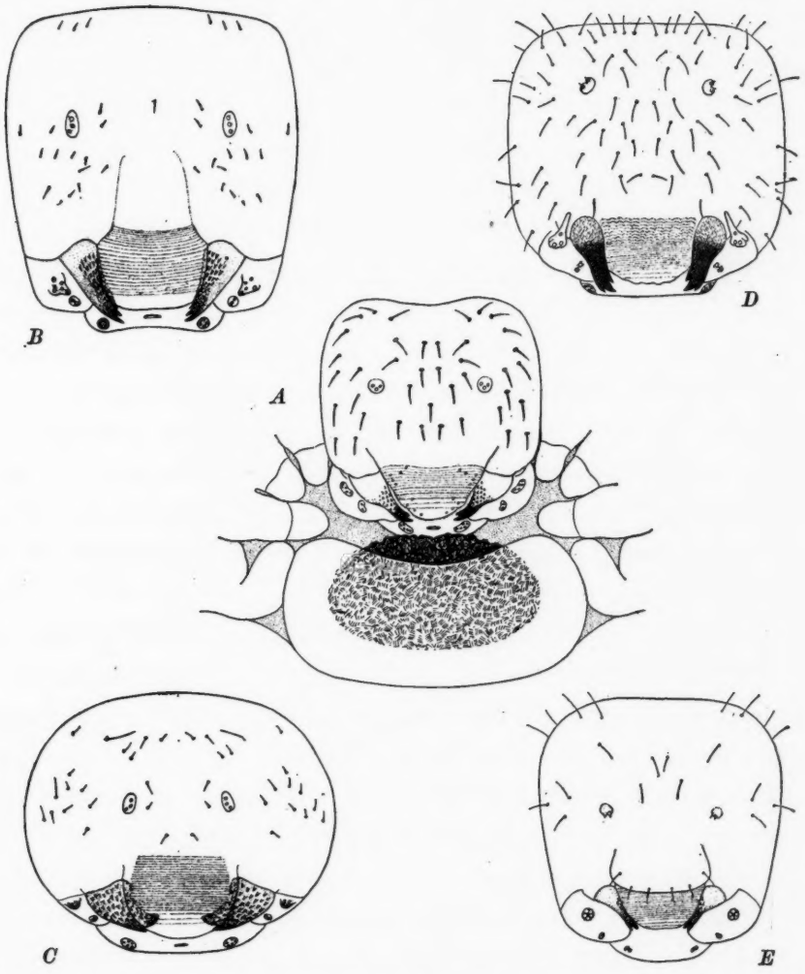

FIG. 4. *A*, Head, trophothylax and exudatoria of larva of *Pseudomyrma gracilis* Fabr. *B*, Head of *Pachysima aethiops* F. larva. *C*, Head of *P. latifrons* Emery larva. *D*, Head of *Tetraponera allaborans* Walker larva. *E*, Head of *Viticicola tessmanni* Stitz larva.

transverse ridges. This peculiar structure, the *trophorhinium*, will be described in greater detail below. Each thoracic segment bears a rounded papilliform exudatorium ventrally on each side next to the head. The sternal portion of the first abdominal segment is transversely elliptical, swollen, protuberant and furnished with a food-

pouch, the *trophothylax*, opening forward, *i.e.*, towards the mouth-parts (Text-figs. 3 and 4*A*).

The hairs on the body of the larva are of three kinds: first, short, stiff, very acute hairs, generally and rather evenly distributed over the whole surface (*microchaetæ*); second, much longer, stouter, more gradually tapering, lash-like and somewhat curved hairs of unequal length, singly or in a row or loose cluster on each ventrolateral surface of each abdominal segment (*acrochaetæ*), and third, long hairs, of uniform length, only slightly tapering, with hooked tips (*oncochaetæ*). These are normally present in transverse rows of four to eight on the dorsal surfaces of the three thoracic and first three to eight abdominal segments. On the more posterior segments they are often represented by simple, *i.e.*, pointed hairs.

In the genera *Tetraponera*, *Viticicola* and *Pseudomyrma* the youngest larvæ, apart from their proportionally longer and more conspicuous oncochaetæ and acrochaetæ and more protuberant trophothylax, have essentially the same structure as full-grown individuals. In the two species of *Pachysima*, however, as the senior author has shown (1918), the youngest larvæ are very unusual in possessing long, stout, blunt bristles in the place of the oncochaetæ and extraordinary exudatoria which may have the form of appendages on the three thoracic and first abdominal segments. The following generic and specific modifications of the principal characters detailed in the preceding paragraphs were noticed in the various Pseudomyrmine larvæ examined:

Tetraponera allaborans (Text-fig. 4*D*).—Head rather rounded behind. Anterior maxillary sense-organs produced into slender, anteriorly directed points. Oncochaetæ straight, without sigmoidal flexure, in four rows, a pair in each row, on each segment from the prothoracic to the sixth abdominal. Similar hairs, but without hooks, occur on the seventh to ninth abdominal segments. Acrochaetæ absent. Microchaetæ short and sparse, much longer on the head and somewhat longer on the prothorax than on the more posterior segments.

Viticicola tessmanni (Text-fig. 4*E*).—Larva very slender, with very prominent trophothylax and large rounded exudatoria on each side of it. Oncochaetæ straight, four in number, in two pairs on each segment from the prothoracic to the sixth to eighth abdominal. Acrochaetæ very long but unequal, numerous, on the side of each abdominal segment. Microchaetæ short, unequal, scattered.

Pachysima aethiops (Text-fig. 4*B*).—In large larvæ there seem to be no traces of oncochaetæ or acrochætæ. Microchaetæ short, of unequal length. Head rather subrectangular, mandibles broader than in the three other genera and the sense-organs of the maxillæ appearing as distinct though low tubercles. Labrum trapezoidal, its anterior border truncate, entire.

Pachysima latifrons (Text-fig. 4*C*).—In large larvæ the oncochaetæ are present, but very short, stout, curved or feebly sigmoidal, eight on each segment from the metathoracic to the seventh abdominal. Acrochaetæ absent; microchaetæ much longer than in *aethiops*. Head transversely elliptical. Mandibles broader and flatter, with broad, blunt apical tooth. Papillæ bearing the maxillary sensillæ less prominent. Labrum with more rounded anterior border.

Pseudomyrma (Text-figs. 3 and 4*A*).—Head distinctly subrectangular, as long as broad. Exudatoria present as rounded papillæ on the ventrolateral borders of the three thoracic and first abdominal segments, and much alike in the youngest and oldest larvæ. Trophothylax more protuberant in the youngest stages, but throughout larval life with essentially the same structure as in the three other genera. Mandibles small, with 2–3 strongly chitinized, rather blunt teeth. Upper surface of mandibles with regular rows of subimbricate papillæ as in the other genera. Trophorhinium well-developed. Oncochaetæ slender, always sigmoidal, or with one or more flexures, four, very rarely six to a segment, in two pairs on the dorsal surface of the three thoracic and a variable number of basal abdominal segments; rarely lacking on the pro- and mesothorax. Acrochaetæ long, single or in rows, on the first five or more abdominal segments. Microchaetæ much as in the other genera. While the morphological characters seem to be very constant, the pilosity differs somewhat in the different species, as shown in the following series:

Ps. gracilis and varieties (Text-figs. 3 and 4*a*).—Oncochaetæ occasionally lacking on the pro- and mesothorax, but usually present on all the thoracic and four basal abdominal segments, replaced by simple bristles on the three succeeding segments. Acrochaetæ single on first abdominal, two on second to fifth segments, the more ventral hair smaller. Microchaetæ acute, bristle-like.

Ps. rufomedia.—Acrochaetæ five or six on the side of each abdominal segment, a few smaller homostichous hairs also on the thoracic segments.

Ps. caroli.—Oncochaetæ on all the thoracic and first to fifth abdominal segments. Acrochaetæ single. Microchaetæ very small and delicate so that the integument seems to be very smooth.

Ps. filiformis.—Oncochaetæ lacking on prothorax, rather short and delicate on the meso- and metathorax and basal abdominal segments. Acrochaetæ single, not very stout. Microchaetæ rather long, sparser than in *gracilis*.

Ps. belti var. *fulvescens*.—Oncochaetæ six on pro- and mesothorax, three on each side of the middorsal line. Acrochaetæ three or four on each side of the abdominal segments, in a transverse row. Microchaetæ sparse as in *filiformis* but coarser and more conspicuous.

Ps. flavidula. Acrochaetæ flagelliform, rather delicate, single. Microchaetæ sparse, distinct.

Ps. sericea var. *fortis.* Oncochaetæ slender, on the three thoracic and first to third abdominal segments. Acrochaetæ stout, single, on the first to sixth abdominal segments. Microchaetæ extremely short and sparse so that the integument appears to be very smooth. Hairs on head scattered and inconspicuous.

Ps. championi var.—Oncochaetæ delicate, present on three thoracic and first to fourth abdominal segments. Acrochaetæ slender, single, on first to sixth abdominal segments. Microchaetæ small, inconspicuous.

Ps. decipiens.—Very similar to *championi*, but the oncochaetæ even more delicate and the acrochaetæ longer and stouter. Antennal rudiments and maxillary sense-organs larger and more heavily chitinized.

Ps. elegans.—Antennal rudiments small. Exudatoria, especially of the prothorax, larger than in the other species. Oncochaetæ long and moderately stout, on the three thoracic and first to fifth abdominal segments. Acrochaetæ long, four or five in a regular transverse row on each side of all the abdominal segments. In the young larvæ these hairs are very long and form an uninterrupted transverse row on the ventral and lateral surfaces of each abdominal segment.

Ps. elongata.—Pilosity very much as in gracilis. Acrochaetæ long, delicate, single. Microchaetæ short and sparse.

We may now turn to an analysis of the pellets dissected out of the trophothylaces of the various species of Pseudomyrminæ. As all of these corpuscles (400 in number) contain more or less insect fragments, whereas the latter are frequently absent or very scarce in the infrabuccal pellets of the cospecific workers, we have been led to assume that the corpuscle given to the larva is compounded of accumulated strigil-sweepings (spores, mycelium, pollen, particles of dirt, etc.) together with pieces of freshly captured insect prey. In all probability the adult ant, while malaxating the latter, consumes portions of it before depositing the remainder together with the heterogeneous contents of the infrabuccal pocket in the trophothylax. In other words, the Pseudomyrminæ combine the contents of the dust-bin and garbage-can and serve up the mixture as appropriate food for their young—a truly remarkable example of food-conservation and one certainly very rarely, if ever, exhibited by the ants of the other subfamilies!

While it is rather easy to identify most of the vegetable and inorganic substances in the tropholthylax pellets, this is by no means true of their insect components. The latter comprise small irregular bits of chitin, hairs, pieces of facetted eyes, legs, antennæ, etc. No entomologist is sufficiently expert to refer such fragments even

to the order, much less to the family or genus to which they belong, especially when, as in the case of the pellets of the Pseudomyrminæ, he is dealing with fragments of small tropical insects. Only in a few instances, where the fragments were exceptionally large or of insects possessing unique peculiarities of structure (Coccids, Lepidoptera, some Coleoptera) has it been possible to refer them to their family or order.

The species of Pseudomyrminæ are here discussed in the same sequence as in the list on p. 250.

1. *Tetraponera allaborans* (35 pellets).—Practically all of the pellets of this species consist of fragments of small insects, among which pieces of chitin, detached mandibles, and pieces of eyes and hairs can be recognized. Pollen, fungus spores and pieces of mycelium occur in very few instances. Plate IV, Fig. 29 shows pollen grains and numerous small insect fragments, among which portions of the compound eyes of at least two species are clearly discernible.

2. *Viticicola tessmanni* (13 pellets).—The insect substances in the pellets of these larvæ resemble the yolk of ants' eggs and the fat-body of the larvæ themselves, suggesting that some of the brood had been used as food for the more vigorous progeny. In one pellet pieces of the skin of a *Viticicola* larva could be clearly recognized. There are also spores and bits of hyphæ in many cases and particles that seem to be pith and callous tissue. This ant forms very large colonies which nest only, so far as known, in the hollow stems of a singular Verbenaceous liana, *Vitex staudtii*. The pale color of the adult ants, even of the males, the relatively small eyes, the often wingless or subapterous females and the peculiar relations of the ants to the host plant as described in another paper (1921?) by the junior author, all indicate a condition of symbiosis more intimate even than that obtaining between certain species of *Pseudomyrma* (*belti, spinicola, canescens*) and the bull-horn acacias. In this connection, the absence from the *Viticicola* pellets of any clearly recognizable insect material obtained outside the myrmecodomatia may be very significant.

3. *Pachysima aethiops* (34 pellets).—Like the preceding, this large black ant is definitely associated with a host plant, in this instance *Barteria fistulosa*, in the internodes of which it lives. It has precisely the same geographical range as the *Barteria* in the western part of the Ethiopian Region. Practically every pellet examined contains pieces of Coccids or the crumpled-up bodies of entire young Coccids. Fungus spores and pieces of mycelium are often abundant as are also pieces of plant-tissue, evidently gnawed from the walls of the cavities (myrmecodomatia) inhabited by the ants. In a few of the pellets the junior author also found small Nematodes resembling the species of *Pelodera* described by Janet (1893*b*, 1893*c*) as living both as parasites in the pharyngeal glands of certain European ants and as free organisms in the

detritus of the nest. Plate I, Fig. 1 shows an entire mite (?) taken from one of the pellets; Fig. 3 a cluster of spores mixed with the soft tissues of a Coccid. Plate II, Fig. 10 shows what is an unmistakable piece of a Coccid, and Fig. 13 an entire specimen of small size but so perfectly preserved that Prof. Robert Newstead was able to identify it as a larval *Stictococcus formicarius* Newst. In Fig. 14 there is a collection of spores, plant-hairs and other detritus, and similar materials make up the bulk of the pellet, part of which is shown in Plate V, Fig. 37.

4. *Pachysima latifrons* (2 pellets).—This species is much rarer than the preceding but seems to have very similar habits. Of the two pellets examined, one contains fragments of Coccids, some spores, bits of mycelium, pollen grains and some pith tissue, with amber-colored cell-contents; the other contains much the same substances together with a few Nematodes.

5. *Pseudomyrma gracilis* (10 pellets).—Half of the pellets are from the larvæ of colonies inhabiting dead twigs, the other half from colonies nesting in the large thorns of acacias. In the former the pellets consist of bits of insects, often showing the hairs, legs, claws, antennal and tarsal joints and groups of ommatidia, more or less abundant spores, hyphæ and pieces of plant-tissue, especially plant-hairs. Plate IV, Fig. 28 shows a portion of one of these pellets with its coarse insect fragments. The pellets of the larvæ from acacia thorns are very different in that they lack the bits of insects and contain instead pieces of the food-bodies (Beltian bodies) of the host-plant, mingled with spores and in some instances with a few pollen grains. The occurrence of the Beltian bodies in the larval food-pellets is interesting, as it shows that *Ps. gracilis*, which nearly always nests in the dead twigs of various trees and bushes, is quite as able, when inhabiting acacias, to utilize the Beltian bodies as food as are the obligatory Pseudomyrmas (*belti, spinicola, canescens*) of these trees.

6. *Pseudomyrma gracilis* var. *mexicana* (43 pellets).—The long series of pellets of this variety contains an extraordinary collection of the most diverse insect fragments, varying considerably in color and evidently belonging to numerous species. Some of the pieces are rather large chunks, others finely triturated and are mingled with more or less abundant collections of spores, occasional hyphal fragments and pollen grains. Among the pollen grains those of pines are easily recognized. We figure portions of six of the pellets on Plates II to V. Fig. 11 shows an assortment of spores and pollen grains, Fig. 17 very coarse bits of insects with pollen grains and Fig. 22 a considerable portion of a small crushed (beetle?) larva. In Fig. 27 the insect fragments are more finely divided and mixed with fungus hyphæ, in Fig. 30 much coarser and retaining their bristly hairs. Fig. 36 represents a small portion of a pellet made up entirely of a great variety of spores, with some insect hairs.

7. *Pseudomyrma gracilis* var. *dimidiata* (29 pellets).—All the pellets contain insect fragments and most of them a variable number and variety of spores and pollen grains. Some of the latter could be recognized as belonging to pines. Of the bits of insects, some are very coarse, as shown in Plate IV, Fig. 31. The fungus spores, of which several are shown in Plate V, Fig. 34, are very peculiar. One of them is represented under a higher magnification in Fig. 32 on the same plate.[1]

8. *Pseudomyrma gracilis* var. nov. (5 pellets).—The components of the pellets of larvæ of this variety taken from large acacia thorns are similar to those of *dimidiata*. Bits of insects, spores and pollen grains, some of which were from pine trees, predominate. Two of the pellets also contain small pieces of plant-tissue. Plate I, Fig. 5 shows large fragments clearly recognizable as portions of a small Curculionid beetle. In trimming the photograph the snout was cut off. Plate IV, Fig. 23 shows a mixture of pollen grains and insect appendages.

9. *Pseudomyrma rufomedia* (21 pellets).—Nearly all the pellets contain numerous bits of insect material, some in chunks. Spores and pollen are also present, being in some cases very abundant, as shown in Plate V, Fig. 33, from a pellet made up entirely of these components.

10. *Pseudomyrma elegans* (2 pellets).—Only two pellets were obtained from a small number of larvæ of this species collected by Mr. Emerson in British Guiana in an earthern termitarium. One contains considerable fragments of a small larva (apparently a Myrmeleonid, judging from a nearly entire eye), a few fungus spores, Lepidopteran scales and pieces of algal filaments. The other, much less voluminous pellet contains bits of an unidentified insect, with small pieces of vegetable tissue (bits of moss leaves?). In neither pellet was there anything that could be identified as pieces of termites or of their eggs.

11. *Pseudomyrma belti* (32 pellets).—In the pellets of this ant, which is an obligatory inhabitant of the thorns of the bull-horn acacias, insect fragments are less abundant than in the various preceding species; spores are also scarce in most cases;

[1] These spores, or rather spore-masses, evidently belong to a species of *Ravenelia*, a singular genus of leaf-fungi, the interesting morphology of which was elucidated by Parker many years ago (1886). The specimens were forwarded to Prof. J. C. Arthur, who sent the following comments on them: "I am able to make out that the very interesting rust spores on the slide, which you sent, are those of *R. distans* Arth. & Holw. Not only are the teliospores present but the very characteristic urediniospores also, a half dozen or more of which show in excellent condition. They are unusually small, thin-walled and pointed, with four equatorial pores, making them a very characteristic spore. The host is some Mimosaceous plant, but unfortunately has not determined, as you will see on page 424 of my Uredinales of Guatemala. Only one collection of the rust is known, which came from Retalhuleu. Of course an ant would pick up spores of the rarest fungus possible, and in this case it is not only that the *Ravenelia* is little known, but there are numerous other spores present, which appear to be some species of *Diorchidium*, which I am quite unable to recognize. The genuine Diorchidiums are found on Mimosaceous hosts, but I know of no species from North America and none with spores quite like this."

pollen grains much more abundant. In nearly all cases pieces of Beltian bodies, with occasional bits of other vegetable tissues, are present. Plate II, Fig. 9 shows some of the Beltian bodies, each of which seems to have been merely broken into a few pieces by the workers before being placed in the trophothylax of the larva. Plate V, Fig. 38 shows an abundance of pollen grains, probably derived from the host plants which were in flower when the ants and their larvæ were collected.

12. *Pseudomyrma belti* var. *fulvescens* (21 pellets).—Since the habits of this variety are the same as those of the typical *belti*, it is not surprising to find essentially the same constituents in the trophothylax pellets. There are, however, in addition to the sparse bits of insects, spores, pollen-grains and fragments of Beltian bodies, numerous plant-hairs and bits of miscellaneous vegetable tissues. In a few of the pellets the pollen is largely that of pines. In a series of larvæ taken from acacia thorns in the dry Zacapa region of Guatemala no insect fragments were found and nearly all the pellets contain some pine pollen.

13. *Pseudomyrma spinicola* (1 pellet).—The single pellet obtained from a larva of this Acacia-inhabiting species defies analysis. It consists of a soft, apparently coagulated substance, possibly of vegetable origin, but no cellular elements can be detected in it.

14. *Pseudomyrma sericea* var. *fortis* (6 pellets).—All the pellets taken from the larvæ of this ant, which was found nesting in the internodes of a large palo santo (*Triplaris macombii*), contain numerous large fragments of insects (Plate I, Fig. 7), more or less abundant fungus spores, with bits of hyphæ and of medullary tissue evidently gnawed from the walls of the myrmecodomatia by the workers.

15. *Pseudomyrma championi* var. (2 pellets).—One of the pellets contains a crushed insect (Plate IV, Fig. 25), a 5-toothed mandible of which is left intact, together with numerous spores and pollen grains. The other pellet consists in great part of spores and pollen, with some soft material of unknown origin.

16. *Pseudomyrma filiformis* (9 pellets).—This ant, as the senior author has shown (1919), lives in dead branches in rather shady places. The food of the adults and larvæ appears to be very similar to that of other species of the genus dwelling in dead twigs. The insect fragments are frequently large and coarse, as in the portion of a pellet represented in Plate IV, Fig. 24, and are mingled with fungus spores, often abundant as in this case. Traces of pollen grains and the parenchyma tissue of plants are occasionally present.

17. *Pseudomyrma decipiens* (12 pellets).—Pieces of insects were found in all the pellets, sometimes as more or less hairy bits of chitin, sometimes as chunks or single hairs. Spores are common to very abundant in all but one pellet; pollen grains and

bits of hyphæ are less numerous. Plate V, Fig. 39 shows a portion of the single pellet referred to, which consists almost entirely of enormous numbers of small spores.

18. *Pseudomyrma caroli*.—No pellets were obtained from the rather small number of larvæ of this species in the collection.

19. *Pseudomyrma elongata* (22 pellets).—In all cases the pellets contain bits of insect material, often in rather large chunks. In two pellets small entire mites were found. Fungus spores are more or less abundant in nearly all cases, pollen grains are less numerous and fungus hyphæ very scarce.

20. *Pseudomyrma flavidula* (66 pellets).—In the long series of pellets examined, bits of insects, varying from chunks or whole appendages to minute particles of chitin, scales and hairs are constantly present. In 35 of the pellets from larvæ collected on Fish Hawk Key, Andros Island, fungus spores, bits of mycelium and pollen are very scarce, in the remaining 31 pellets, from Mangrove Key, Andros Island, spores and bits of hyphæ and miscellaneous vegetable tissue are rather abundant. Plate III, Fig. 20 shows a pellet fragment made up of soft insect tissues interspersed with fungus spores of many kinds. In Plate IV, Fig. 26 there are many insect fragments, spores and bits of mycelium.

21. *Pseudomyrma flavidula* var. *delicatula* (22 pellets).—On the whole, the components of the pellets are very similar to those of the typical *flavidula*, but some contain scales of Lepidoptera and in several whole mites were found. Bacteria could also be recognized among the spores and hyphal elements, which are somewhat less abundant than in the typical form of the species. Plate I, Fig. 4 shows part of a pellet in which fragments of a small ant can be identified. Plate II, Fig. 15 shows fungus spores of a singular type mingled with insect-hairs and other débris.

22. *Pseudomyrma arboris-sanctæ*.—Unfortunately no pellets were obtained from the larvæ of this ant, which is an obligatory symbiont of a palo santo (*Triplaris cummingiana*). Nothing is known of the nature or source of its food.

23. *Pseudomyrma* sp. from Patulul, Guatemala (7 pellets).—All the pellets contain bits of insects, sometimes recognizable as bits of legs, heads of larvæ, eyes, pieces of cuticle, hairs, scales, etc. The vegetable components, comprising spores, hyphæ, pollen-grains and miscellaneous pieces of tissue, vary from mere traces to large accumulations. In some pellets there is also a considerable quantity of dirt or very fine detritus, as shown in Plate V, Fig. 35.

24. *Pseudomyrma* sp. from Antigua, Guatemala (2 pellets).—The constitution of these pellets is like that in the preceding species. In one, fragments of plant epidermis were identified.

25. *Pseudomyrma* sp. from Escuintla, Guatemala (2 pellets).—One pellet contains a few spores, the other fragments of chitin.

26. *Pseudomyrma* sp. from Cartago, Costa Rica (3 pellets).—All the pellets contain bits of chitin, eyes, hairs, etc. and two of them also moderate quantities of spores, bits of mycelium and pollen.

It will be seen that the ingredients of the pellets of the various Pseudomyrminæ as briefly summarized in the foregoing paragraphs, show considerable uniformity in nearly all the species and considerable diversity in the individual pellet. In other words, all the species supply their larvæ with both insect and vegetable substances, but of many different kinds. There can be no doubt that small miscellaneous insects furnish the most important ingredient of the pellets in most species, and that this ingredient, which supplies the most easily assimilable proteids for the growth of the larvæ, is rarely completely lacking even in the acacia-inhabiting species. In the latter the Beltian bodies are unquestionably important sources of food for both the adults and the young. Nor would the spores, hyphæ and pollen grains, which are in most cases at least merely strigil-sweepings, be so constantly fed to the larvæ, unless they could be at least partly utilized as food. That these ingredients, and especially the spores and pollen, contain substances of high nutritive value, is certain, and it is not improbable that the larvæ can triturate them by means of the trophorhinium and thus render them assimilable. This must, indeed, be true, if the spores are actually ingested, for none of them can be detected as whole bodies among the stomach contents as is the case in the above-described Myrmicinæ (*Leptothorax*, *Cryptocerus* and *Cataulacus*), which have no trophorhinium and swallow entire insect fragments and spores.

The constant occurrence of a variety of spores in the infrabuccal pockets of the Pseudomyrminæ is not surprising, when we consider that these very active, large-eyed, wasp-like, arboreal ants, owing to their habit of incessantly patrolling the surfaces of bushes and trees in the tropics, could not fail to accumulate great numbers of the most diverse fungus-germs on their bodies and appendages. The possibility of their behaving as very active and deleterious agents in the spread of many of the fungus-diseases of tropical plants is apparent, but the precise extent of the injury thus caused would depend on whether the workers usually or always feed their infrabuccal pellets to the larvæ or whether, like other ants, they often rid themselves of these corpuscles outside their nests and on the surfaces of plants where the contained spores and bits of hyphæ might germinate. The solution of such problems can, of course, be undertaken only in the tropics.

The trophorhinium, an organ apparently overlooked by previous observers, is beautifully developed in all the Pseudomyrminæ, but seems to show little variation within the subfamily (Text-fig. 4). As stated above, it consists of numerous trans-

verse, parallel, very minutely spinulose ridges in the chitinous cuticle lining the flattened mouth-cavity. If the mouth be carefully opened with the dissecting needles and the dorsal and ventral portions spread apart as in Fig. 5, it will be seen that the dorsal surface or portion (*a*), corresponding anteriorly to the ventral surface of the

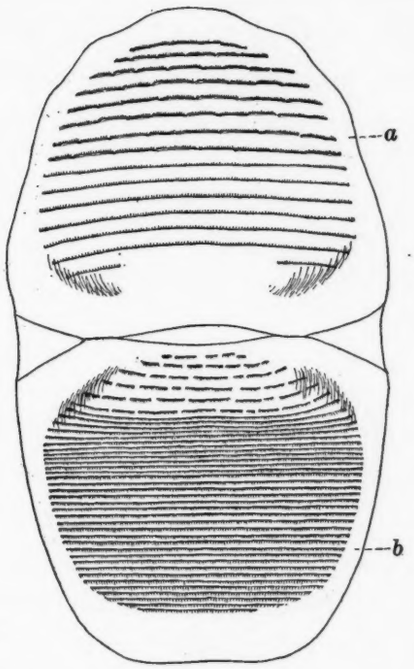

FIG. 5. Trophorhinium of *Pseudomyrma gracilis* Fabr. *a*, roof. *b*, floor of mouth.

labrum, begins near its anterior border and extends back nearly to the opening of the gullet. The more anterior ridges are made up of rather arcuate sections, whereas the posterior are straight and more even. The ventral surface or portion (*b*), corresponding to the floor of the mouth, is of similar structure, except that the ridges are much more numerous and closer together, especially anteriorly. Posteriorly, in the neighborhood of the gullet, they are interrupted and much further apart. On both surfaces the very fine, hair-like spinules point towards the oral orifice. The cuticle of both surfaces is slightly darker than elsewhere on the body. It is not necessary, however, to separate the walls of the buccal cavity as in Text-fig. 5, in order to determine the peculiarities of the two surfaces. This can be done very readily by focussing with the fine adjustment of the microscope on the head of a larva from which all the soft parts have been removed by treatment with caustic potash, or by the study of sections such as the one represented in Text-fig. 6 and 6*a*. Here the difference in spacing between the fine ridges of the dorsal and ventral surfaces is distinctly seen, the latter

beginning just behind the orifice of the salivary duct on the labium and the former extending further forward on the ventral surface of the labrum.

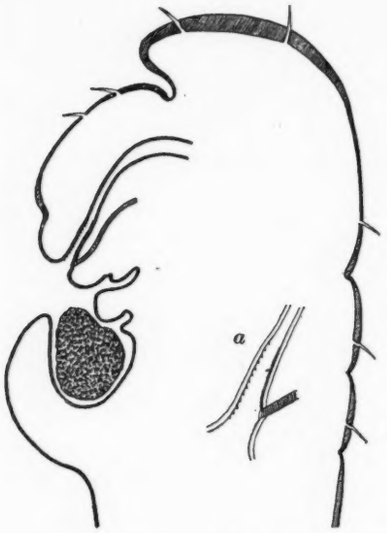

FIG. 6. Sagittal section through anterior end of larva of *Viticicola tessmanni* Stitz. *a*, lining of mouth in section, enlarged.

That an organ of such structure would be admirably adapted to triturating particles of food and sifting or straining out the coarser and harder pieces seems to us to be extremely probable. The process of feeding would appear to be as follows: The pellet placed in the trophothylax by the worker nurse and consisting of the strigil-sweepings, etc. taken from her own infrabuccal pocket *plus* some fragments of insect prey, is probably bathed or saturated with the saliva of the larva secreted into the trophothylax from the orifice of the labial duct. The proteolytic ferment of the secretion would evidently peptonize the softer portions of the particles which could then be drawn back by the mandibles in installments between the two surfaces of the trophorhinium where the indigestible chitinous fragments could be separated out and the remainder turned over to the gullet and swallowed. As a matter of fact, the senior author has seen numerous particles, like those in the trophothylax, spread out between the two apposed surfaces of the trophorhinium in some larvæ that had been suddenly killed by immersion in strong alcohol while apparently in the very act of feeding.

The senior author's study of the trophorhinium in many other ant-larvæ belonging to most of the subfamilies, shows that its delicate transverse ridges have developed from the minutely granular or reticular structure so characteristic of the general chitinous integument. This is beautifully seen in certain Ponerinæ, *e.g.*, in the Ethio-

pian *Megaponera foetens* Fabr. (Plate III, Fig. 18), where the transverse ridges are still represented by a reticulate areolation like that of the general integument, except that the meshes are drawn out transversely. Among all the ants studied, the trophorhinium seems to attain its most elaborate development in the Ectatommiini among the Ponerinæ and the higher genera of Formicinæ. Fig. 19, Pl. III shows the larval mouth-parts of a species of the former subfamily, the Australian *Rhytidoponera cristata* Mayr. The trophorhinium may be discerned as an arcuate system of extremely fine, parallel ridges extending across the space between the bases of the mandibles. It will be noticed also that the upper surfaces of the latter are very finely and regularly imbricated. This condition likewise obtains, although in some species less conspicuously, in larval Pseudomyrminæ, as stated on p. 256 and as shown in Text-fig. 4. When the mandibles are opened or closed, their imbricated surfaces would naturally rub against the dorsal plate of the trophorhinium, and in forms with large mandibles, like *Rhytidoponera* the latter must overlap more or less when opening and closing and thus also rub against one another. Not improbably, therefore, the rough surfaces of the mandibles may reinforce the triturating and sifting functions of the trophorhinium.

All of these structures, however, seem to have still another function. Comparison of the trophorhinium with the stridulatory organs at the base of the mid-dorsal aspect of the postpetiole and first gastric segment in adult ants of most of the subfamilies (Pseudomyrminæ, Ponerinæ, Dorylinæ and Myrmicinæ) suggests that it may also have a sound-producing function, when it is not being used as a mill or strainer and the two apposed surfaces can be rubbed directly against one another, *i.e.*, without the intervention of food particles. Owing to the small size of the organ and the extreme delicacy of its parallel ridges, the tones produced would be very feeble and of very high pitch, like those produced by the striated surfaces on the bases of the postpetiole and first gastric segment of adult ants. Similarly, the imbricated dorsal surfaces of the mandibles would probably produce faint, shrill sounds when rubbed against one another or against the dorsal surface of the trophorhinium. The resemblance of the latter to the stridulatory organs of the adult becomes even more striking when we recall that the very fine parallel ridges of these organs also arise, as Sharp (1893) and Janet (1893*a*) have shown, by a simple modification of the general reticular surface-sculpture of the chitinous integument.

That the trophorhinium has the grinding and sorting function we have ascribed to it, is also indicated by certain other facts. The stomach contents of Pseudomyrmine as well as other larvæ possessing a trophorhinium consist of such finely divided particles that their nature cannot be determined, whereas the particles fed

to the larvæ (Ponerinæ) or placed in the trophothylax are, as has been shown, very coarse and in most cases easily recognizable as insect or vegetable. On the other hand as stated on p. 255, certain Myrmicine larvæ, like those of *Leptothorax*, *Cryptocerus* and *Cataulacus* have no trophorhinium and swallow their coarse food-particles whole. In fact, the stomachs of these ant-larvæ always contain a collection of insect fragments so large and angular that it is difficult to see how they can pass through the slender gullet. Similarly, the Doryline larvæ have no trophorhinium and take into their stomachs crude though dechitinized pellets of insect tissue.

General Considerations.

The feeding of the larvæ among ants exhibits a much greater diversity than in any other group of social insects. We were able to distinguish the following methods:

1. Feeding with whole insects or pieces of insects (Ponerinæ, and some Myrmicinæ and Formicinæ);

2. With pellets made of the flesh of insects (Dorylinæ);

3. With the contents of the infrabuccal pocket, either alone or with the addition of fresh insect fragments (Pseudomyrminæ and possibly some Myrmicinæ, such as *Cryptocerus* and *Leptothorax*). In the acacia-inhabiting species of *Pseudomyrma* portions of the Beltian bodies of the host plant are also fed to the larvæ;

4. With pieces of seeds (Granivorous Myrmicinæ);

5. With fungus hyphæ, normal or modified as "kohlrabi," or bromatia. (Tribe Attiini among the Myrmicinæ);

6. With liquids regurgitated from the ingluvies, or crop of the worker (Dolichoderinæ, Formicinæ and many Myrmicinæ).

It is evident that the first method is the most primitive and, owing to the fact that the pieces of insects are often given to the larvæ without malaxation, apparently an even more ancient form of feeding the young than we find in the social wasps. The second method, however, as employed by the Dorylinæ, seems to be very much like that of the higher Vespidæ. All the other methods are highly specialized and are evidently derived secondarily from specializations in the feeding habits of the adults. This is obvious in the granivorous, fungus-growing and honey ants, which represent peculiar adaptations to life in arid or desert environments or to regions in which, during long periods of the year, insect food is very scarce. The conditions in the Pseudomyrminæ are unique, owing to the development in the larvæ of a special post-oral receptacle (trophothylax) for the reception of a food-pellet provided by the worker and consisting of the strigil-sweepings compacted in her infrabuccal pocket *plus* a certain amount of freshly captured and dismembered insect prey. In a study

undertaken by the senior author and Mr. George C. Wheeler of the larvæ of a large number of other ant genera, no structure comparable to the Pseudomyrmine trophothylax has been found, except in certain species of *Camponotus* of the subgenus *Colobopsis*. In all the species of the latter subgenus examined the larva is very hypocephalic and the ventral portion of the first abdominal segment projects considerably beyond the thoracic segments and presents a pronounced concavity or basin in the mid-ventral region precisely in the position of the trophothylax of the Pseudomyrminæ. A feeble vestige of the same structure occurs in many *Camponotus* larvæ belonging to other subgenera. No solid pellet is deposited in the basin of *Colobopsis*, but it may, perhaps, be used to hold a supply of the liquid food regurgitated by the workers or of the saliva secreted by the larva itself for the benefit of its attendants.[1]

Some interesting questions are suggested by the composition of the pellet formed in the infrabuccal pocket of imaginal ants. Much of it undoubtedly consists of strigil-sweepings, as Janet observed and as we have found from examination of species of the most diverse genera from widely different stations (deserts, tamarack-bogs, mesophytic and rain-forests, prairies, etc.) and geographical regions. We should, of course, expect insects like the Formicidæ, which are more or less hairy and sculptured, with strongly articulated bodies, nesting in plant cavities or in the ground and constantly moving over the dusty soil and vegetation, to accumulate on the surfaces of their bodies a most heterogenous collection of minute particles and eventually to gather them into their infrabuccal pockets by licking with the tongue or using the strigils. As we have shown, the analysis of the pellets exhibits this diversity very clearly. Many of the components, such as sand-grains, particles of earth and wood, plant and insect hairs, and bits of chitin are inert structures of no further significance, except as they may serve as substrata for the growth of the numerous fungus spores which are such surprisingly constant constituents of the pellets. The junior author has discussed the fungus elements and their possible economic importance in other papers (1920, 1921?). A cultivation of the pellets on artificial media will very probably show that the spores and pieces of hyphæ are quite viable after their sojourn in the infrabuccal pocket. In fact, this is a certainty in the case of the Attiine ants, the recently fecundated queens of which, as von Ihering, J. Huber and Bruch have shown, carry the germs of the fungus gardens of their prospective colonies as pellets in their infrabuccal pockets. The fact that the pellets, even of other species of ants, are cast out by the workers somewhere in their environment, either on the kitchen middens of the nest or outside its precincts, *i.e.*, in situations where the spores may readily germinate, is of no little ecological and economic significance, for it shows that

[1] Cf. the senior author's discussion of the exudates (1918).

ants may be very important agents, or vectors in the distribution of many kinds of fungi in general and of various phytopathogenic fungi in particular. Moggridge (1873) long ago demonstrated the important rôle of the grain-storing ants as distributors of the seeds of the higher plants in arid or desert regions, and Sernander (1906) has shown that many herbaceous plants (myrmecochores) in our northern mesophytic forests are disseminated by some of the common species of *Tetramorium*, *Leptothorax*, *Lasius* and *Myrmica*. Further researches in this field of investigation cannot fail to bring to light many facts of both theoretical and practical value.

We have not had an opportunity to study the mouthparts of larval and imaginal insects of the various orders with a view to determining whether organs similar to the trophorhinium can be detected. It would seem that the fine ridges or striæ on the tongues of many imaginal ants and the delicate rows of minute spinules on their buccal and pharyngeal linings, are analogous structures. Somewhat similar structures seem also to have been seen by Carpenter and MacDowell (1912) and Carpenter (1913) on the maxillulæ of certain Dascyllid larvæ among the Coleoptera and by De Gryse (1915) on the homologous organs of Lepidopteran larvæ. Still more striking is the resemblance of the mouth-lining of spiders, even in certain minute details, to the trophorhinium of ant-larvæ. Lyonet (1832, Pl. 10, Figs. 4 and 20) and Kessler (1849, Pl. 9, Figs. 11 and 12) long ago described the buccal cavity in spiders as flattened, with apposed palatal (dorsal) and lingual (ventral) plates, or surfaces, but did not describe the chitinous structures in sufficient detail. Both surfaces are transversely ridged, the lingual much more finely and densely than the palatal. Moreover, the lingual ridges, at least towards the lateral corners of the buccal cavity, are beset with minute spinules precisely like those on the trophorhinium of ant-larvæ. The whole subject, however, of the finer details in the chitinous lining of the Arthropod mouth, requires a special investigation which would lead far beyond the restricted scope of the present paper.

It was suggested above that the trophorhinium might have a stridulatory function. This was inferred from its peculiar structure and from comparison with the stridulatory surfaces on the middorsal aspect of the postpetiole and first gastric segment in adult ants of the subfamilies Dorylinæ, Ponerinæ, Pseudomyrminæ and Myrmicinæ, as described and figured by Sharp (1893) and Janet (1893, 1900). A study by the senior author and Mr. G. C. Wheeler shows that the trophorhinium is present in the larvæ of all the subfamilies, except the Dorylinæ, Cerapachyinæ and some Myrmicinæ, and that it exhibits in the various tribes and genera numerous, very interesting modifications in detail. A full account of the organ with illustrations is reserved for publication in the near future. A glance at the sketches and photographs

of the Pseudomyrmine trophorhinium accompanying this paper (Text-figs. 4, 5 and 6, Plate III, Figs. 18 and 19) shows that the transversely striated dorsal and ventral linings of the larval mouth when rubbed on one another, without intervening particles of food, must produce a tone, extremely faint, to be sure, but not improbably loud enough to be perceived by the worker nurses. And we are not indulging in fancy when we say that this tone may be a hunger cry or cry of distress analogous to that of the human infant.[1]

But in our opinion the trophorhinium is not the only stridulatory organ in ant larvæ. In many species, notably in Ponerinæ, Pseudomyrminæ, some Myrmicinæ (especially the Attiini) and the Formicinæ, the anterior or outer surfaces of the mandibles, as has been stated, are wholly or partly covered with fine, subimbricated papillæ or projections. When the mandibles are merely opened or closed sounds could be produced in two ways, either by a rubbing of their upper surfaces against the dorsal striæ of the buccal lining or by the sharp teeth of one mandible scraping the subimbricated surface of the other. It is highly probable, therefore, that many ant larvæ can produce at least three different sounds, each perhaps associated with a different larval need and capable of eliciting an appropriate response from the attendant workers. If this supposition is correct, the stridulation of the larval brood of a formicary, at least when inadequately supplied with food, would resemble a symphony, or perhaps, more closely, a jazz concert, inaudible to our ears, owing to the very high pitch of its component tones, but perceptible and urgently significant to the workers entrusted with preparing and distributing the larval rations. Perhaps the use of an extremely delicate microphone placed among great masses of large and hungry ant-larvæ may enable us to hear at least some of the lower notes of this shrill chorus. It may also be suggested that the singular habit exhibited by many ants of sorting their larvæ according to size in different parts or chambers of the nest—a habit which has reminded some authors of the division of school-children into classes—may have some connection with stridulation, for if the pitch varies with the size of the larvæ, as there is every reason to suppose, there might be obvious advantages to the nurses in keeping the various stages in groups instead of intermingled.

Although in imaginal insects stridulatory organs are of frequent occurrence they are so rare in larvæ that doubts may arise concerning the accuracy of our interpreta-

[1] In this connection the following remarks on the human infant by Mrs. Blanton, quoted by Watson (1919), are interesting: "The cry of one baby can be distinguished with some practice from the cries of another even in a nursery of 25, the overtones varying just as in older people. . . . The 'hunger cry' has generally a well-marked rhythm, the first syllable of preliminary sound coming on the first part of the first beat, the second or accented syllable on the second part of the first beat and a quick intake of breath as the third beat. This measure is most often repeated in groups of 5 or 6, each slightly more forceful than the preceding ones until the fourth or fifth, the last one being softer. Thus also will the groups be repeated. Each measure is also a trifle higher in pitch than the one preceding."

tion of the sound-producing function of the trophorhinium and subimbricated mandibular surfaces. Stridulation is known to occur in some Lepidopteran caterpillars, but according to Prochnow (1912) their sound-producing organs have a very low development "being little different from the sculpture of the integument on other parts of the caterpillars' body," even in *Rhodia fugax* Butl., "which emits a clear and rather loud tone." Concerning the ethological meaning of stridulation in these insects so very little is known that they need not be further considered in this connection. Much more interesting is the case of the large Lamellicorn beetles of the genus *Passalus*, which is widely distributed through the tropics of both hemispheres and is even represented by a species (*P. cornutus*) in the United States as far north as Illinois and Massachusetts. Ohaus (1899–1900, 1909) was able to ascertain from a study of several Brazilian species that these beetles live in rotten logs in families each comprising a male and female with their larval offspring. The senior author has also frequently observed the same composition of these communities in the United States, Central America and Queensland. Ohaus found that the beetles make spacious galleries, comminuting the wood and probably treating the particles with some digestive enzyme (extraintestinal digestion?) so that they can be eaten by the larvæ which follow along the galleries just behind their tunneling parents. Owing to the structure of their mouthparts the larvæ are quite unable to comminute the wood, and when removed from their parents soon die. The beetles not only guard their greenish eggs and diligently provide food for their larvæ but also protect the pupæ and feed the imaginal young till their chitinous integument is completely hardened.

It has long been known that the *Passalus* larva has beautiful stridulatory organs in the form of a broadly elliptical, finely striated area on each of the middle coxæ. The hind leg is reduced to a small, single-jointed appendage, shaped like a mammal's fore paw, with very short digits and claws. The latter are drawn over the striated area as the appendage is worked up and down and produce an audible tone. Schiödte (1862–1873) and Sharp (1899) have published excellent drawings of the organ in our American *Passalus cornutus* and a Bornean species. Plate II, Fig. 12, from a photograph of a Costa Rican species observed by the senior author, is interesting as showing how the fine striæ are made up by a fusion in rows of the minute papillæ of the general integument. We have already called attention to the origin of the stridulatory striæ as modifications of the general integumentary sculpture in the trophorhinia of certain Ponerine ant larvæ and in the postpetiolar and gastric stridulatory areas of many adult ants belonging to the lower subfamilies.

The stridulatory organs of the adult *Passalus* differ greatly in structure and position from those of the larva. The accounts of them given by Leconte (1878),

Ohaus (1900) and Babb (1901) do not, however, agree. Babb clearly describes the organs as a couple of patches of minute denticles on the dorsal surface of the fifth abdominal segment, which rub against specialized areas consisting of ridges and denticles on the ventral surfaces of the folded wings. As in the case of the larva, the tone produced by this apparatus is clearly audible.

The social habits of *Passalus*, as described by Ohaus, clearly suggest that the stridulatory organs of both adults and larvæ must represent means of communication, capable of keeping the members of the family together and in mutual coöperation. That in highly social insects like the ants the even more helpless larvæ should possess elaborate stridulatory organs capable of apprising the attendant workers of such organic needs as those of food, of change of position when the temperature and moisture conditions are unfavorable, of pupation, of the secretion of exudates or of the evacuation of meconium, is not surprising. It would, indeed, be strange if some such method of communication had not developed in so extraordinarily resourceful a group of insects as the Formicidæ.

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DESCRIPTION OF PLATES.

PLATE I.

FIG. 1. Portion of larval pellet of *Pachysima aethiops*, showing mite (?) and triturated plant tissue. × 300.

FIG. 2. Strigil, or toilet-organ of *Harpagoxenus sublaevis* Mayr. × 116.

FIG. 3. Portion of larval pellet of *Pachysima aethiops*, showing spores and soft tissue of a Coccid (*Stictococcus formicarius* Newst.) × 180.

FIG. 4. Portion of larval pellet of *Pseudomyrma flavidula* var. *delicatula*, showing fragments of insects (ants). × 58.

FIG. 5. Portion of larval pellet of *Pseudomyrma gracilis* (var. nov.), showing fragments of a weevil. × 78.

FIG. 6. Portion of stomach contents of *Cataulacus egenus* larva, showing spores of different types. × 96.

FIG. 7. Portion of larval pellet of *Pseudomyrma sericea* var. *fortis*, showing fragments of chitin. × 78.

FIG. 8. Portion of stomach contents of *Leptothorax* larva, showing fragments of insects. × 96.

PLATE II.

FIG. 9. Portion of larval pellet of *Pseudomyrma belti*, showing fragments of Beltian food-body. × 58.

FIG. 10. Fragment of Coccid (*Stictococcus formicarius* Newst.) from larval pellet of *Pachysima aethiops*. × 208.

FIG. 11. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana* showing pollen and various types of spores. × 330.

FIG. 12. Stridulatory organ of *Passalus* sp. from Costa Rica. × 82.

FIG. 13. Larval *Stictococcus formicarius* Newst. from larval pellet of *Pachysima aethiops*. × 58.

FIG. 14. Portion of larval pellet of *Pachysima aethiops*, showing spores, plant hairs and other detritus. × 330.

FIG. 15. Portion of larval pellet of *Psyeudomyrma flavidula* var. *delicatula*, showing spores of different types. × 330.

FIG. 16. Portion of larval pellet of *Pachysima latifrons*, showing spores, pollen and dirt. × 330.

PLATE III.

FIG. 17. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of insects and pollen. × 78.

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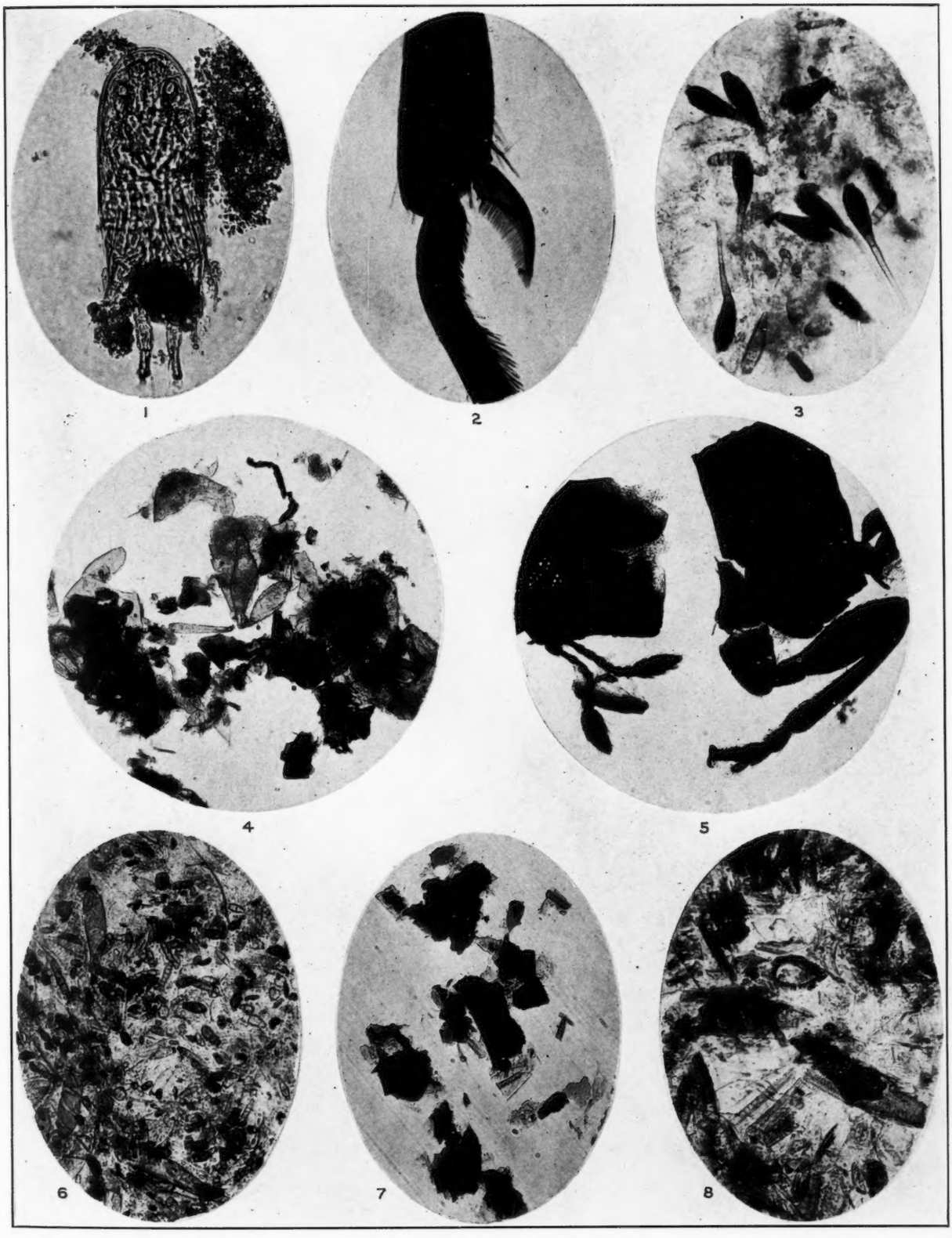
FIG. 19. Mouthparts of larva of *Rhytidoponera cristata* Mayr showing trophorhinium. × 100.

FIG. 20. Portion of larval pellet of *Pseudomyrma flavidula*, showing soft insect tissue and spores of different types. × 78.

FIG. 21. Portion of larval pellet of *Pseudomyrma flavidula*, showing fragments of mites and spores. × 78.

FIG. 22. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of larva. × 58.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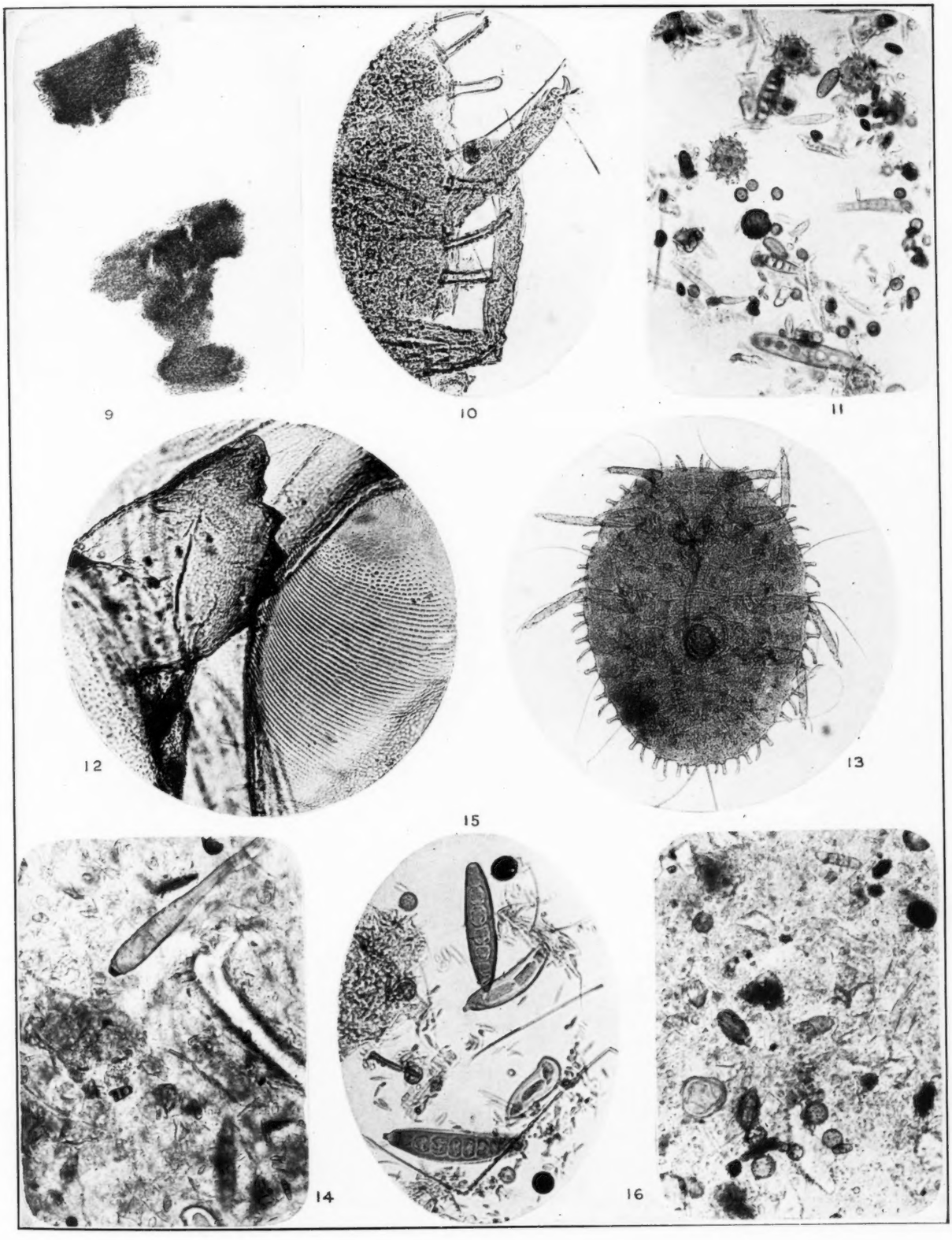
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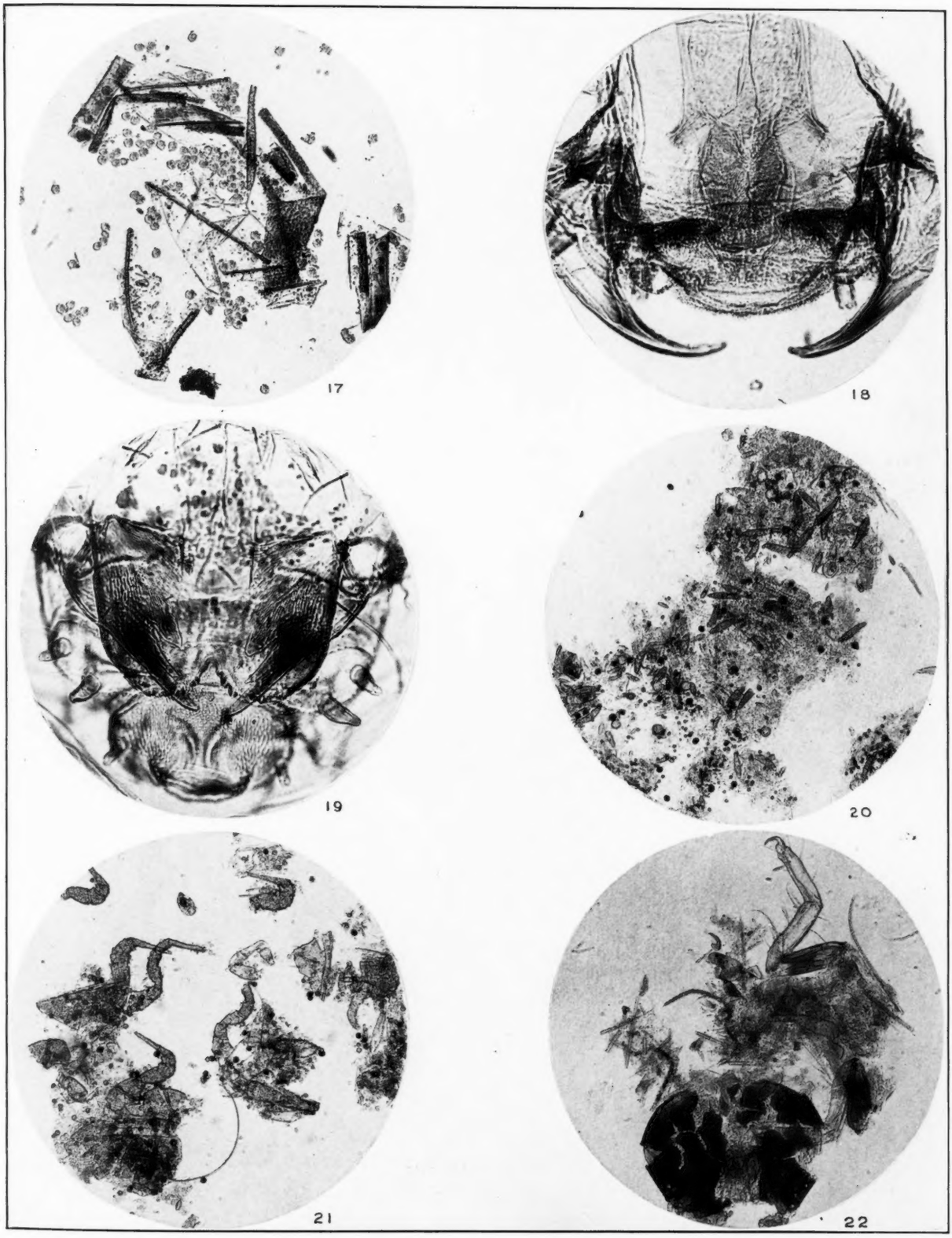
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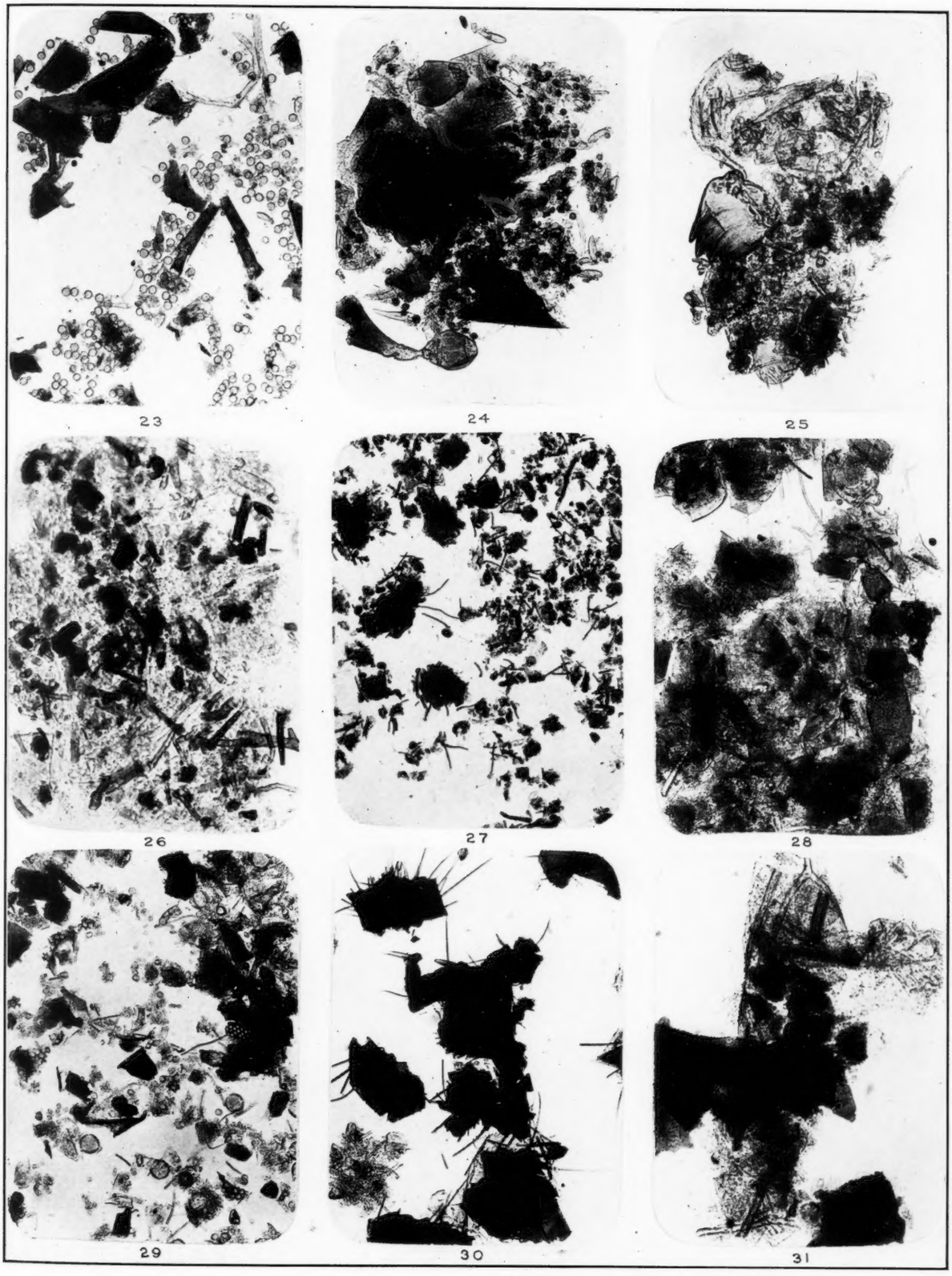
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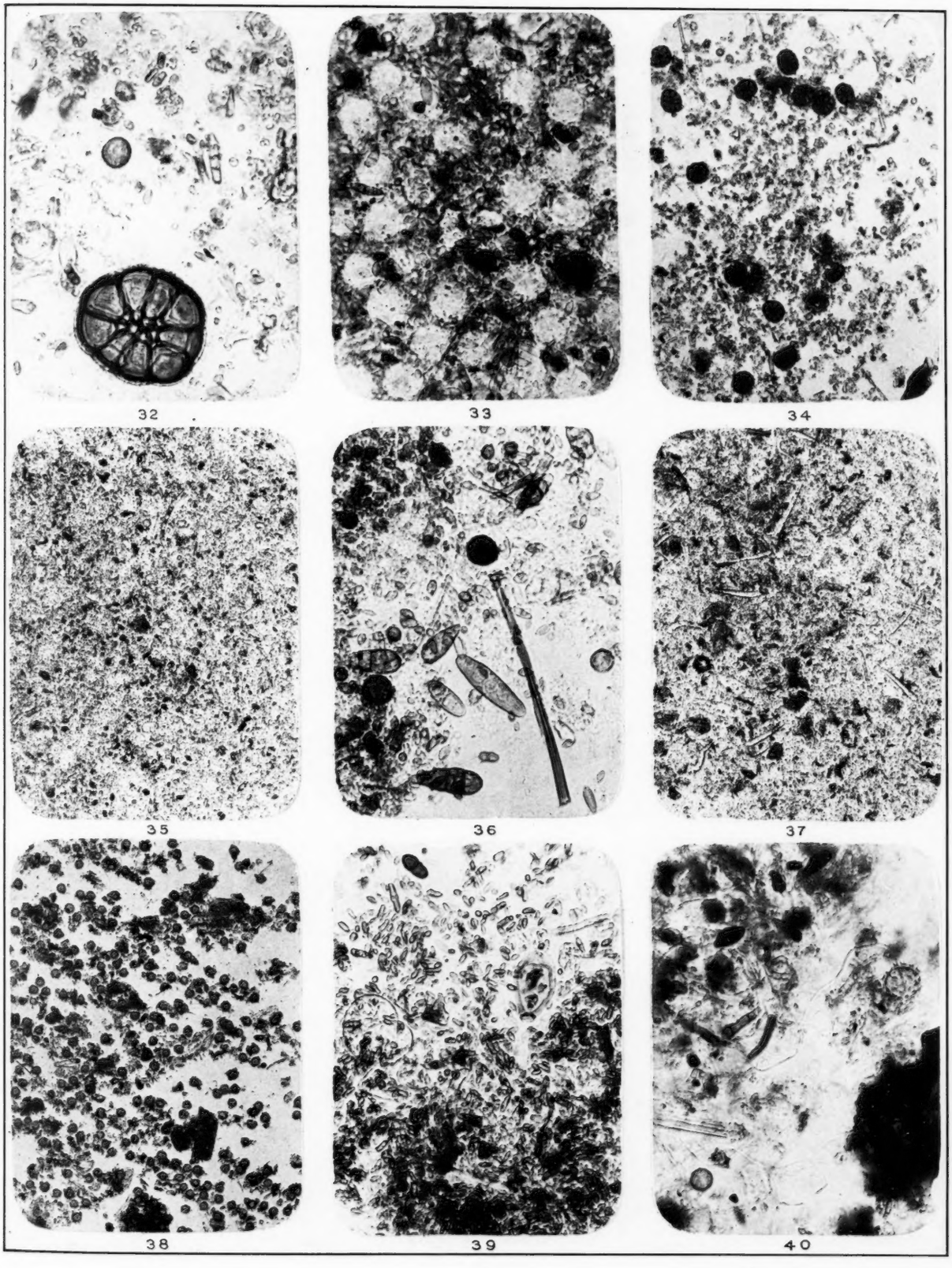

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Plate IV.

Fig. 23. Portion of larval pellet of *Pseudomyrma gracilis* (var. nov.), showing fragments of insects and pollen. × 58.

Fig. 24. Portion of larval pellet of *Pseudomyrma filiformis*, showing fragments of insects and spores. × 78.

Fig. 25. Portion of larval pellet of *Pseudomyrma championi*, showing fragments of insects, spores and pollen. × 78.

Fig. 26. Portion of larval pellet of *Pseudomyrma flavidula*, showing fragments of insects, hyphæ and spores. × 330.

Fig. 27. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of insects and hyphæ. × 58.

Fig. 28. Portion of larval pellet of *Pseudomyrma gracilis*, showing fragments of insect. × 78.

Fig. 29. Portion of larval pellet of *Tetraponera allaborans*, showing fragments of insects and pollen. × 78.

Fig. 30. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of insect × 78.

Fig. 31. Portion of larval pellet of *Pseudomyrma gracilis* var. *dimidiata*, showing fragments of insects. × 58.

Plate V.

Fig. 32. Portion of larval pellet of *Pseudomyrma gracilis* var. *dimidiata*, showing spores of different types among others those of *Ravenelia distans* A. & H. × 330.

Fig. 33. Portion of larval pellet of *Pseudomyrma rufomedia*, which is composed entirely of pollen and spores. × 330.

Fig. 34. Same as Fig. 32. × 58.

Fig. 35. Portion of larval pellet of *Pseudomyrma* species, from Patulul, Guatemala, showing dirt and other detritus. × 58.

Fig. 36. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, which is composed entirely of spores and hairs. × 330.

Fig. 37. Portion of larval pellet of *Pachysima aethiops*, showing plant hairs, spores and dirt. × 58.

Fig. 38. Portion of larval pellet of *Pseudomyrma belti*, showing pollen. × 58.

Fig. 39. Portion of larval pellet of *Pseudomyrma decipiens*, which is composed almost entirely of spores. × 330.

Fig. 40. Portion of larval pellet of *Pseudomyrma sericea* var. *fortis*, showing hyphæ and fragments of medullary tissue from myrmecodomatia of *Triplaris macombii*. × 330.

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NEW SERIES—VOLUME XXII

PART 5

A Review of the Doradidæ, a Family of South American Nematognathi, or Catfishes.

By Carl H. Eigenmann

PHILADELPHIA:
THE AMERICAN PHILOSOPHICAL SOCIETY
104 South Fifth Street
1925

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FIG. 23. Portion of larval pellet of *Pseudomyrma gracilis* (var. nov.), showing fragments of insects and pollen. × 58.

FIG. 24. Portion of larval pellet of *Pseudomyrma filiformis*, showing fragments of insects and spores. × 78.

FIG. 25. Portion of larval pellet of *Pseudomyrma championi*, showing fragments of insects, spores and pollen. × 78.

FIG. 26. Portion of larval pellet of *Pseudomyrma flavidula*, showing fragments of insects, hyphæ and spores. × 330.

FIG. 27. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of insects and hyphæ. × 58.

FIG. 28. Portion of larval pellet of *Pseudomyrma gracilis*, showing fragments of insect. × 78.

FIG. 29. Portion of larval pellet of *Tetraponera allaborans*, showing fragments of insects and pollen. × 78.

FIG. 30. Portion of larval pellet of *Pseudomyrma gracilis* var. *mexicana*, showing fragments of insect. × 78.

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PLATE V.

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FIG. 39. Portion of larval pellet of *Pseudomyrma decipiens*, which is composed almost entirely of spores. × 330.

FIG. 40. Portion of larval pellet of *Pseudomyrma sericea* var. *fortis*, showing hyphæ and fragments of medullary tissue from myrmecodomatia of *Triplaris macombii*. × 330.

ARTICLE 5.

A REVIEW OF THE DORADIDÆ, A FAMILY OF SOUTH AMERICAN NEMATOGNATHI, OR CATFISHES.[1]

(Plates I–XXVII.)

By CARL H. EIGENMANN.

INTRODUCTION.

The present is one of a series of monographs dealing with the families and subfamilies of the freshwater fishes of South America. Preceding monographs are: (1) A revision of the South American Nematognathi, or Catfishes. *Occasional Papers California Academy Sciences*, 1890, Vol. I. In coöperation with Rosa S. Eigenmann; (2) The Gymnotid eels of tropical America. Max Mapes Ellis, *Mem. Carnegie Mus.*, 1913, Vol. VI; (3) The Cheirodontinæ, a subfamily of minute characid fishes of South America. *Mem. Carnegie Mus.*, 1915, Vol. 7; (4) The American Characidæ. *Mem. Mus. Comp. Zoöl.*, 1917–1925, Parts 1, 2, and 3 issued so far; (5) The Pygidiidæ, a family of South American Catfishes. *Mem. Carnegie Mus.*, 1918, Vol. 7.

The present paper is based principally on the collections in the Museum of Indiana University.

The material was gathered during (*a*) my visit to British Guiana, 1908; (*b*) to Colombia, 1912; (*c*) the trip of Dr. W. R. Allen to the Peruvian Amazon, and (*d*) the trip of Mr. N. E. Pearson with the Mulford Expedition to Bolivia. In part it was received in exchange from other museums. In 1888 I examined the collection of Doradids in the Museum of Comparative Zoölogy (*Occasional Papers Cal. Acad. Sci.*, I, 1890), and more recently those of the Carnegie Museum. I am indebted to the Carnegie Museum, the Museum of Comparative Zoölogy, the Museum of the Philadelphia Academy of Natural Sciences and the Tropical Research Station under the direction of Dr. William Beebe for the privilege of examining some of the Doradids in their collections. Dr. Thomas Barbour was kind enough to reëxamine *Hoplodoras uranoscopus* described by Mrs. Eigenmann and myself thirty seven years ago. Dr. Alipio Ribeiro generously had Sr. E. Cruzlima make drawings of the types of *Megalodoras libertatis* and *Anadoras insculptus*. Unless otherwise indicated the photographs were made by the author, mostly based on anatomical preparations made by him. The drawings were made by Eugene N. Fischer (Plate XII), Dr. Clarence

[1] Contribution from the Zoölogical Laboratory of Indiana University, No. 198.

Kennedy (various details in the text figures), and particularly by W. S. Atkinson of Stanford University.

The numerous anatomical preparations enabled me to resurrect and re-define many of Bleeker's abandoned genera and to describe a number of other genera as new. The examination of the air-bladders demonstrated such an unexpected, amazing and riotous divergence that it is regrettable that not all of the species could be examined. So many species are known only from the types that probably many other forms are undiscovered, which makes rather elaborate if not complete illustrations justifiable and desirable.

Position of the Doradidæ Among Other South American Fishes.

South America is very rich in freshwater fishes. As in North America, Europe, Africa and India, the Ostariophysi are dominant in South America. There are series of families of Heterognaths or Characins, another series of peculiar eels, the Gymnonoti and another series of Nematognaths or Catfishes. Of the fifteen families of catfishes found in South America, one family extends north to Central America, one is found in warm seas, the rest are all peculiar to South America.

Class PISCES
Superorder or Series OSTARIOPHYSI (Plectospondili)
Orders NEMATOGNATHI—HETEROGNATHI—GYMNONOTI.
South American Families of Nematognathi and their distribution:

- Diplomystidæ (Temperate South America),
- Ariidæ (Warm seas and rivers),
- Auchenipteridæ (Tropical South America),
- **Doradidæ** (Tropical South America),
- Ageneiosidæ (Tropical South America),
- Helogenidæ (Tropical South America),
- Hypophthalmidæ (Tropical South America),
- Pimelodidæ (Central America and Tropical South America),
- Bunocephalidæ (Tropical South America),
- Aspredinidæ (Tropical South America, in part marine),
- Pygidiidæ (South America),
- Nematogenyidæ (Chile),
- Cetopsidæ (Lowlands of Tropical South America),
- Astroblepidæ (Andes from Panama to Titicaca),
- Callichthyidæ (Tropical South America, chiefly lowlands),
- Loricariidæ (Tropical South America, lowlands and mountains).

The Nature of the Doradidæ.

Diagnostics.—The Doradidæ are a family of catfishes, Nematognaths, peculiar to tropical South America. They are distinguished from all other Nematognaths by the presence of a series of plates along the sides, each with a strong, median, backward-directed spine, sometimes supplemented by smaller spines on the surface of the plate. They are furthermore distinguished from all but the Auchenipteridæ by the exaggerated development of the parapophyses of the fourth vertebra, associated with

Fig. 1. The fourth vertebra of *Pterodoras granulosus*. *A*, From below, showing the anterior (*A.* IV) and posterior (*P.*) lateral processes. The anterior process with the oval disks which fit into pockets of the air-bladder. *B*, From above, the body of the vertebra partly cut away to show the Weberian bones. *T* = tripus or malleus. Compare with Fig. 11.

the air-bladder and the Weberian apparatus. The anterior lateral process of the fourth vertebra arises as a thin, firm, flexible plate and ends in a large, circular, disk-like or conical plate so closely attached in an indentation of the air-bladder that it frequently parts company with the basal part and remains attached to the air-bladder, when the latter is removed. The disk is of very different texture from that of the flexible, compact, basal part to which it is attached. The Doradidæ further agree with the Auchenipteridæ in the solidly united bones of the skull, nuchal shields and dorsal plate. The nasal openings are remote. The gill-membranes united with the isthmus.

The species reach an extreme length of a meter. Most of them are much smaller.

Structural radiation.—There are two groups of species arranged under "*a*" and "*aa*" in the key to the genera. In the broad-breasted species, "*a*," in which the head is more depressed, the premaxillary is normal, provided with bands of teeth, the mental barbels simple. In the narrow-breasted species the head is compressed,

rather than depressed, the snout narrow, sometimes prolonged, conical. In these the premaxillary is subcircular, with very few or no teeth. Frequently the barbels are fimbriated and can be folded forward and used as a screen over the small mouth.

In the broad-mouthed species the anterior nostril is very near the lip, in the conical, prolonged-snouted ones the anterior nostril is always a considerable distance from the lip. There are no nasal barbels in any of the species.

The species best fitting the North American conception of a catfish is *Pterodoras granulosus* (Plate VIII). It has a broad ethmoid and broad premaxillaries, which bear wide bands of teeth. While in general appearance this species seems most conventionalized, the structure of its air-bladder indicates that it is far from primitive. Judging by the air-bladder and by the distribution, *Centrochir crocodili* and *Franciscodoras marmoratus* are the oldest and most nearly like the original Doradids. A very close rival to them is the widely distributed *Platydoras costatus* (Plate IX).

Fins.—The dorsal is usually composed of one spine and six rays. In the species of *Nemadoras bachi*, *Agamyxis pectinifrons*, and in *Acanthodoras*, it is I, 5, even I, 4 in some specimens of *A. cataphractus* and in *castaneoventris*, which is probably a synonym of the *cataphractus*. It is situated immediately behind the nuchal shield. The dorsal spine may be slender or large and heavy, serrate on both edges, serrate only in front, or without serræ. The spine, if without serræ, has longitudinal grooves and ridges. The anal is short, with from 10 to 16 rays. The adipose fin is present in all but *Physopyxis*. Usually it is well defined and short. In the larger, heavily armored species it is usually not well defined, prolonged forward as a heavy and scarcely flexible keel, extending from the adipose toward the dorsal. The caudal may be rounded, truncate, emarginate or deeply forked. The ventrals are rounded, without marked peculiarities. The pectoral spine is in some species notably heavy and large, in others long and slender. It is always serrate on both margins, sometimes also on the dorsal surface.

Bones of the head.—The bones of the upper surface of the head are granular or striated, covered with very thin skin. They are united by sutures into a solid shield or plate. The sutures can frequently be seen only with a lens. In the numerous figures the sutures were traced and marked under a binocular dissecting microscope. The bones are uniformly numbered in the different figures. The bones of the top of the skull united by sutures are (2) the ethmoid, (3) lateral ethmoid, (5) frontals, (6) sphenotic, (7) pterotic, (8) supraoccipital, (9) epiotic and three elements (*X*, *Y*, *Z*) of the dorsal scutes, which are expanded ends of interneural spines.

The skull varies from nearly flat in *Acanthodoras* to sharply roofshaped in *Trachydoras* (Plates II and IV). The suprascapula (10) is ankylosed to the pterotic

(7) and epiotic (9). It entirely excludes the epiotic from the margin of the skull in the broad-snouted species and in *Pseudodoras*. In all the species in which the epiotic (9) is connected with the first lateral scute by a process (9*a*) it forms part of the margin of the shield and the supraclavicle (10) is but narrowly in contact with the epiotic (9).

In the broad-headed species the fontanel is a small, oval opening between the anterior end of the frontals (Plates II to IV), sometimes extending into the ethmoid. In extreme cases the fontanel is reduced to sub-circular (Plate II, Fig. 3) and may be entirely occluded. In the narrow-headed species the fontanel is more elongate. It

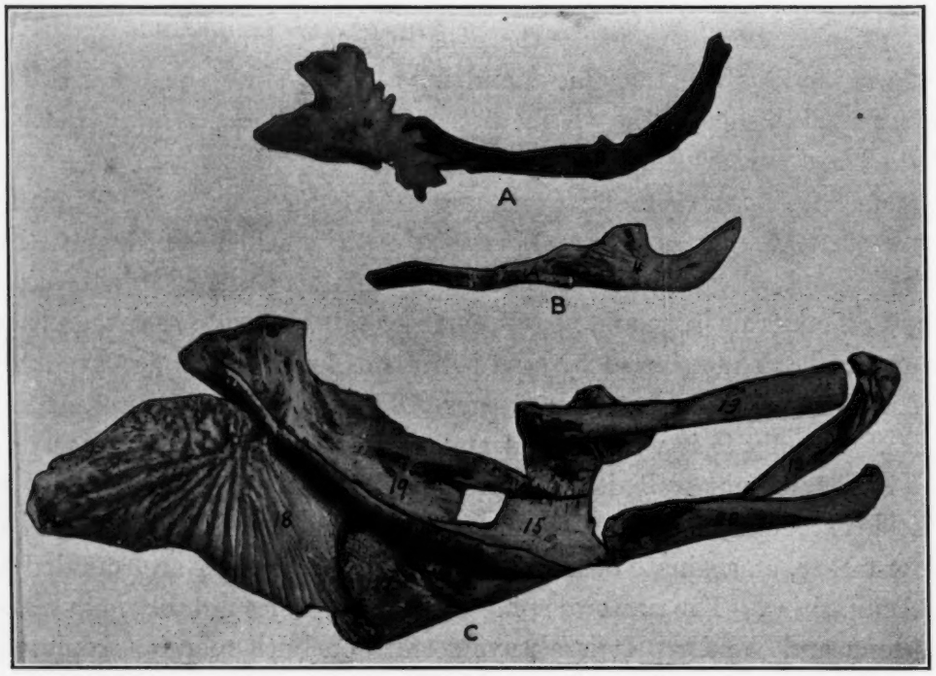

FIG. 2. *A*, Preorbital and suborbitals of *Astrodoras asterifrons*. *B*, Preorbital and suborbitals of *Pseudodoras niger*. *C*, Membrane bones of the right side of *Pseudodoras niger*. Notations: 4, Preorbital; *A* and *B*, suborbitals; 12, maxillary; 13, palatine; 14, mesopterygoid; 14a = metapterogoid; 15, quadrate; 16, preopercle; 17, interopercle; 18, opercle; 19, hyomandibular.

moid and into the occipital, with a narrow bridge in the posterior third of the frontal. reaches its maximum in *Leptodoras* (Plate II, Fig. 2) where it extends both into the eth-

A very slender rod, the nasal, is attached by ligament to the margin of the ethmoid, between the ethmoid and the nasal cavity. It is marked *D* in the plates, but is mostly obscure.

There is but little variation of the same bone in the different species. The greatest variation is found in the epiotic and in the region in front of the eye, in the

preorbital bone (4) and the suborbital chain. The preorbital bone (4) is always more or less fan-shaped. The pointed anterior end of the bone may be in contact with the maxillary and premaxillary. It may not reach so far. In a few species, *Pseudo-*

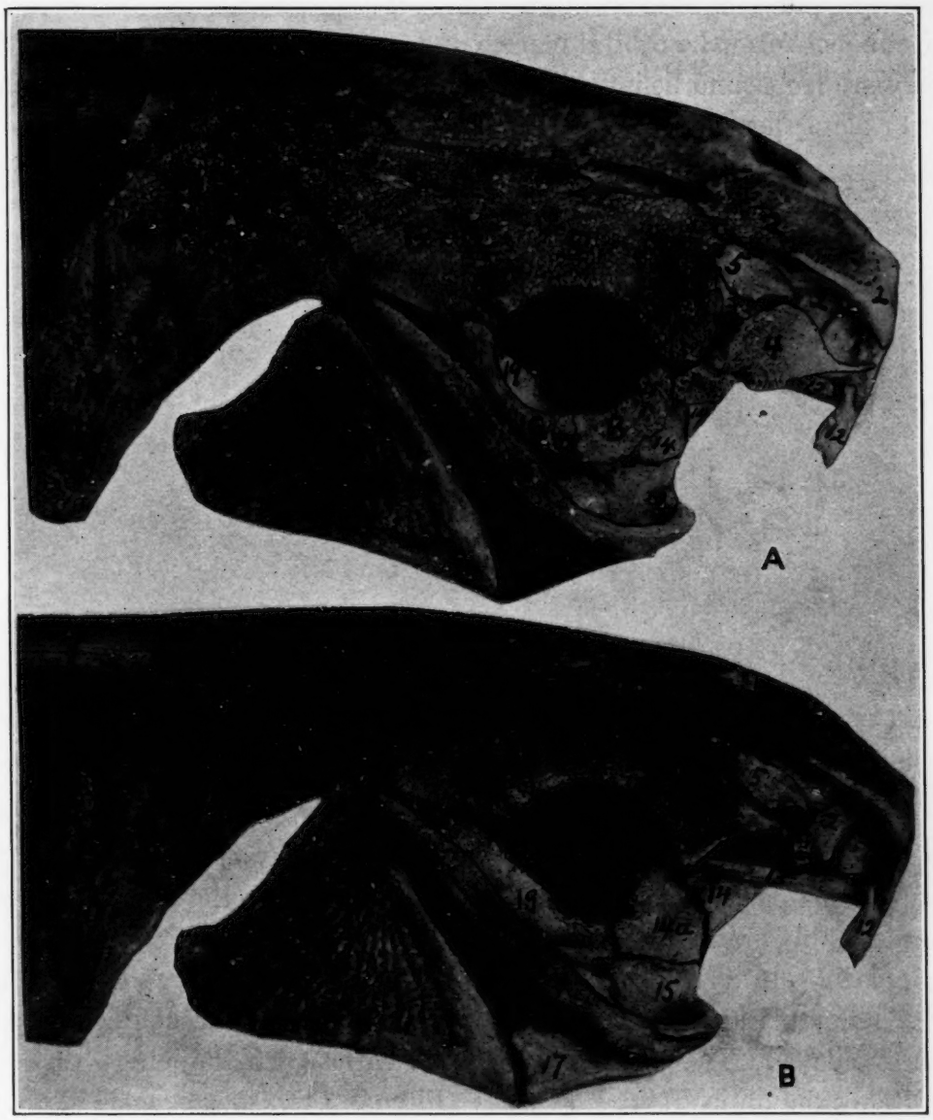

FIG. 3. *Platydoras costatus*. *A*, With the preorbital and suborbitals. *B*, With the preorbital and suborbitals removed. 1, premaxillary; 2, ethmoid; 3, lateral ethmoid; 4, preorbital; 5, frontal; 6, sphenotic; 7, pterotic; 8, supraoccipital; 9, epiotic; 10, supraclavicle; 12, maxillary; 13, palatine; 14, mesopterygoid; 15, quadrate; 16, preopercle; 18, opercle.

doras niger, it is joined to the lateral shoulder of the ethmoid, the ectethmoid. It is procumbent in the larger species, its expanded edge smooth (*Megalodoras*, Plate VI, Figs. 3 and 4) or scalloped and covered with skin. In other species it is serrate or finely

pectinate (*Agamyxis*, Text-fig. 14, *Astrodoras*, Plate III), the pectinate edge being raised to form a crest in front of the posterior nostril. The suborbital varies with the size of the eye. In some species, those with minute eyes, the first suborbital lies under the preorbital and does not form part of the orbital margin. In other cases all three suborbital bones form the orbital margin and are of about uniform width. In some cases they are feeble and lie imbedded in the skin; in others they are of unequal size and one or all of them thick, granular.

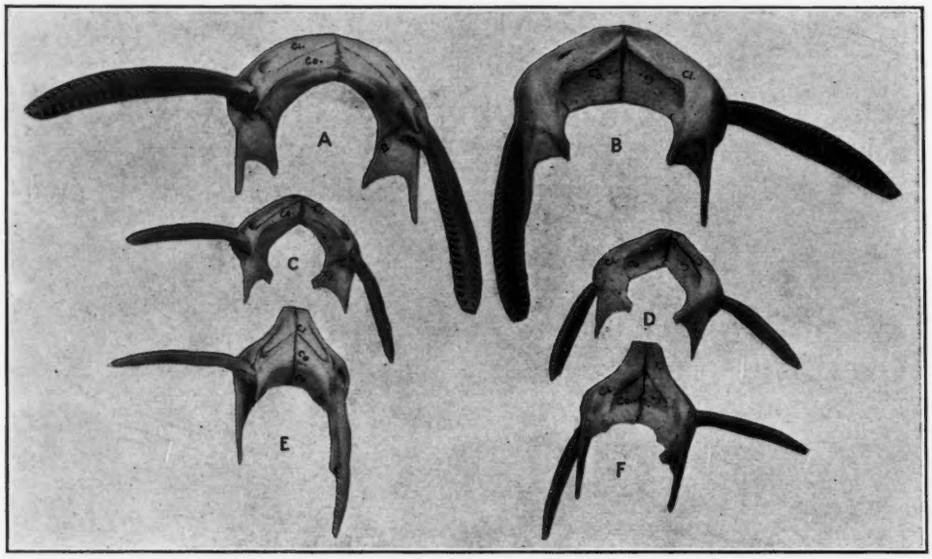

FIG. 4. Clavicle, coracoid and pectoral spine in *Megalodoras irwini: A*, from below; *B*, from above; *Pterodoras granulosus: C*, from below; *D*, from above; *Doras micropœus: E*, from below; *F*, from above.

The opercles may be covered with smooth skin, but usually the opercle (18 in the figures) is at least partly granular or striate, and sometimes the interopercle and preopercle are notably granular (*Trachydoras*, Plate XVIII).

The palatine (13) is rod-shaped, in contact with the maxillary in front (12) lying above the mesopterogoid (14), which connects the metapterogoid (14*a*) with the lateral ethmoid (3). The size of the palatine varies with the length of the snout, reaching its maximum in *Doras micropœus* (Plate XXIV), *Leptodoras* (Plate XXIV) and *Pseudodoras* (Plate XVII).

The shoulder girdle.—The clavicle and coracoid are always united with their fellows of the opposite side by an interlocking suture into a solid plate, provided with a process extending from the coracoid backward below the pectoral, and another process usually longer than the coracoid from the clavicle backward above the pectoral, the so-called humeral process.

Rarely the coracoid process is exaggerated and extends as far as the humeral process (*Trachydoras paraguayensis, cl.* and *co.*, Plate XVIII), or even much further (*Physopyxis lyra,* Plate 16). The surface of the humeral process may be granulate or the granules may be enlarged into spines along a series, particularly toward the tip. In the more exaggerated forms the spiny humeral process and the pectoral spine form a scissors-like apparatus (Plate XXV, Fig. 3).

The upward projecting flanges of the coracoid and the inward flanges of the clavicle are very different in different species. In the narrow-breasted genera they are extensive and form a dividing wall between the gill-cavity and the parts behind.

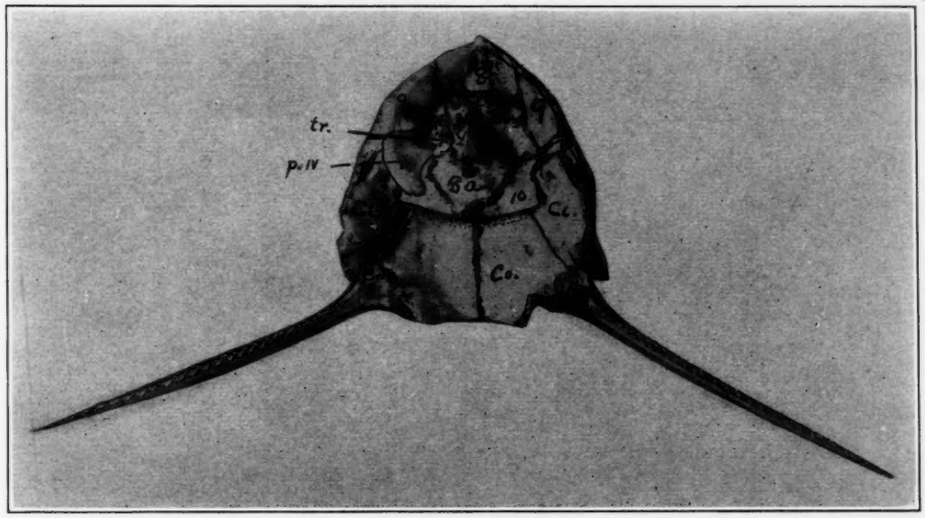

FIG. 5. Cross section of *Trachydoras atripes* (I. U. M. 16006) just behind the bony diaphragm formed by the flanges of the coracoid, *co.*; clavicle, *cl.*, supraoccipital 8, exoccipital 8*b*, basioccipital 8*a*; epiotic 9, supraclavicle 10; the broken vertebra *v*, the process of the fourth vertebra *X*.

In these species the effect is heightened by the expansion of the basioccipital into a thin plate extending downward. The partition or diaphragm formed by the flanges of the clavicle, coracoid and basi- and exoccipital reaches its maximum in the species of *Trachycorystes* (Text-fig. 5). The opening is just sufficient for the passage of the alimentary canal, being much smaller than the pupil. In *Trachydoras paraguayensis* the downward plate of the basioccipital and supraclavicle actually overlap, passing behind the processes of the clavicle and coracoid. In this species the opening is not visible looked at from straight behind; the alimentary canal is a siphon. The partition is least developed in *Megalodoras* and *Lithodoras.*

In most of the species the lower surfaces of the coracoid-clavicle are covered with the erector muscles of the pectoral spine and these with skin (*cl.* and *co.* in the ventral views on many of the plates); in a series *Anadoras grypus, weddelii* (Plate XV), *Ambly-*

doras monitor (Text-fig. 15), *A. hancockii* (Plate XIII) and *Physopyxis lyra* (Fig. 1, Plate XVI), there is a decreasing space for muscles and an increasing granulation of the bones, covered with very thin, scarcely perceptible skin.

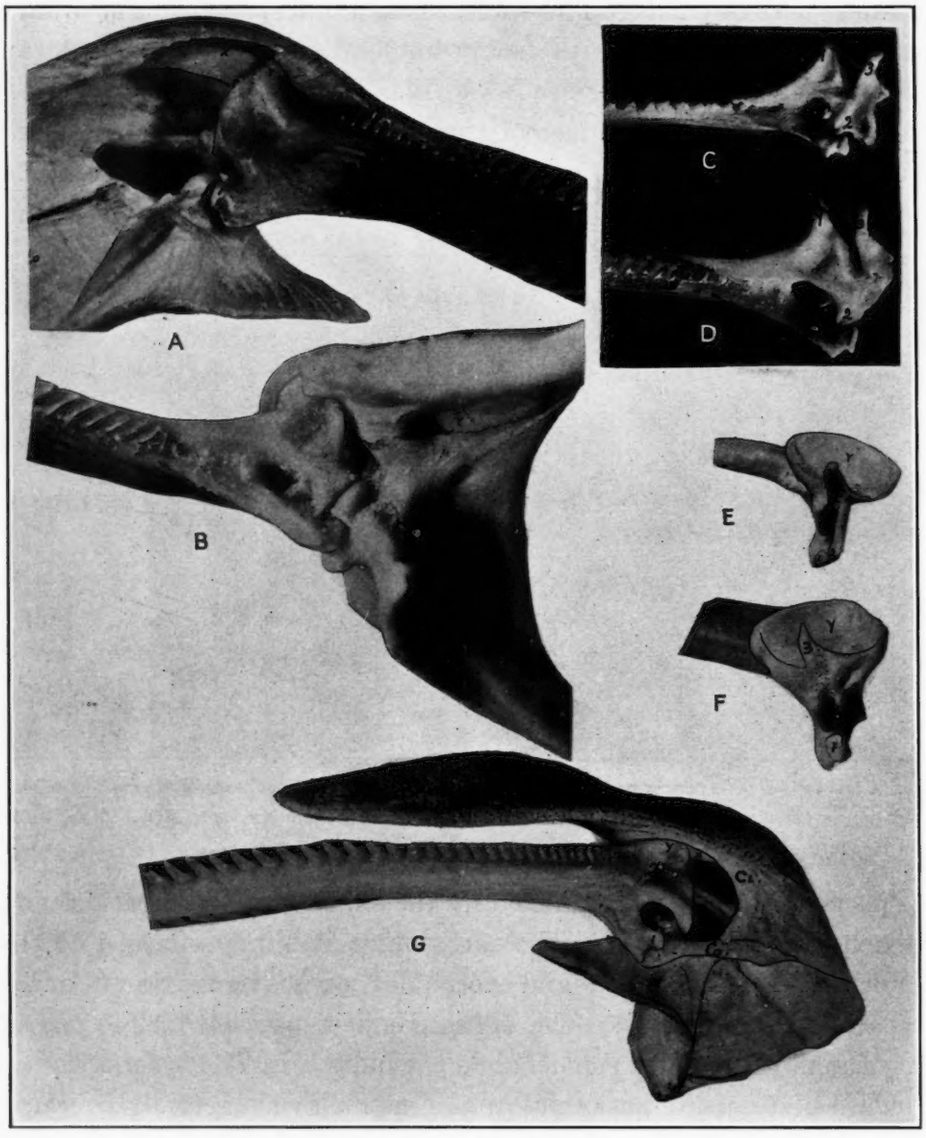

FIG. 6, *A*, *B*, *D*. Pectoral lock of *Megalodoras irwini*. *A*, Pectoral spine depressed; *B*, the same erect; *D* and *F*, the basal end of the pectoral spine, *D* from the axillary edge, *F* looking directly at the base of the spine. Part of the clavicle cut away to the left of the spine in *A*, to show the groove in which the outer flange of the spine moves. *C* and *E*, the basal end of the pectoral spine of *Pseudodoras niger*, to correspond with *D* and *F*. *G*, the right pectoral spine of *Platydoras costatus*.

Lock of the pectoral spine.—A most striking feature of the Doradidæ is the elaborateness of the pectoral spine and everything associated with it, its muscles, and the adaptation of the bones for their attachment, the coracoid and clavicle.

As in other Nematognaths the pectoral spine can be erected and locked at any stage of erection. Though in type the lock is not different from that in some other Nematognaths it may not be out of order to redescribe it.

The left pectoral joint and lock can best be imagined by using the right hand and arm as a model. The arm would represent the spine (Text-fig. 6), the thumb (1) and pointer finger (2) would represent two processes engaging between them a portion of the coracoid, the tip of the thumb turning in a socket in the lower outer surface of coracoid as a fixed point, the tip of the forefinger moving in a short groove of the inner surface of the coracoid.

The middle finger is grooved along its entire lower surface, fitting over a ridge of the coracoid, the end of the middle finger (3) impinging upon the clavicle.

The ring and little fingers (4 or *y*) are greatly expanded sidewise into a curved blade projecting around the dorsal surface of the spine, moving in a deep, *curved* groove in the expanded, laterally-projecting portion of the clavicle. In erection and depression the spine revolves on two axes. In the main movement, the erection and depression, the spine moves about the axis provided by the ridge over which the grooved lower surface of the spine fits. The second axis extends from the thumb to the tip of the spine and movement about it results in the tortion of the spine. The thumb (1) moves, twists in the socket, the pointer finger moves in a short, shallow groove in the coracoid, the expanded flange (4) moves in a much longer, deeper, curved groove in the clavicle. In order to depress, the spine must not simply revolve on the transverse (middle finger) axis; it must also undergo a slight tortion about the longitudinal axis of the spine with the end of the thumb as the pivot. The moment the twisting in the longitudinal axis stops, the spine becomes locked in whatever stage of erection, because the outer groove is curved.

The spine cannot be depressed in a preserved or living specimen rotated about the ridge of the coracoid by simply employing a force directly toward depression. The spine will break off sooner than go down. It can easily be depressed by twisting the spine, key-fashion; the left one clockwise, the right one contrary, at the same time that force is being applied to depress it.

The above description was made with the left side of *Pseudodoras niger* in mind. There are small modifications from this type in other species.

The pectoral spines and everything connected with them, the serration, the high development of the humeral process, frequently with spines opposed to those of the pectoral spine, the great development of the clavicle and coracoid to provide attachment for the pectoral muscles, the expansion of the clavicle to provide the elaborate groove in which the flange of the spine moves, all point to functions of the spine beyond

and in addition to that as a defensive organ. It appears as though the entire evolution of the Doradidæ centered in the pectoral spine, which is well shown in Plate XXV.

Lock of the dorsal spine.—The dorsal spine may also be locked in various positions. It can always be erected, but depressed only when the "latch" is raised. Its articular surface and lock involve two interneurals, that below the spine itself, and that below the "latch," the modified spine in front of it, and the modified first ray or spine which forms the latch. In *D. micropœus* the anterior interneural involved (Fig. 7, *E, INT. I.*) consists of a median plate (*m.pl*) and lateral flanges (*f*) continuous with similar flanges (the parapophysis) of the fourth vertebra. The tip of the first interneural in front of its flange is expanded to form the "*Y*" element of the dorsal plate.

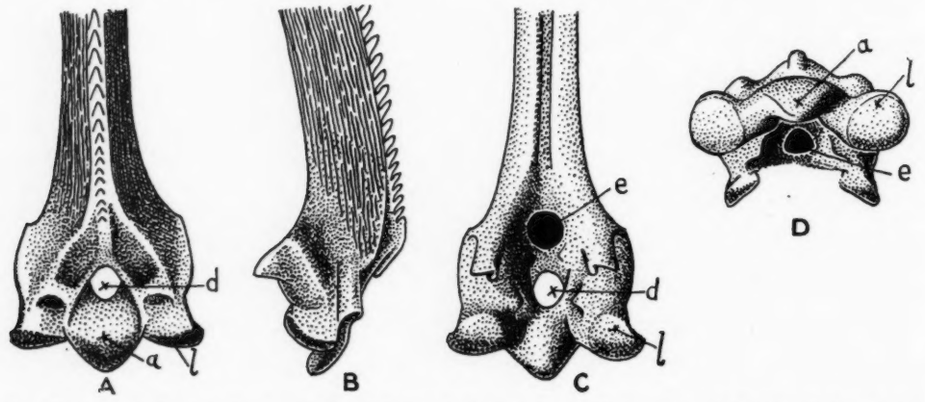

FIG. 7. The base of the dorsal spine of *Pseudodoras; A*, from in front; *B*, from the right side; *C*, from behind; *D*, from below. Kennedy, del. *a*, the anterior articular surface in contact with the second interneural which passes through the opening *d; e*, the opening through which blood vessels, nerves, etc., enter the spine; *l*, the lateral articular surfaces bearing on the lateral flanges of the second interneural.

Behind the flange the tip of the first interneural is transformed into a two-faceted, smooth, articular surface (*b*, Text-figs. 7 and 8) facing forward and upward. Looked at from above and in front it resembles very much a partially unfolded young leaflet. The two-pronged fulcrum (marked *O* and *E* or *D.S.I*), the modified first spine of the dorsal, straddles this. The second interneural has a pair of similar median lateral ridges which are expanded at the tip and provided with articular surfaces (*K*, Text-fig. 8) on which the lateral ends (*l*) of the dorsal spine move. The anterior part of middle plate of the second interneural is developed into a concave articular surface (*c*, Figs. 8 and 9) in which the anterior, mesial end (*a*) of the dorsal spine moves.

A finger-like prolongation from the middle process of the interneural is crooked backward through an opening in the dorsal spine (*B*, Text-fig. 7). It meets a similar

forward-crooked process from the posterior part of the second interneural, the two uniting to form a ring by which the dorsal spine is fastened to the second interneural. As long as the spine is erect and the crooked fulcrum is down as in Text-fig. 9, *A*, *E*, and *G*, the spine is locked, erect. The spine can be depressed when the fulcrum *O* is raised as in *D* and *F*.

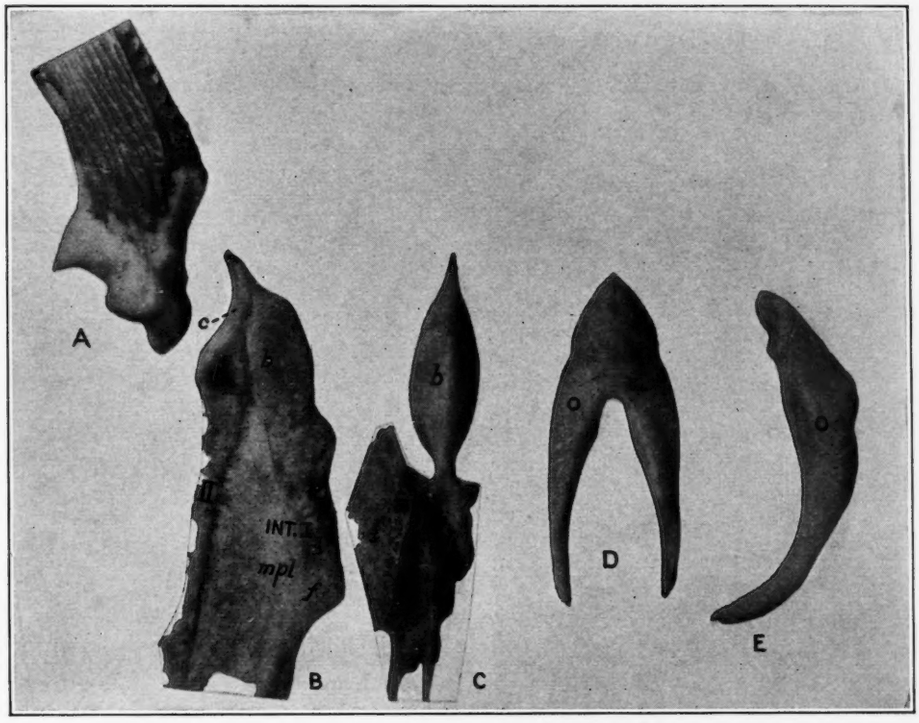

FIG. 8. *A*, Base of dorsal spine, and *B* to *E*, elements of the dorsal lock of *Pseudodoras niger*. The right in figures *A*, *B*, and *E*, are anterior. *A*, base of spine from the right; *B*, tip of first and second interneural from the side; *C*, tip of first interneural from in front. The tip of the second neural spine that passes through the dorsal spine at *d* is broken off. *D*, the "latch," the fulcrum, or first dorsal spine from in front. *E*, the same from the side.

In *Megalodoras irwini* a median partition or interneural septum extends down from the "*X*" element of the dorsal plate (*F* and *G*, Fig. 9). In *D. micropœus* there is no such partition (*D* and *E*, Fig. 9). There are many gradations in the development of the median partition between these two extremes.

An interneural septum below the "*X*" element of the dorsal plate is complete or nearly complete in *Centrochir crocodili*, *Platydoras costatus*, *Lithodoras dorsalis*, *Anadoras weddelii* and *grypus* and in the adult of *Pseudodoras niger*.

The septum is not complete in *Pterodoras granulosus*, *Astrodoras asterifrons*, *Acanthodoras spinosissimus*, *Amblydoras hancocki*, *Doras carinatus*, *D. punctatus*, *Opsodoras humeralis*, *Leptodoras linnelli* and *Trachydoras paraguayensis*.

Judging from the miscellaneous species in which it is complete or nearly complete, and those in which it is incomplete, this character cannot be given much weight in classification.

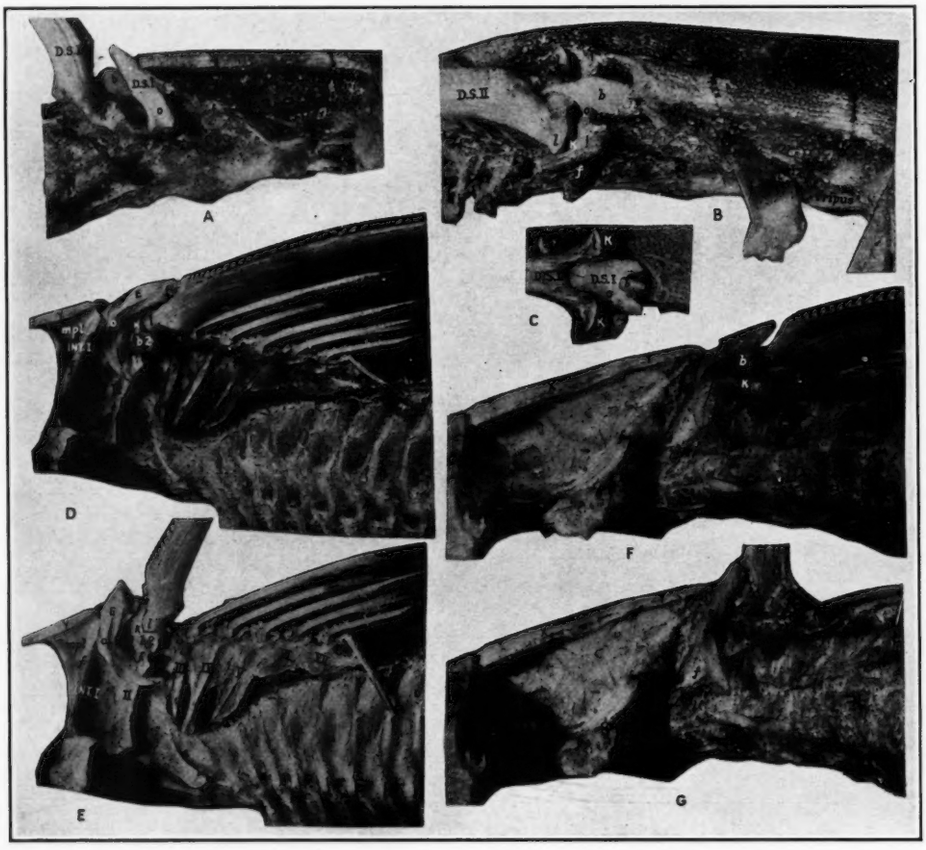

FIG. 9. *A*, Dorsal lock in *Platydoras costatus* from the side. *B*, The same three-quarter view. *C*, The same from the top. *D* and *E*, Dorsal erect and depressed in *Doras micropœus*. *F* and *G*, Dorsal depressed and erect in *Megalodoras irwini*. *b*, articular surface of the first dorsal spine; *D. S.* I, first dorsal spine; *D. S.* II, second or main dorsal spine; *E*, first dorsal spine; *f*, lateral flange of the I and II interneurals; *K*, articular surface of the lateral flange of the second interneural; *l*, lateral articular surface of the dorsal spine; *mpl*, middle plate of the interneural; *o*, first dorsal spine.

Surface plates.—Immediately above the humeral process the air-bladder is in contact with the skin to form a tympanum. The air-bladder space is separated internally from the muscled region above, either by a membranous septum or by a bony process or plate (9*a* in Plates XVIII, XX and XXIV) from the epiotic to a downward-directed process of the last element of the dorsal plate and the first rib.

The series of lateral scutes consist of one to three small bones without hooks near the middle of the tympanum, and from 16 to 40 hooked bones along the sides. The first one to three hookbearing scutes are in contact with the dorsal plate above

and may or may not reach the humeral process. The last plate is on the middle of the base of the caudal. The plates are sometimes minute, isolated and imbedded in the skin (*Hassar*, Plate XXII; *Doras*, Plate XXVII, Fig. 4). Sometimes the plates are very wide, nearly covering the entire sides, *Platydoras* (Plate IX), *Megalodoras*

FIG. 10. The humeral region of a specimen of *Pseudodoras niger* from La Merced, showing the supplementary lateral scute at *X*.

(Plate VI), and *Acanthodoras* (Plate XI). In the extreme the plates of opposite sides may meet along the middle of the back (*Acanthodoras spinosissimus*, Fig. 2, Plate XI).

Rarely, plates are developed along the middle of the back between the dorsal and the adipose (*Lithodoras dorsalis*, *Hemidoras hemipeltis* and *morrisi*). In one species, *Hypodoras forficulatus*, a large regular plate covers the anterior half of the adipose (Plate IV, Fig. 1, and Plate XXV).

In an extreme case, *Lithodoras dorsalis*, dermal plates may develop at any point on the surface and, in very old, they may form a protective armor over the entire body.

The caudal fulcra are always well developed, frequently they are laminate, extending forward on the caudal peduncle. Sometimes the fulcra become spread out forward, forming isolated plates, or plates in contact over the entire region to the adipose and anal (Plate III, Figs. 1, 5, and 6; Plate IV, various figures).

Air-bladder.—(Plate I.) The greatest amount of variation is found in the air-bladder. It is indeed difficult to reconcile the fact that the various types of air-

bladders belong to related species. There are five general types. Of these there are various modifications and combinations only generally correlated with other structures.

The simplest, most primitive type is found in *Centrochir crocodili*, *Hoplodoras uranoscopus* and *Franciscodoras marmoratus* (*A*, *B*, *C*, Text-fig. 12). In these species the air-bladder, externally at least, is a simple subconical or subglobular bag.

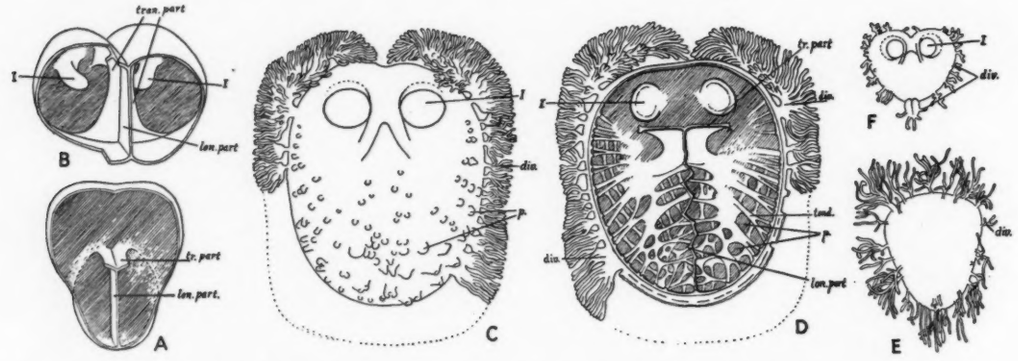

FIG. 11. Air-bladders of: *A*, *Doras carinatus*, the upper wall cut away to show the median partition in the posterior half and the partial anterior partition at its anterior end. *B*, *Doras carinatus*, transverse section, on the left immediately behind the transverse partition on the right some distance further back. Compare with Plate I, Figs. 22 and 31. *C*, *Pterodoras granulosus*, dorsal view. Compare with Plate I, Fig. 3. *D*, The same with the ventral wall cut away to show the partitions and the tendinous stays on the inner face of the dorsal wall. Compare with Plate I, Fig. 1. *E*, *Doras humeralis*, dorsal view, with many slender diverticulæ. *F*, *Megalodoras irwini*, ventral view of the young, 90 mm. long. For the adult see Plate I, Fig. 5. *div.*, diverticula; *i.*, pocket for the process of the vertebra; *lon. part.*, longitudinal partition; *tend.*, tendinous stays; *tr. part.*, transverse partition; *p.*, ventral pockets.

Internally the cavity of the bladder is partially divided into three sections. A median partition divides the posterior two thirds or three fourths into a right and left half. In front of this a partition runs partly across the bladder. A horizontal section of these partitions gives the letter "T." In front of this "T" the bladder is a cavity extending from side to side. Into this anterior cavity project the disks and prolongations of the parapophyses of the fourth vertebra connected with the Weberian apparatus. The anterior cavity of the bladder is continuous with the posterior sections along the outer wall of the bladder (Text-fig. 12 and Plate I).

Another simple type occurs in *Platydoras costatus* (Figs. 14–18, Plate I). In these species there are two simple bladders connected with each other by a very short, thin canal. The anterior bladder resembles that of *Centrochir*, the posterior (cœcum) is similar but much smaller, elongate, egg-shaped.

The third type is a modification of the second. The posterior bladder is double or split lengthwise, the two halves sometimes recurved (Text-fig. 12, *E* et al.).

A fourth type was observed once. This consists of the double air-bladder, with a third division behind the second in *Scorpiodoras heckeli* (Text-fig. 12, *D*).

The fifth type of bladder is provided with a number of radiating tubes or cœca. The walls of this type are firm, not flexible, provided with internal ridges varying greatly in number and size. In *Pterodoras* they are not unlike the ribs of a boat (Text-fig. 11, Plate I, Figs. 1–3).

The tubes vary greatly in number, number of rows, in size and in character. Some are simple, narrow, sausage-shaped diverticula around the sides of the bladder. Others are branched simply or in an arborescent fashion, or even feather or plume like, and imbedded in masses of fat (Figs. 1 to 4 and 6, Plate I).

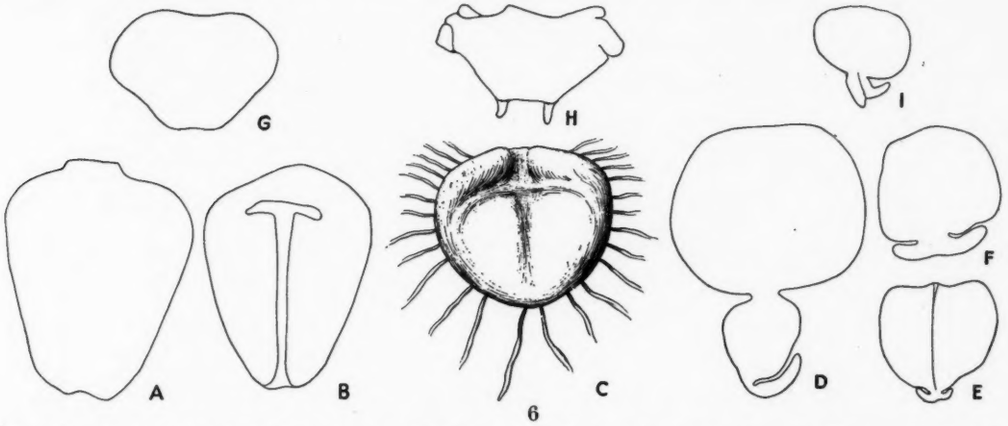

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FIG. 12. Air-bladders of: *A*, *Centrochir crocodili* (Humboldt); *B*, *Franciscodoras marmoratus* (Reinhardt); *C*, *Hoplodaros uranoscopus* (Eigenmann and Eigenmann); *D*, *Sorpiodoras heckeli* (Kner); *E*, *Doras punctatus* (Kner); *F*, *Astrodoras asterifrons* (Heckel); *G*, *Acanthodoras spinosissimus* (Eigenmann and Eigenmann); *H*, *Leptodoras linnelli* (Eigenmann); *I*, *Hypodoras forficulatus* (Eigenmann).

The beginning of the development of cœca in the double air-bladder occurs in *Pseudodoras niger* (16, Plate I). In this species the air-bladder has circular, thin places in the side wall of the depressed bladder. It would seem that the thin walls of these disks are capable of acting bellows-like with fluctuating pressure. The walls themselves seem inflexible. The height of modification of this type is found in *Megalodoras irwini* (4, Plate I) in which the number, size and elaborateness of the branching of the diverticula reaches the truly marvelous.

The diverticula in the single air-bladder go through a similar, though not identical, gamut of development. There may be many very thin tubules in various of the species with fimbriated maxillary barbels or there may be a few larger, short out-pocketings as in *Leptodoras linnelli*. The height of elaborateness is found in *Pterodoras granulosus* (1 to 3, Plate I) and is described under that species and in species of *Opsodoras* (especially in *O. humeralis* (6, Plate I)).

The modifications in the second air-bladder are preserved step-wise in various species. In its simplest form it is a minute protuberance, *Doras carinatus* (Fig. 8, Plate I), next an elongate, ovate bladder with pointed end, with or without diverticula. The beginning of the double posterior air-bladder is preserved in *Lithodoras dorsalis*. In this species an external constriction partially divides the bladder. The end is divided into two lateral horns. An internal septum completely divides the rest of the posterior bladder into two unequal lateral parts.

The modifications of this type can best be described by a series of figures of bladders actually observed.

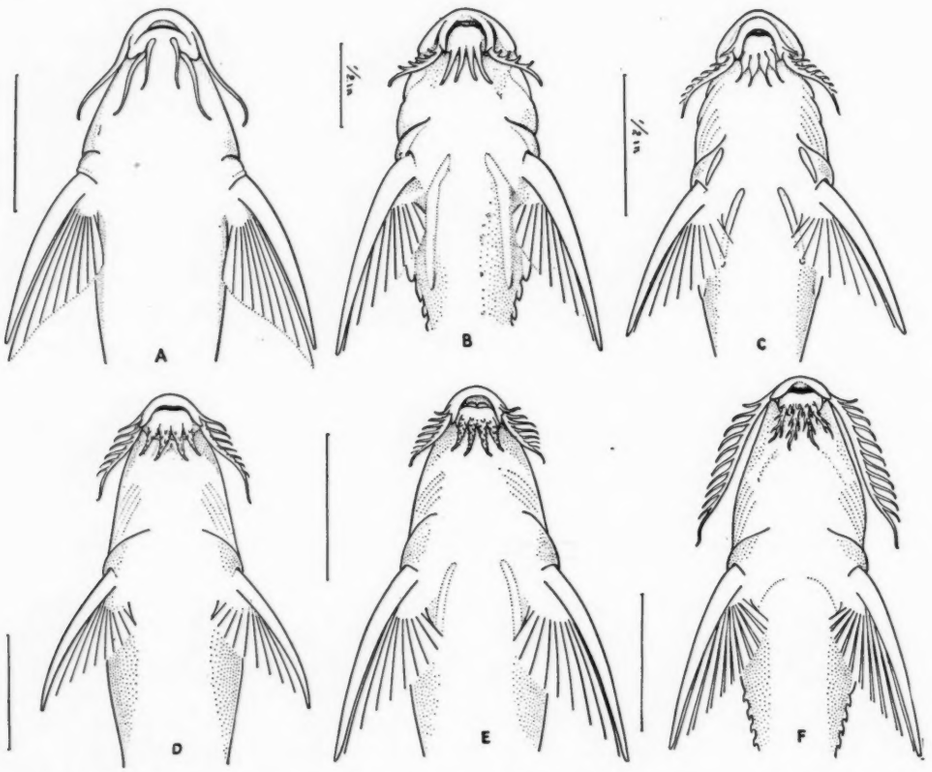


FIG. 13. Lower surfaces of the heads of different species under different magnification. The lines at the margin equal one inch unless otherwise indicated. *A, Rhinodoras d'orbignyi; B, Trachydoras paraguayensis; C. Trachydoras atripes; D, Hassar wilderi; E, Opsodoras humeralis; F, Hemidoras morrisi.*

Barbels.—The barbels are but moderately developed; there is always one maxillary and two pairs of mental barbels. In no species do the maxillaries extend any great distance beyond the tip of the humeral process. Usually they are shorter. In many species the mental barbels are warty, or carry supplementary barbels. In these the bases of the mental barbels, which are in a series close behind the lip (not

paired), are frequently united by a broad basal membrane. The height of the complication in the barbels is found in *Leptodoras linnelli* (Fig. 2, Plate XXIV). In this species the maxillary barbel is divided beyond the bone into an inner and an outer branch.

In the species with fringed mental barbels in which the bases of the barbels are united by a membrane, the mental barbels can be bent forward to form a screen, or sieve over the mouth.

Color scheme.—The underlying scheme of coloration consists of a light band along the spines and darker bands above and below the light band. The darker bands are continued to the tips of the caudal lobes. The parallel bands on the caudal are frequently present when there is no indication of a longitudinal stripe elsewhere (Figs. 1 and 2, Plate IX; 1, 2 and 5, Plate XI; 1 and 4, Plate XII; 1 and 2, Plate XV). The lateral stripes are best developed in *Platydoras costatus.* In a few species (*Hassar* and *Platydoras*) there is a black spot on the dorsal. The most strongly marked species is *Acanthodoras spinosissimus.* It is black, with light spots and streaks and has a banded caudal.

Weberian apparatus.—The Weberian apparatus consists of a long, narrow, oval bone, *scaphium* or *stapes*, fitting over the oval opening, *fenestra "a,"* into the lymph spaces of the internal ear. Between it and the *tripus*, imbedded in tendonous tissue, lies the curved *incus* or *intercalarium;* the *tripus* or *malleus* is a flat blade extending backward from the *incus* to below the lateral process of the fourth vertebra.

In some species the *tripus* remains a flat blade to its posterior end, in others it is folded upon itself from its inner edge; at a greater or lesser distance from its posterior end it comes in contact with the fourth vertebra by one or two short processes forming the pivot on which it moves.

A slender, unnamed, rod-shaped bone extends from the anterior, raised, lateral edge of the fourth vertebra backward and attaches by a fan-shaped tendon to the *tripus.* The anterior process of the fourth vertebra arises as a thin, sinuous bone from the basal half of the neural spine. It extends outward and more or less downward, being notched to arch over the *tripus.*

At the outer edge it is continued, or attached, to a disk or cone, fitting into an indentation in the anterior or upper wall of the air-bladder. In contact with the bone the wall is thin, delicate. If conical the apex of the cone is indented into the air-bladder (Plate XIX, Fig. 4, *i;* XX, Fig. 1, *i*).

The bones are so delicate, so small and so variable that I am not able to state positively that these structures are built on this plan in all the species. The small rod from the *tripus* to the anterior edge of the fourth vertebra seems to be especially

variable. The part of the process of the fourth vertebra which is hooked over the outer edge of the *tripus* is also variable.

The body of the fourth vertebra is very long, its lower surface smooth and intimately associated with the air-bladder. Sometimes (in *A. spinosissimus*) a deep median septum is developed in the mid-ventral line. The anterior end of the vertebra is always well marked, sometimes by a thickening or general enlargement,

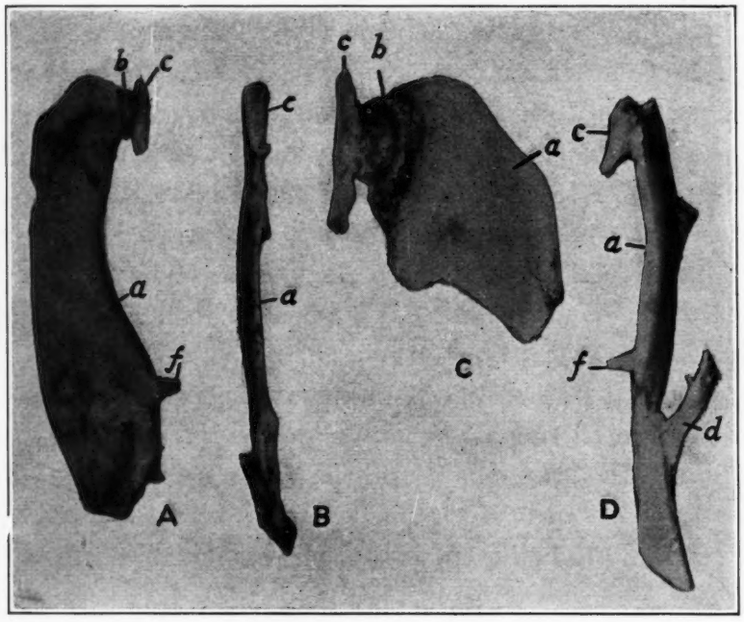


FIG. 14. The auditory, or Weberian ossicles of *Pseudodoras niger*. *A*, The right ossicle seen from below; *B*, the same from the vertebral viewpoint; *C*, the anterior end of *A* seen from above, more enlarged. The fulcrum, *f*, situated near the middle in *A* and *B*, is broken and restored in outline in *A*. *D*, the ossicles in another individual showing the supplementary bone, *d*, attached at the right; *a*, tripus or malleus, supposed to be the modified rib of the third vertebra; *b*, intercalarium or incus, supposed to be the modified neural arch of the second vertebra; *c*, scaphium or stapes, supposed to be the modified neural arch of the first vertebra. The claustrum, not shown in the photograph, is supposed to be the spine of the first vertebra. More probably the "scaphium and claustrum" represent the first neural arch and rib.

sometimes by a sharp ridge. The latter is much exaggerated in *Leptodoras*, in which it forms a deep pocket divided by a median septum. Into these pockets the anterior ends of the air-bladder fit.

DISTRIBUTION AND RADIATION OF THE DORADIDÆ.

The species of the Doradidæ belong prevailingly to the lowlands of the Amazons, ascending in the Chanchamayo to at least 2,500 feet, and in the Urubamba to 3,000 feet.

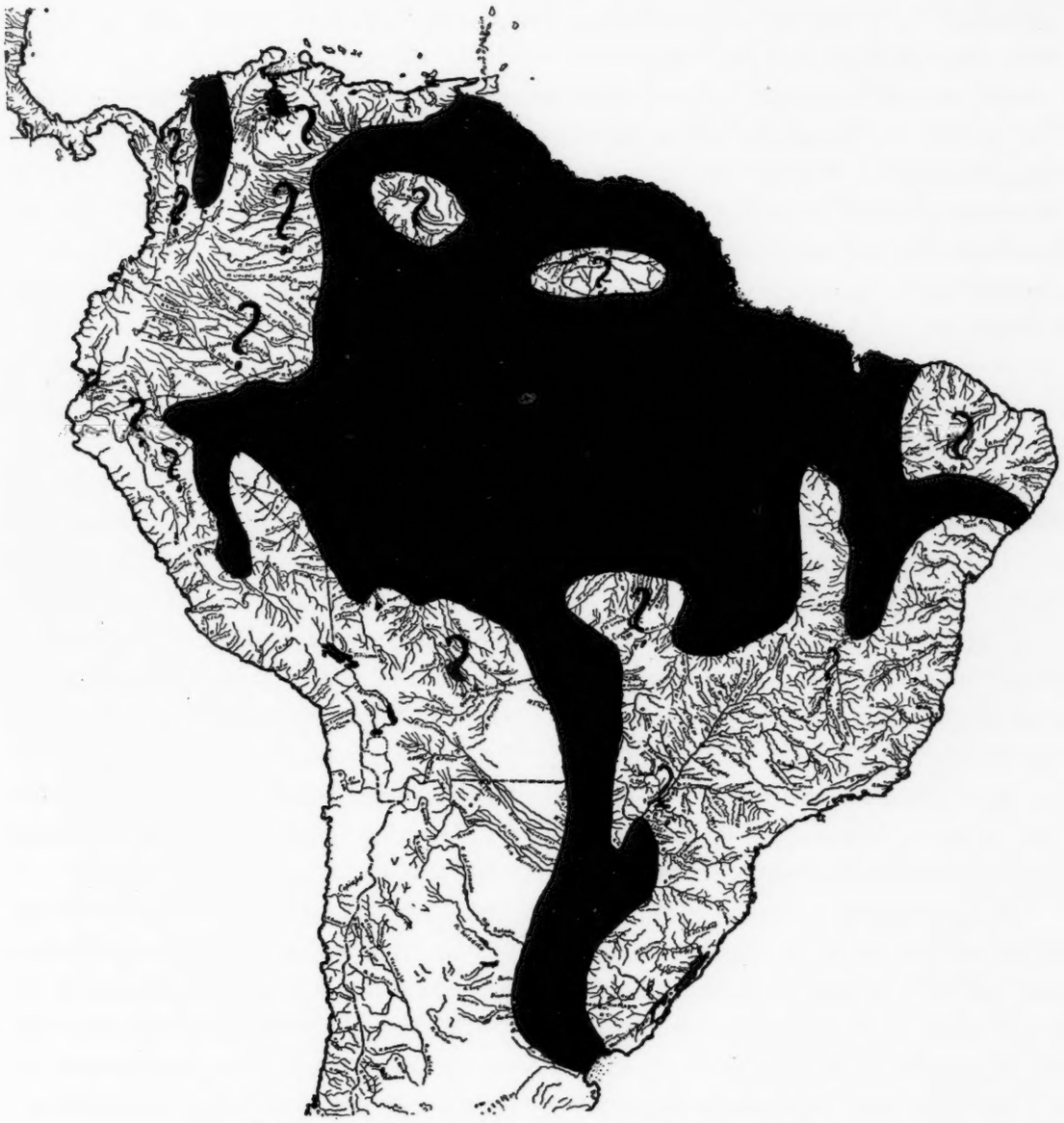


FIG. 15. Distribution of the Doradidæ. Areas in doubt marked ?.

They extend in a long pseudopodium down the Paraguay to the La Plata. The number of species that extend into the La Plata-Paraguay basin is but five, of which two, the genera *Oxydoras* and *Rhinodoras*, are peculiar to that system. Two other species extending into the La Plata system, *Pterodoras granulosus* and *Platydoras costatus*, are the most widely distributed species of the family. The fifth, *Trachydoras paraguayensis*, occurs on both sides of the divide separating the Paraguay and the Amazon. The La Plata is beyond the normal range in climate of the species and

their southward extension is evidently due to the facilities for easy movement furnished by the Paraguay-La Plata river.

At that, no member of the Doradidæ has been recorded from the upper Parana. Nor do they occur in any of the rivers emptying between the La Plata and the Rio San Francisco. Their point of origin is evidently the Amazonian basin; all but *Oxydoras*, *Rhinodoras* and *Franciscodoras* occur in the Amazon basin. The Rio San Francisco has provided means of migration to points not otherwise accessible and is responsible for another pseudopodium-like extension of habitat. They are found throughout the San Francisco but not east of it as far as is known.

One of the species of the San Francisco, *Franciscodoras marmoratus*, with a simple air-bladder is peculiar to it. *Platydoras costatus* and *Pseudodoras niger* are the other species of the San Francisco. They are widely distributed species.

I found no members of the Doradidæ above the Kaieteur falls and on the basis of that fact I have left the highlands of the Guianas blank in the distribution map.

But one species is found west of the Andes, *Centrochir crocodili*. It is a species without strongly marked characters; it, like *Franciscodoras*, has a simple air-bladder.

The Brazilian Amazons are most abundantly supplied with members of the Doradidæ. Next comes the Marañon. As far as the evidence goes at present, the species of the genus *Doras* prevail in the Brazilian Amazons while those of its ally, *Opsodoras*, prevail in the Marañon.

As far as the evidence goes, it points to the conclusion that the Doradidæ antedate the present configuration of the country and that they were in what are now the Magdalena, San Francisco, and La Plata basins from early times.

They evidently formed part of the fauna segregated by the formation of the Cordillera of Bogota and have not had access to the Magdalena since these mountains were formed. *Franciscodoras*, the peculiar genus of the Rio San Francisco, is evidently the original member inhabiting the Rio San Francisco. *Platydoras* (and *Pseudodoras?*), found everywhere, may be part of the original fauna; but as they are not found in the Magdalena, they may be later products which have migrated into the San Francisco by way of the Tocantins, Somo and Rio Preto.

Similarly, the genera peculiar to the La Plata basin, *Rhinodoras* and *Oxydoras*, may be looked upon as the more slightly modified descendants of the original fauna. *Platydoras costatus* may also be an original inhabitant but has less claim since it would involve the supposition that it has retained its character, both in the San Francisco and the Plata, since early time. It seems clear that *Trachydoras paraguayensis*, belonging to an otherwise Amazonian genus and to a species also found in the Amazon basin, is an interloper in the Paraguay.

Centrochir of Trans-Andes, *Franciscodoras* peculiar to the San Francisco, *Rhinodoras* and *Oxydoras* peculiar to the La Plata, are of very long-standing and a composite of these and of their nearest, and at the same time most widely distributed, relatives probably presents a generalized portrait of the earliest Doradid. See *Centrochir crocodili* (Plate VIII); *Franciscodoras marmoratus* (Plate XXIX); *Rhinodoras d'orbignyi* (Plate XXVI, Fig. 5); *Pseudodoras niger* (Plate XVII); *Platydoras costatus* (Plate IX). To these should probably be added *Pterodoras granulosus* (Plate VII, *a*), generalized Doradid externally but with an incomprehensible air-bladder.

From these generalized species there have been specializations along some definite lines, the modifications of one character frequently not correlated with modifications in other characters.

Megalodoras has gone but a short distance. It has emphasized the spines of the fins. *Lithodoras* has reached the maximum in the development of the number of dermal plates, which, however, have also increased in *Hypodoras* and *Hemipeltis*. The size of the lateral scutes has gone both up and down from the normal. They have reached their minimum, not in number but in general development, in *Doras micropœus* and *Opsodora hemipeltis*. I was inclined to place a higher value on the reduction in the size of the plates than it merits. The species with reduced plates are more nearly related to different species with well-developed plates than they are to each other. The reduction of the plates has taken place independently in different species. The pairs *Doras carinatus* to *Doras micropœus* and *Opsodoras parallelus* to *hemipeltis* are evidently more closely related to each other than the last of each of these pairs with minute plates is to species of *Hassar* which have minute plates.

The upward variation in the size of the plates reached its maximum in *Acanthodoras*. In this and the nearest related forms the scutes are variously provided with spines. The caudal fulcra, well developed in the typical species, have undergone development in several independent lines. In several independent lines the anterior ones have become enlarged and cover the caudal peduncle more or less completely.

The modifications of the pectoral musculature, the granulation of the coracoid and the development of a "diaphragm" have been mentioned above.

The mouth, the dentition, the barbels and the food habits have been associated, correlated modifications, directed distinctly away from the generalized *Centrochir* and *Franciscodoras*. In a measure each genus and species is the end product of series of modifications from the generalized Doradid. Among those that have gone furthest afield are *Trachydoras* with its associated *Hemidoras* and *Doras*, and *Leptodoras* with its associated *Nemadoras*, *Hassar* and *Opsodoras*.

Table of Distribution.

The following table indicates the species not seen by the author, the species known from the types only (marked *), the length in millimeters of the largest recorded specimen and the recorded distribution of the various species.

	Largest Specimen Known.	La Plata.	Paraguay.	Bolivia.	Bahia to Rio Grande do Sul.	Rio San Francisco.	Brazilian Amazon.	Marañon.	Guianas.	Orinoco and Venezuela.	Magdalena.
I. *Megalodoras irwini*	612						×	×	×		
libertatis (not seen)†	112						×				
II. *Centrodoras brachiatus*	410						×	×			
III. *Hoplodoras uranoscopus**	530						×				
IV. *Pterodoras granulosus*	230	×	×	×			×	×	×		
*lentiginosus**	357						×				
V. *Centrochir crocodili*	210										×
VI. *Platydoras costatus*	217		×	×		×	×	×	×	?	
albomaculatus (not seen)*	85									×	
helicophilus (not seen)*	350								×		
dentatus (not seen)*	135								×		
VII. *Franciscodoras marmoratus*	360					×					
VIII. *Lithodoras dorsalis*	960						×	×	×		
IX. *Acanthodoras calderonensis* (not seen)	103						×	×			
cataphractus	170						×		×		
spinosissimus	150						×				
X. *Agamyxis pectinifrons* (not seen)*	76							×			
flavopictus (not seen)*	110							×			
XI. *Astrodoras asterifrons*	100			×			×				
XII. *Scorpiodoras heckelii*	173						×				
XIII. *Amblydoras monitor**	38							×			
hancocki	123						×	×	×		
XIV. *Anadoras regani* (not seen)*	90						×				
grypus	150						×	×			
nauticus (not seen)*	82							×			
weddellii	113			×			×	×			
*insculptus**	100						×				
XV. *Hypodoras forficulatus**	123							×			
XVI. *Physopyxis lyra**	35							×			
XVII. *Pseudodoras niger*	840					×	×	×	×		
huberi (not seen)*	400						×				
XVIII. *Oxydoras kneri*	360		×								
XIX. *Rhinodoras d'orbignyi*	160	×	×								
XX. *Trachydoras paraguayensis*	93		×	×			×				
nattereri	130?						×	×			
atripes	103							×			
trachyparya (not seen)*	93						×	×			
XXI. *Doras fimbriatus* (not seen)*	125						×				
brevis (not seen)*	125						×				
punctatus	125						×				

	Largest Specimen Known.	La Plata.	Paraguay.	Bolivia.	Bahia to Rio Grande do Sul.	Rio San Francisco.	Brasilian Amazon.	Marañon.	Guianas.	Orinoco and Venezuela.	Magdalena.
XXII. *Doras eigenmanni*	95		×				×				
microstomus	52						×		×		
carinatus	267						×		×		
*micropœus**	270								×		
lipophthalmus (not seen)*	190						×				
XXIII. *Hemidoras stenopeltis*	160			?			×	×			
morrisi	147							×			
XXIV. *Opsodoras hemipeltis**	143							×			
*parallelus**	151							×			
humeralis	154						×	×			
*orthacanthus**	133							×			
trimaculatus	77						×		×		
leporhinus	81						×			×	
ternetzi	129						×				
boulengeri (not seen)*	167						×				
stübeli (not seen)*	120							×			
morei (not seen)*	125						×				
steindachneri (not seen)*	235						×				
XXV. *Hassar affinis* (not seen)*†	255										
orestis	196						×				
*wilderi**	215						×				
*notospilus**	70								×		
XXVI. *Leptodoras linnelli*	209						×		×		
acipenserinus	223						×?	×			
juruensis (not seen)*	235						×				
hasemani (not seen)*	132						×				
XXVII. *Nemadoras elongatus* (not seen)*	105						×				
bachi (not seen)*	90						×				
TOTALS		2	5	5		3	47	26	15	2	1
							58				

† Paranahyba and Itapicuru.

Along one line, the dorsal spine, originally serrate, has become less and less serrate till it has lost all serration in *Astrodoras*. The preorbital, procumbent and smooth in the original Doradid has become serrate and pectinate, the marginal teeth becoming erect, a line of development reaching its maximum in *Agamyxis* and *Astrodoras*.

Along another series the eye has become reduced in size, a modification indicated by peculiar adjustments in the orbital bones.

Conclusion.—Members of the Doradidæ are found from Buenos Aires through the Paraguay, Madeira, the Amazons to the Guianas and from the mouth of the Rio

San Francisco and Para to an elevation of about three thousand feet at the base of the Andes in Bolivia and Peru. One member of the family, *Centrochir longispinis*, is found in the Magdalena river, across the first chain of the Cordilleras in Colombia. As far as is known, they do not occur in the rivers between the mouth of the Rio San Francisco and the Rio Plata or in the upper courses of the Rio Parana.

KEY TO THE GENERA OF THE DORADIDÆ.

a. Head depressed; width at the clavicle greater than the length of the head to the end of the bony opercle; eye, if small, far in front of the middle of the head; if large or medium, near middle of the head; mouth terminal, wide; premaxillary wide, fixed, with a band of teeth; epiotic bordered by the occipital, two nuchal plates, the supraclavicle and pterotic; process of epiotic not reaching the first rib and first hook-bearing lateral scute, except in *Pterodoras;* barbels all simple.

b. Adipose fin continued forward as a low keel, longer than anal fin; snout smooth, preorbital bones not pectinate or serrate; caudal forked.

c. Caudal peduncle naked above and below, or the posterior half covered with laminate caudal fulcra.

d. A second air-bladder or cœcum; both depressed and fringed with numerous diverticula (not examined in *libertatis.*

*. Less than 23 lateral scutes; fewer thorns on posterior face of the dorsal spine than on the anterior. . . . . 1. **Megalodoras** Eigenmann, gen. nov.

. Scutes about 38–40 on a side. . . . . 2. **Centrodoras Eigenmann, gen. nov.

dd. Only one air-bladder.

e. Lateral scutes 18; air-bladder wider than long, with a marginal row of long, diverticula; thorns on back of dorsal spines fewer and smaller than on anterior margin.

3. **Hoplodoras** Eigenmann, gen. nov.

ee. Lateral scutes 25 to 30, very narrow, strongest over end of anal; eye entirely in the front half of the head; about 20 thorns on the posterior face of the dorsal spine, nearly of equal size with those covering the opposite side or anterior face; no posterior air-bladder, the anterior depressed, margined with fimbriated diverticula on the posterior and lateral edge, a long plume-like diverticulum on the outer anterior angle, curved around to the front; a bony stay behind the tympanum. . . . . 4. **Pterodoras** Bleeker.

eee. Lateral scutes 30. Fontanel not constricted; skull finely granular, arched; posterior hooks of the dorsal spine stronger than the anterior; three hook-bearing scutes in contact with the dorsal plate, the rest of the scutes graduate to the last, narrow; depth of the air-bladder more than half its width, without diverticula, no second air-bladder or cœcum. (See also *bb.*) . . . . . 5. **Centrochir** Agassiz.

cc. Caudal peduncle entirely covered with plates above and below (modified fulcra); scutes very deep, leaving only a narrow naked area along the back; three scutes in contact with the dorsal plate; posterior thorns of the dorsal spine disappearing with age; eye in middle of the head; air-bladders without diverticula, large; depth of the anterior air-bladder half its width; posterior carrot-shaped. . . . . 6. **Platydoras** Bleeker.

bb. Adipose fin high, longer than anal, not continued forward as a keel, or continued for a short distance only; dorsal spine serrate in front only; eye nearer snout than to end of opercle; caudal peduncle largely covered with fulcra above and below, increasing in size forward. Scutes about 30, about

one third or a little less of the depth; caudal forked or deeply emarginate; fontanel without groove, its depression ending abruptly; anterior air-bladder large, divided by a median partition to very near the anterior end, without diverticula; no posterior bladder or cœcum.

7. **Franciscodoras** Eigenmann, gen. nov.

bbb. Adipose fin shorter than anal, not shading into a keel in front (*A. calderonensis*?).

f. Dorsal spine serrate in front and behind, the posterior serræ stronger than the anterior; lateral scutes 16 to 20; caudal forked. . . . . 8. **Lithodoras** Bleeker.

ff. Dorsal spine serrate in front, not behind; caudal round or truncate.

g. Caudal peduncle without plates above and below; nasal bone short, with less than ten spines; eye minute, about 4 in interorbital, far in advance of the middle of the head; fontanel subcircular; breast entirely covered with skin or the tips of the coracoid process granular; sides of dorsal spine and upper surface of the pectoral spine serrate, except in *A. calderonensis*. Caudal round or truncate. (Species with light stripe along the sides.)

9. **Acanthodoras** Bleeker.

gg. Caudal peduncle with spiny plates above and below; sides of the dorsal and pectoral spines serrate; nasal plate erect, pectinate; eye moderate, in advance of the middle of the head; dorsal and pectoral spines with spines along the side. (Species without a light line along the spines.) . . . . . 10. **Agamyxis** Cope.

ggg. Caudal peduncle with prominent laminate caudal fulcra covering about half its surface above and below; dorsal and pectoral spines with ridges on the sides.

h. Nasal plate erect; width at clavicle greater than distance from snout to end of dorsal plate; posterior air-bladder simple or forked. . . . . 11. **Astrodoras** Bleeker.

hh. Nasal plate procumbent; width at clavicle about 1.25 in distance between snout and end of dorsal plate; second air-bladder heart-shaped, with a simple cœcum.

12. **Scorpiodoras** Eigenmann, gen. nov.

fff. Dorsal spine grooved, without spines on sides, front, or back, except in *Physopyxis*.

i. Adipose fin present; coracoid process shorter than humeral process.

j. No plates between dorsal and adipose; caudal truncate.

k. Granulations of coracoid and clavicle confluent into a large buckler; nasal serrate.

13. **Amblydoras** Bleeker.

kk. Granulations of coracoid and clavicle not confluent, a space covered with skin between the granulations; nasal not serrate. . . . . 14. **Anadoras** Eigenmann, gen. nov.

jj. A large plate over anterior part of adipose; caudal truncate; nasal serrate.

15. **Hypodoras** Eigenmann, gen. nov.

ii. No adipose; coracoid processes lyre-shaped, reaching ventrals, much larger than humeral processes which do not reach the first lateral scute; caudal rounded.

16. **Physopyxis** Cope.

aa. Width at the clavicle less than the length of the head; snout conical or subconical; mouth relatively small; premaxillaries attached to the front of the maxillary, not joined to each other, not joined to the ethmoid, which is pointed and without lateral wings in front. Teeth feeble or none. Caudal forked.

l. Nuchal shields without a foramen.

m. Barbels simple. Mental barbels not united by a membrane joining their bases. Adipose fin continued forward as a low keel, longer than anal fin; epiotic bordered as under *a* of the Key, not joined by process with the first hook-bearing scute; first nostril remote from the lip.

n. Eye behind middle of head; caudal peduncle naked above and below. Serræ on posterior margin of the dorsal spine antrorse, or at right angles to the spine, less numerous and much more feeble than those of the anterior margin; fontanel with a bridge.

o. Lateral scutes 16–23; rows of tentacles on roof of mouth.....17. **Pseudodoras** Bleeker.

oo. Scutes 34–40; no tentacles on roof of mouth.....18. **Oxydoras** Kner.

nn. Eye in middle of head; caudal peduncle entirely covered above with laminated caudal fulcra, nearly entirely covered below; serræ on posterior face of dorsal spine retrorse, much longer than those on anterior; fontanel not divided; lateral scutes about 30.

19. **Rhinodoras** Bleeker.

mm. Maxillary barbels fringed, the bases of the mental barbels united by a membrane; eye behind the middle of the head; dorsal serrate on both margins; caudal forked; adipose short; epiotic forming part of the edge of the skull between the supraclavicle and the nuchal plate, with a process to the first hook-bearing scute; second air-bladder minute or absent.

p. Opercle, preopercle and coracoid-process granular; first lateral scute large; fontanel not continued as a groove; adipose not continued as a ridge.

20. **Trachydoras** Eigenmann, gen. nov.

pp. Opercle, preopercle and coracoid process covered with skin. Back and ventral surface without plates. Adipose not continued as a ridge.....21. **Doras** Lacépède.

ll. Nuchal shields with a foramen on each side.

q. Maxillary barbel fringed with minute barbels, otherwise as under *ll.*

r. A series of plates between the dorsal and adipose and sometimes between the ventrals and anal.....22. **Hemidoras** Bleeker.

rr. Back without scutes.

s. Origin of ventrals nearer caudal than to snout; adipose not continued forward as a ridge.

t. Eye normal; scutes usually well developed.....23. **Opsodoras** Eigenmann, gen. nov.

tt. Eye elongate; snout very long and slender; humeral process large and rounded.

24. **Hassar** Eigenmann and Eigenmann.

ss. Slender, origin of ventrals nearer snout than to caudal; air-bladder very short, fitted into cupped downward processes of the coalesced vertebræ; adipose continued forward as a keel.....25. **Leptodoras** Boulenger.

qq. Maxillary barbels simple; mental barbels connected by a basal membrane; adipose short.

26. **Nemadoras** Eigenmann, gen. nov.

Genus I. MEGALODORAS Eigenmann, gen. nov.

Type, *Megalodoras irwini* Eigenmann.

Lateral plates few; adipose fin continued forward as a keel; head depressed, snout not conical; serræ of posterior margin of dorsal spine fewer and weaker than the anterior, obsolete with age; eye in middle or near middle of the head; suborbital chain (Fig. 4, Plate 6) consisting of three bones, the anterior in contact with the lower surface of the preorbital; the second borders about half the eye, is much wider than the posterior and has small spines at the upper border, its anterior end in contact with the first and with the preorbital; preorbital large, granular along its upper margin, in contact with the premaxillary and maxillary in front; mouth subterminal; occipital region somewhat roof-shaped, with or without median groove.

Air-bladder (of *Megalodoras irwini*, Fig. 4, Plate I and Text-fig. 12) much depressed, in two parts. The anterior part heart-shaped, nearly as wide as long, with about a dozen tubular cœca or diverticula in a row from the outer posterior angle around the sides to near the center in front; the cœca largest behind, smallest in front, all of them profusely branched; a few much smaller cœca in a row behind the outer row in front; the posterior bladder as long as the anterior, a constriction between the two; much narrower than the anterior and tapering backward. About seven lateral diverticula along the sides of the second bladder, decreasing in size backward and branched like the posterior ones of the anterior bladder. The chief features of the bladder are developed in a specimen 90 mm. long; but the posterior bladder shorter.

The air-bladder has not been examined in *libertatis*.

KEY TO THE SPECIES OF MEGALODORAS.

a. Lateral scutes, 17 or 18.

b. Eye 2.5 to 3 in the interorbital.

c. Fontanel narrow, not continued as a groove; dorsal spine smooth behind; scutes 2 + 17; fins spotted; lower half of sides mottled and spotted. 700 mm. over all.....1. **irwini** Eigenmann.

cc. Fontanel continued to the middle of the occipital as a wide shallow groove, to the end of the dorsal plate as a narrow groove; eye nearly 3 in the interorbital; scutes 15 to 17. Gill-opening extending to middle of the pectoral spine; dorsal spine with few serræ behind.

2. **irwini** Eigenmann, spec. nov.

bb. Eye 1.6 in the interorbital; scutes 18; dorsal spine strongly serrate on both margins. Fontanel continued to the end of the dorsal plate.....3. **libertatis** Ribeiro.

1. **Megalodoras irwini** Eigenmann.

Plate 25, Fig. 1.

T. R. S. No. 2567; type, skin; Kartabo, Bartica District, British Guiana, September 11, 1919; William Beebe.

Measurements in life: length, 612 mm.; length entire, 700 mm.; width, 170 mm.; depth, 110 mm.; head, 140 mm.; interorbital, 48 mm.; weight, 9 pounds; D. I, 6; A. $12\frac{1}{2}$; scutes 2 + 17; width of head equal to its length; eye 3.5 in snout, 6.5 in head, 2.6 in interorbital; dorsal spine 110 mm. long, striate on the sides, smooth behind, with about 40 serræ on the anterior margin, increasing slightly toward the tip; pectoral spine 155 mm. long, striate on the sides with about 50 serræ on the anterior margin, 38 on posterior; humeral process narrow, two thirds as long as pectoral spine; fontanel a narrow slit, ending abruptly over middle of eye, not continued as a groove, not constricted; two angular plates in the tympanum, in a line with the spines of the plates; the third to ninth plates highest, a little more than snout and eye, the hooks increasing in size to above the anal. The following is taken from the field notes: Ground color pinkish buff, becoming clear cream buff on the under side of the head. Top of the head indistinctly mottled with black. Side plates, ventral, anal and caudal

fins irregularly mottled or spotted with black. Pectoral fins almost entirely black, pinkish buff at base. Interplate space of the back black, as is also the skin just back of the side of the top of the head, and anterior to the large side plates. Belly indistinctly mottled with black, the mottling more concentrated at the sides.

The stomach of this fish was entirely empty, while the intestine had seven specimens of the common large brown river snail, evenly spaced along its length.

The Creole name of this fish is "Key-way-mamma," which translated means "mother of the snails," in allusion to its habit of feeding upon snails. The fish are not eaten by the natives, their flesh being too tough and not considered wholesome. This specimen was captured upon a hook.

This large specimen was originally given a distinct name. No. 16168 intermediate in size, 395 mm., shows the Guiana specimen to be but an over-large specimen of *M. irwini* and the name proposed for it has therefore been suppressed.

2. **Megalodoras irwini** Eigenmann, spec. nov.

Plate 1, Fig. 4; Plate 3, Fig. 3; Plate 6, Figs. 1–4; Plate 25, Fig. 2; Text-fig. 11, F.

I. U. M. 15427, 5, 90, 220, 278, 300 and 315 mm. The 278 mm. specimen, the type. Iquitos, Allen 1920 and Morris 1922.

I. U. M. 16168, 2, 275 and 395 mm. Santarem market, 1924. Carl Ternetz.

Head 4; depth 4–4.5; D. I, 6; A. 12 or 13; scutes 2 + 15 to 17; width of head equal to its length; interorbital 2.75–3; eye equidistant from snout and end of opercle; its diameter 8 in the head, 3 in the interorbital; width of mouth about equal to the interorbital; teeth minute but firm, in well-developed narrow bands; snout depressed; profile along median line to occipital straight; fontanel continued as a groove to the dorsal; opercle, suborbital, and nasal granular; head granular to near the anterior nostrils; maxillary barbels in the three larger extending beyond tip of humeral process; outer mental barbels reaching beyond base of pectoral spine or entire base of pectoral fin, the inner mental barbel about half as far; dorsal spine about two-thirds as long as the pectoral spine, about equal to the length of the head less half the opercle, its sides striate, serrate in front (32 to 35) and behind (22 to 25); pectoral spine very strong, considerably longer than the head, its upper and lower surfaces striate, its anterior and posterior edges very strongly serrate; humeral process very rough, reaching to or beyond the middle of the pectoral spine; two small free plates above the humeral process, in a line with the first four hooks; the first lateral plate in contact with the dorsal plate and humeral process; the anterior lateral plates crowded, those over the anal wide, the spines increasing in size to those above the anal; surfaces of the plates above the ventrals spinulose, the spinules decreasing in number to the

caudal peduncle, the plates on the caudal without spinules, but with a prominent median spine; a hard keel forward from the adipose; many caudal fulcra, but no plates on caudal peduncle above or below. Skin of back and sides with hard papillæ; a narrow light band from the dorsal to the tip of the upper caudal lobe, a wavy light band from above the eye, following the edge of the skull then back along the upper half of the lateral plates; a similar band on the humeral process and back on the lower half of the plates, the two lateral bands, irregular, with interruptions and unions; lower surface with spots decreasing in size to the middle of the belly; fins spotted, the pectoral dark anteriorly.

3. **Megalodoras libertatis** (Ribeiro).

Plate VII.

Doras libertatis Ribeiro, Comm. Linhas. Telegr. Estrat. Matto Grosso as Amazonas Annexo 5, 1912, p. 20 (Manaos).

Scutes 18; head 4, depressed forward, mouth equal to distance between the outer borders of the nasal crests; maxillary barbel beyond basal third of pectoral; anterior nares on lips; groove of fontanel to end of occipital process; low, radiating ridges from center of scutes to the margin of the scutes where they end in a spiny tubercle; dorsal spine reaching the 10th scute; adipose small, over posterior part of the anal; two rows of contiguous spots from the snout to the base of the upper caudal lobe; a more or less interrupted band from the snout, through eyes, along the spines to the caudal; another series, less distinct, from the angle of the mouth to the ventrals; spots on the belly. After Ribeiro. This species is known from the type only. 112 mm. long in the museum at Rio de Janeiro.

Genus II. CENTRODORAS Eigenmann, gen. nov.

Type, *Doras brachiatus* Cope.

Eye in middle of head; fontanel not constricted, not continued as a groove; three scutes in contact with the dorsal plate, rapidly graduate to about the 6th, then narrow; dorsal spine with antrorse hooks in front, with straight to retrorse hooks behind; bones of the head striate. Width a trifle more than head; plates very numerous, 38–40; air-bladder cardiform, depressed, with numerous, fimbriated or tufted diverticula; a narrower second bladder or cœcum with very profusely tufted lateral diverticula which are brittle.

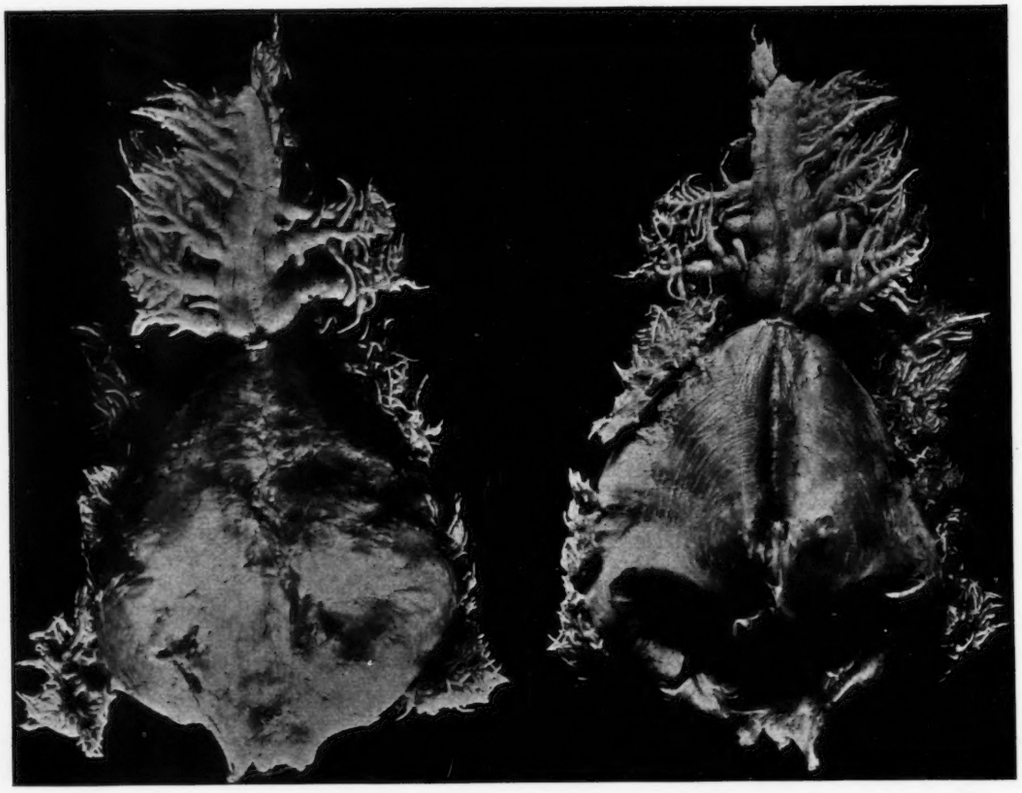


FIG. 16. Air bladder of *Centrodoras brachiatus* (Cope). Left from above, right from below.

4. **Centrodoras brachiatus** (Cope).

Doras brachiatus Cope, Proc. Acad. Sci. Phila., 1871, p. 270 (somewhere between mouth of Rio Negro and the Huallaga); Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 234; Ribeiro, Peixes, IV, 1911, p. 216.

Rhinodoras amazonum Steindachner, Sb. Ak. Wiss. Wien, LXXI, 1875, p. 141, Pl. II, 1875 (Amazon near Teffé); Ribeiro, Peixes, IV, 1911, p. 197, Fig. 94.

Oxydoras amazonum Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 159; Fisher, Ann. Carnegie Mus., XI, 1917, p. 421 (between Santarem and Pará).

Known from the types of *C. amazonum* and *C. brachiatus* the specimen recorded by Fisher and 16166 and 16167, 367 and 410 mm. Santarem market. Carl Ternetz.

Genus III. HOPLODORAS Eigenmann, gen. nov.

Type, *Doras uranoscopus* Eigenmann and Eigenmann.

This genus is very similar to *Megalodoras* in all outer appearances, but the air-bladder differs. The air-bladder is single, wider than long and with a single series of simple cœca or diverticula. As in *Megalodoras* the adipose fin is prolonged forward as a keel and the lateral scutes are large, few in number.

5. **Haplodoras uranoscopus** (Eigenmann and Eigenmann).

Plate 12, Figs. 1 and 2, Text-fig. 12C.

Doras uranoscopus E. & E., Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 159; Occasional Papers Cal. Acad. Sci., I, 1890, p. 228.

Known from the type in the Mus. Comp. Zoöl.; a specimen 530 mm. long from Lake Hyanuary.

Caudal fulcra laminate, the first a plate.

Snout 57 mm.; eye 19 mm.; postorbital part of head 50 mm.; distance from snout to dorsal fulcrum 185 mm.; length of dorsal 49 mm.; distance from end of dorsal to end of adipose 136; distance from adipose to end of caudal lobe 160 mm.

No groove on dorsal plate; a groove from the fontanel to the dorsal plate, interrupted at base of occipital process.

Head greatly depressed, the eyes directed upward more than sideways; fins with many small spots; scutes 18; dorsal spine serrate in front, with a few hooks on its posterior margin; gill-opening extending to a point midway between upper angle of gill-opening and eye.

Genus IV. PTERODORAS Bleeker.

Pterodoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, pp. 16 and 86.

Type, *Doras granulosus* Valenciennes.

Eye small, far in advance of middle of the head; palatine bone very small; the first suborbital rudimentary, the second small, attached for its entire length along the lower surface of the thick preorbital; preorbital in contact with the premaxillary and maxillary; adipose fin prolonged forward as a keel; dorsal spine strongly serrate on both margins; lateral scutes low, 22 + 29, directly continuous (in line) with those on the pseudotympanum; caudal forked; caudal peduncle naked above and below; head depressed, snout broad; fontanel continued as a groove to the tip of the occipital process.

No posterior air-bladder (5 and 6, Plate 7); anterior air-bladder heart-shaped, with a wide fringe of tubules along the sides; a plume-like diverticulum with a double series of tubules from the anterior outer angle and curving around the anterior end, the tips of the two feather-like structures meeting near the middle in front; 3 or 4 series of short wart-like prolongations on the upper surface between the lateral diverticula, gradually reduced to a single series on the side; a bony stay or septum from the temporal plate to the process of the dorsal plate (9*a*, Figs. 3, 4, 5, Plate 7), separating the musculature above from the cavity in which the air-bladder lies.

Very similar to *Megalodoras*, differing in having narrower, more numerous lateral scutes, a very different air-bladder, and a bony stay from the epiotic, behind the tympanum (toward vertebræ). This is the only genus with simple barbels having a bony stay connecting the epiotic with the first spine-bearing scute.

6. **Pterodoras granulosus** (Valenciennes).

Plate 1, Figs. 1–3; Plate 3, Fig. 6; Plate 8, Figs. 1–5. Text-fig. 17.

Doras granulosus Valenciennes, in Humboldt, Rec. d'Observ. de Zoöl. et d'Anat. Comp., II, 1811, p. 184; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 229 (Arary?, Uruguay; Buenos Ayres; Serpa); Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 392; Mem. Carnegie Mus., V, 1912, p. 185; Fisher, Ann. Carnegie Mus., XI, 1917, p. 419 (Santarem; Rio Mamoré).

Pterodoras granulosus Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 15 (name only); Silures de Suriname, 1864, p. 36 (Surinam).

Doras maculatus Valenciennes, in d'Orbigny, Voy. Am. Mer., V, II, 1847, p. 7, Pl. 5, Fig. 3; Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 281 (Buenos Ayres); Müller and Troschel, in Schomburgk, Reisen, III, 1848, p. 629; Steindachner, Denkschr. Akad. Wiss. Wien, XLI, 1879, p. 47 (Rio de la Plata); Eigenmann and Eigenmann, Proc. Cal. Acad. Sci. (2), I, 1888, p. 150 (Arary?); Occasional Papers Cal. Acad. Sci., I, 1890, p. 229 (Arary?; Uruguay; Buenos Ayres; Serpa); Ribeiro, Peixes, IV, 1911, p. 214.

Doras murica (ex Natterer, MS.) Kner, Sb. Akad. Wiss. Wien, XVII, 1885, p. 129 (Cujaba).

Doras muricus Günther, Catalogue, V, 1864, p. 202 (Demerara?).

Specimens Examined.

9836, I. U. M. 1, 127 mm. Asuncion, Paraguay. J. D. Anisits.
16175, I. U. M. 3, 213 to 231 mm. Belem (Pará) Fish market. Carl Ternetz. May 1924.
15659, I. U. M. 2, 95–117 mm. Marañon below Pastaza. Allen. Oct. 1920.
15846, I. U. M. 2, 203–216 mm. R. Ucayali, Contamana. Allen. July 1920.
15847, I. U. M. 14, 87–182 mm. Iquitos. Allen. Sept. 1920.
15848, I. U. M. 4, 150–200 mm. R. Puinahua, mouth of R. Pacaya. Allen. Sept. 1920.

Head 4 to 4.33; D. I, 6; A. 13; scutes 1 or 2 + 23 to 28; width at clavicle a little over 3 in the length; eye 7 in the head, 2.1 in the snout, 3 in interorbital in the young, respectively 10, 7.5, 4.5 in the old; distance between the nasal openings from each other at least twice as great as the anterior from the lip; mouth terminal, broad, the upper jaw a little the longer, teeth moderate; gill-rakers minute; maxillary barbel reaching about to tip of humeral process.

Profusely spotted.

The suborbital ring of this species points to an ancestry with a much larger eye than the present.

It is very desirable that the air-bladder of specimens from La Plata be examined.

7. **Pterodoras lentiginosus** (Eigenmann).

Text-figs. 17 and 18.

Doras lentiginosus Eigenmann, Ann. Carnegie Mus., XI, p. 401, 2 figures, 1917. (Santarem; Manaos.)
7049, C. M. paratype, 250 mm. Haseman.

This species is known from two specimens in the Carnegie Museum. It differs but slightly if at all from *granulosus*, the head being slightly longer.

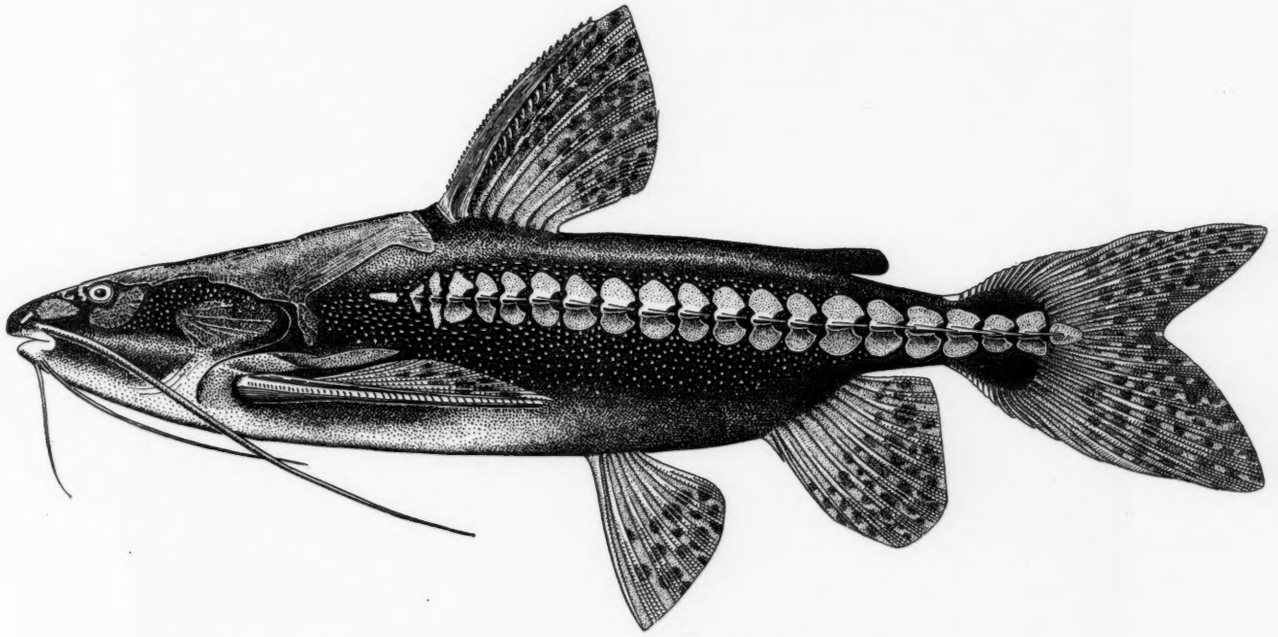


FIG. 17. *Pterodoras lentiginosus* (Eigenmann). From the type in the Carnegie Museum. Kennedy, del.

Genus V. CENTROCHIR Agassiz.

Centrochir Agassiz, Gen. et Spec. Pisc. Bras., 1829, p. 14.

Type, *Doras crocodili* Humboldt.

Width at clavicle more than length of head; lateral plates numerous, the first two or three in contact with the dorsal plate, graduate to the last, each with a simple, central hook; adipose fin continued forward as a keel; head not depressed, arched behind the fontanel, the bones finely granular; snout depressed, conical, mouth narrow, terminal; teeth well developed, in bands; serration on posterior margin of the dorsal stronger than that on anterior; fontanel continued or not continued as a groove; eyes moderate, lateral, in center or a little behind center of head; anterior nostril near the lip; caudal forked; preorbital overlying and in contact with the palatine, not reaching the maxillary and premaxillary in front, its free margin feebly but

regularly notched; first suborbital under the preorbital, the second and third of about equal width, entirely in orbital border.

The air-bladder deep, heart-shaped, without diverticula, no second air-bladder (4, Plate 8; 14, Plate 2).

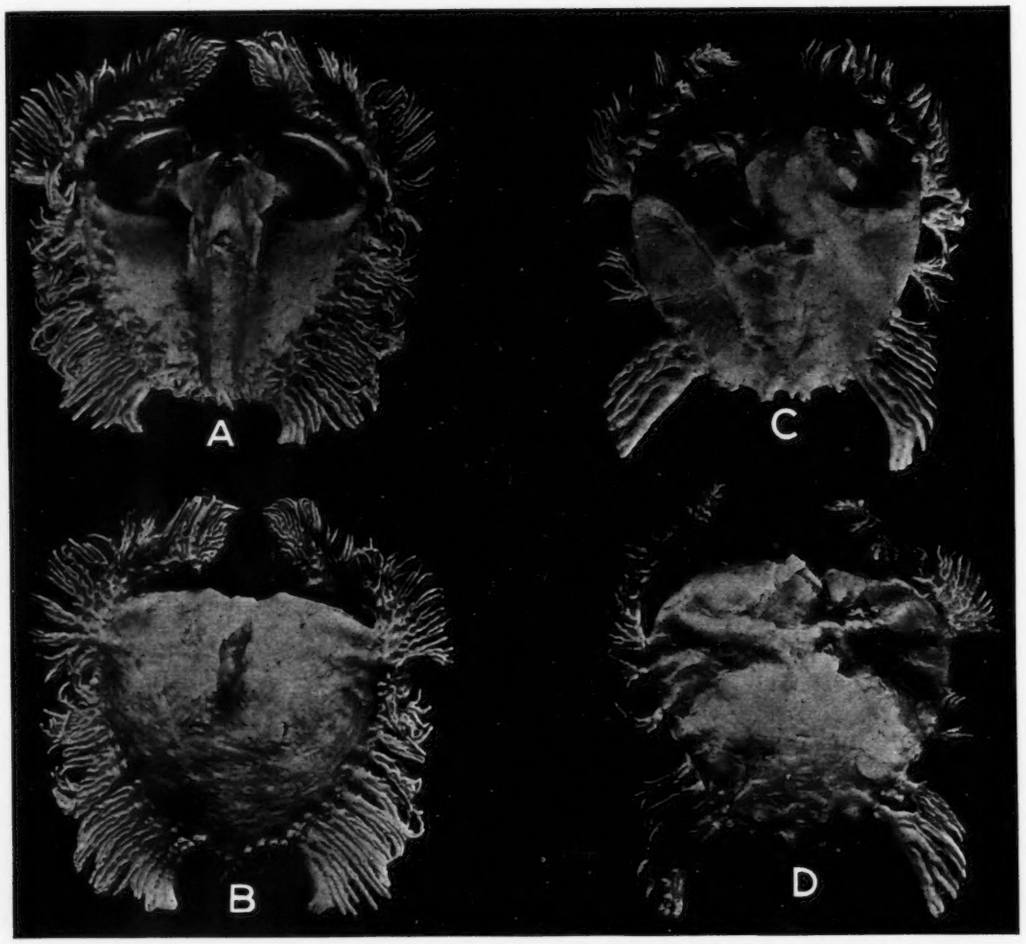


FIG. 18. Air bladders of *Pterodoras*. *A*, *lentiginosus*, dorsal view. *B*, *lentiginosus*, ventral view. *C*, *granulosus*, dorsal view. *D*, *granulosus*, ventral view.

Except on the obvious external characters this description is based on *C. crocodili*.

In the absence of the second air-bladder this genus differs from its nearest relatives.

Habitat: Magdalena and Amazon Basins.

KEY TO THE SPECIES OF CENTROCHIR.

a. Depth about 4.2; scutes about 30; opercle granular; caudal with parallel stripes; dorsal and ventral surfaces of the caudal peduncle partly covered by the fulcra.....8. **crocodili** (Humboldt).

8. **Centrochir crocodili** (Humboldt).

Plate 3, Fig. 4; Plate 5, Figs. 5 and 6; Plate 9, Figs. 4 and 5; Text-fig. 12.

Doras crocodili Humboldt, Rec. d'Observ. de Zoöl. et d'Anat. Comp., II, 1811, p. 181; Eigenmann, Mem. Carnegie Mus., IX, 1922, p. 46 (Soplaviento; El Banco; Puerto Wilches; Puerto Berrio).

Doras longispinis Steindachner, Fisch-Fauna des Magdalenen stromes, 1878, p. 23, Pl. IV, Fig. 2, Pl. V, Fig. 1 (Magdalena Basin).

Specimens Examined.

13555, I. U. M. 154–209 mm. El Banco, Rio Magdalena. Eigenmann.
13556, I. U. M. 141–194 mm. Puerto Berrio, Rio Magdalena. Eigenmann.
13557, I. U. M. 115–210 mm. Soplaviento, Rio Magdalena. Eigenmann.

Genus VI. PLATYDORAS Bleeker.

Doras Lacépède, Hist. Nat. Poiss., V, 1803, p. 116 (*carinatus* and *costatus*).
Doras Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 267 (*carinatus* and *costatus*).
Platydoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, pp. 16 and 86 (*costatus*).

Type, *Doras costatus* Linnæus.

Width at clavicle longer than head, over 3.6 in the length; lateral scutes numerous, covering most of the sides over the anal and forward; in contact with modified caudal fulcra above and below on the caudal peduncle, the three first in contact with the dorsal plate. Adipose fin continued forward as a keel; head not much depressed, arched in the occiput; snout moderately depressed; mouth subterminal, the lower jaw a little the shorter; teeth well developed, in bands, those of the outer row enlarged in the young; serration on posterior margin of the dorsal spine weaker than on the anterior, disappearing with age; bones of the head, opercles, nasals and suborbital pitted-granular; opercle striate; eye lateral, in about the middle of the head; anterior nostrils close to the lips; fontanel without constriction; caudal emarginate; preorbital in contact with the maxillary and premaxillary in front; the first suborbital between the preorbital and the second suborbital; second suborbital little wider than the third, the two forming the lower margin of the orbit.

Air-bladder double (3, Plate 2; 2, Plate 9), the anterior heart-shaped, with a median longitudinal groove, without any cœca; posterior bladder subconical, with a rounded base, without constriction or cœca.

Habitat: San Francisco, Paraguay, Amazon, and Orinoco basins; Guianas. The most widely distributed genus.

KEY TO THE SPECIES OF PLATYDORAS.

a. A conspicuous light band from above the eyes along the middle of the sides through the caudal; scutes 28–31, leaving but a narrow naked area along the back; maxillary barbel reaching second fourth of pectoral spine or tip of humeral process.....9. **costatus** (Linnæus).

aa. Sides plain or spotted.

b. Dark brown, rows of large white spots above and below the lateral line; smaller white spots on belly and caudal; dorsal black with a few large white spots; maxillary with black and white rings; outer surface of humeral process with two series of spines, reaching fourth lateral scute; 29 lateral scutes; maxillary barbel reaching pectoral.....10. **albomaculatus** (Peters).

bb. Uniform blackish; dorsal fin white, its middle black; base of anal and two posterior rays white; humeral process without spines, extending to last third of pectoral; 32 to 34 lateral scutes, those on tail only half as deep as the tail; maxillary barbel reaching middle of pectoral spine; pectoral reaching ventral.....11. **helicophilus** (Günther).

bbb. Brown, lighter below; a dark brown spot behind each lateral hook; caudal lobes with dusky bands; humeral process reaching second lateral scute, with two series of spines; 31 scutes; maxillary barbels to last third of pectoral spine.....12. **dentatus** (Kner).

9. **Platydoras costatus** (Linnæus).

Plate 1, Figs. 5 and 14; Plate 3, Figs. 1 and 2; Plate 5, Figs. 1 and 2; Plate 9, Figs. 1–3.

Silurus costatus Linnæus, Syst. Nat., ed. 12, I, 1766, p. 506.

Cataphractus costatus Bloch, Ausl. Fische, VIII, 1794, p. 82, Pl. 376.

Doras costatus Lacépède, Hist. Nat. Poiss., V, 1803, p. 116, part (South America); Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 268 (Guiana); Castelnau, Anim. Am. Sud. Poiss., 1855, p. 48 (Amazon); Günther, Cat. Fish. Brit. Mus.,V, 1864, p. 201 (British Guiana; River Cupai); Eigenmann and Eigenmann, Proc. Cal. Acad. Sci. (2), I, 1888, p. 161 (Rio Preto; Rio Puty; San Gonçallo; Xingu Cascade; Obidos; Gurupa; Teffé); Occasional Papers Cal. Acad. Sci., I, 1890, p. 231; Perugia, Ann. Mus. Genova (2), 1891, p. 34 (Villa Maria, Paraguay); Kindle, Ann. N. Y. Acad. Sci., VIII, 1895, p. 251 (Trocera on the Tocantins); Eigenmann and Kennedy, Proc. Acad. Nat. Sci. Phila., 1903, p. 500 (Paraguay); Eigenmann, Ann. Carnegie Mus., XXXI, 1907, p. 116 (Corumba; Laguna Ipacarai); Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 393; Ribeiro, Peixes, IV, 1911, p. 210; Eigenmann, Mem. Carnegie Mus., V, 1912, p. 185 (Twoca Pan and Gluck Island, British Guiana; Calobozo); Fisher, Ann. Carnegie Mus., XI, 1917, p. 419 (Maciél, Rio Guaporé; San Joaquin, Bolivia; Santarem; Corumbá; Puerto Suarez; Rio Jaurú, Paraguay Basin).

Platydoras costatus Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 16 (name only); Silures de Suriname, 1864, p. 38 (Surinam).

Doras cataphractus (not of Linnæus) Schomburgk, Fishes Brit. Guiana, I, 1841, p. 158 (Rio Negro).

Doras armatulus Müller and Troschel, in Schomburgk, Reisen, III, 1848, p. 629 (Rupununi; Awaricuar).

Specimens Examined.

16170, I. U. M. 99, 118 and 132 mm. Porto Nacional, Rio Tocantins. Feb. 1924. Carl Ternetz.

16171, I. U. M. 184 mm. Peixe, Rio Tocantins. Carl Ternetz.

16187, I. U. M. 138 to 155 mm. Santarem market. Sept. 1924. Carl Ternetz.

17041, I. U. M. 2, 185 and 217 mm. to end of middle caudal rays; Lake Rogoagua, Bolivia. Pearson.

12030, I. U. M. 1, 100 mm. Twoca Pan, Brit. Guiana. Wm. Grant.

12031, I. U. M. 1, 194 mm. Gluck Island, Brit. Guiana. Eigenmann.

15874, I. U. M. 3, 59–81 mm. Yarinacocha, Peru. Allen. Sept. 1920.

15875, I. U. M. 5, 77–81 mm. Rio Morona, Peru. Allen. Oct. 1920.

10139, I. U. M. 1, 88 mm. Corumba; Paraguay; Anisits.

D. I, 6; A. 11; scutes 2 or 3 + 28 to 30. Eye 4 to 6 in the head, 1.5–2.75 in the snout, 1.3 to 2.75 in the interorbital; bones of head granular; mouth terminal, rather narrow; teeth well developed, in narrow bands, the outer row of the upper jaw enlarged in the young; maxillary barbel to the tip of the humeral process.

A pair of lateral white bands, diverging from the fontanel, passing along the hooks of the scutes to the tip of the middle caudal rays; dorsal with a black spot near the tips of the anterior rays; a pair of black bands along the caudal lobes.

Pectoral spine and innermost rays white, the middle rays black; anal and ventrals with a large black spot; a light line forward from dorsal spine.

10. **Platydoras albomaculatus** (Peters).

Doras albomaculatus Peters, Mb. Ak. Wiss. Berlin, 1877, p. 470 (Calobozo).

Known only from the types, two specimens, the larger 70 mm. to base of caudal; in the Berlin Museum.

11. **Platydoras helicophilus** (Günther).

Doras helicophilus Günther, Proc. Zoöl. Soc., London, XXXVII, 1868, p. 229 (Surinam).

Known only from the types, three specimens, 350 mm. long, in the British Museum.

12. **Platydoras dentatus** (Kner).

Doras dentatus Kner, Sb. K. Akad. Wiss. Wien, XVII, 1855, p. 118, Pl. III, Fig. 3 (Surinam).

Known from the type, about 135 mm. long, in the Vienna Museum. It is possibly the common *P. costatus*.

Genus VII. FRANCISCODORAS Eigenmann, gen. nov.

Type, *Doras marmoratus* Reinhardt.

Adipose fin high, longer than anal, not continued as a keel, or for a very short distance only; dorsal spine serrate in front only; eye in front of middle of head; caudal peduncle nearly completely covered with modified fulcra above and below, the fulcra not in contact with the low lateral plates; fontanel not continued as a groove behind the middle of the head; caudal deeply forked or deeply emarginate; breast covered with skin; scutes large, heavy toward the end of the series.

In the general appearance, the nature of the skull, the single air-bladder, the anterior scutes, etc., this genus resembles *Centrochir*.

13. **Franciscodoras marmoratus** (Reinhardt).

Plate 3, Fig. 7; Plate 22, Fig. 1; Text-fig. 12, B.

Doras marmoratus (Reinhardt, MS.) Lütken, Dan. Vidensk. Selsk. Skr., 1874, p. 30 (Rio das Velhas); id., l.c., 1875, p. 146, Pl. I, Fig. 1 (Rio das Velhas); Steindachner, Sb. Ak. Wiss. Wien, LXXI, 1875, p. 147, Pl. IV; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 237; Ribeiro, Peixes, IV, 1911, p. 204, Pl. XXXVIII, Fig. 1; Fisher, Ann. Carnegie Mus., XI, 1917, p. 420 (Penedo; Cidadedo Barra; Joazeiro).

As far as is known this species is confined to the Rio San Francisco. I have examined some of the specimens in the Carnegie Museum.

Head 4.4; D. I, 6; A. 13; scutes 31 to 32, two scutes in contact with the dorsal plate; eye 2.25 to 2.5 in snout, 6 to 7 in head, 2.33 in interorbital.

Genus VIII. LITHODORAS Bleeker.

Lithodoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 84.

Type, *Doras dorsalis* Cuvier and Valenciennes.

Width at clavicle greater than length of head, about 3.5 in the length; scutes few, 18 to 20, very narrow, with a single hook, the first two in contact with the dorsal plate; adipose fin not continued as a ridge; snout truncate, not depressed, the occiput roof-shaped; mouth terminal, the teeth well developed, in bands; bones almost smooth; fontanel continued as a groove to the end of the nuchal plate; serræ on the posterior margin of the dorsal larger than those of the anterior; eye large, lateral, its center a little in advance of the middle of the head; suborbital bone large, reaching the premaxillary and maxillary in front, with divergent ridges ending in points along its free margin. Second suborbital narrower than the third, the first appearing as a granular ridge below the preorbital; preopercle smooth; all the naked parts of the body becoming more or less covered with granular scales with age.

14. **Lithodoras dorsalis** (Valenciennes).

Plate 2, Fig. 7; Plate 9, Fig. 6.

Doras carinatus Valenciennes, in Humb. Rec. d'Observ. Zoöl. et d'Anat. Comp., 1811, II, p. 184 (not *Silurus carinatus* L.).

Doras dorsalis Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 284 (Cayenne); Kner, Sb. Ak. Wien, XVII, 1855, p. 128 (Para); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 205 (copied); Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 159; Occasional Papers Cal. Acad. Sci., 1890, p. 225 (Pará); Ribeiro, Peixes, IV, 1911, p. 212, Pl. XXXIX (Tabatinga; [1] Pará); Fisher, Ann. Carnegie Mus., XI, 1917, p. 419 (Pará).

[1] Ribeiro records from Tabatinga a specimen 810 mm. to the last scute, 960 mm. to the end of the caudal. This is the largest Doradid on record.

Doras papilionatus Filippi, in Guér. Ménev. Rev. and Mag. Zoöl., 1853, p. 167 (Amazons); Günther, l.c. (copied).

Doras lithogaster Heckel, MS. in Kner, Sb. Ak. Wien, XVII, 1855, p. 132 (Forte do Rio Branco); Günther, l.c. (copied).

Lithodoras lithogaster Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 15 (name only).

Habitat: Para; Rio Negro; Cayenne.

Specimens Examined.

4248, I. U. M. one, 150 mm. Pará. Thayer Expedition?
4250, I. U. M. one, 171 mm. Brazil. Thayer Expedition.
16174, I. U. M. three, 128–164 mm. Belem, Pará, market. 1924. C. Ternetz.

Head 4.33; D. I, 6; A. 13 or 14; scutes 2 + 16 to 20.

Snout short, not greatly depressed; occipital region distinctly roof-shaped.

Eye large, lateral, 5.33 in head, 2 in snout, 2 in interorbital; the posterior margin of the pupil in the middle of the head. End of fontanel a little in advance of the posterior margin of the eye; a distinct groove from fontanel to the dorsal plate.

Gill-opening extending down to the middle of the pectoral spine. Isthmus almost equal to the length of the head; second scute about equal to the snout, 5th about six-tenths as long; two feeble plates in front of the hook-bearing ones on a straight line with the rest; caudal forked; mouth large, terminal; the lower jaw a little shorter than the upper, teeth in bands, strong; width in front of the pectoral a little greater than length of head; adipose equals anal, or shorter.

Genus IX. Acanthodoras Bleeker.

Type, *Silurus cataphractus* Linnæus.

Width at clavicle, 2.3 to 2.75 in the length; lateral plates numerous, covering more than half of the sides; no plates on upper or lower surface of caudal peduncle; adipose fin not continued forward as a keel; head depressed, the nuchal region but slightly arched; mouth terminal, the mouth wider than the interorbital; teeth well developed, in bands; no serræ on posterior margin of the dorsal spine; skull finely granular (the granules along its edge, larger, sharper in *spinosissimus*); fontanel subcircular, not continued as a groove; eyes very small, lateral, far in advance of the middle of the head; anterior nostril very near the lip; caudal rounded; opercle finely striate in *cataphractus*, coarsely granular in *spinosissimus;* preopercle and interopercle similar; preorbital short, in contact with the premaxillary in front, expanded, fan-like behind, with backward directed serræ on the outer half of the expanded edge; first suborbital small, along the lower surface of the nasal, second and third of about

equal size (with sharp granules, similar to those along the edge of the skull above the eye in *spinosissimus*).

Coracoid entirely covered with skin or the tip finely granular.

First air-bladder short, heart-shaped; no trace of a second air-bladder.

Habitat: Amazon and Orinoco Basins; Guianas.

KEY TO THE SPECIES OF ACANTHODORAS.

a. A light line along the median spines of the lateral scutes and forward to, or near to, the eye; no plates on caudal peduncle above or below.

b. Dorsal spine with a series of thorns along its anterior margin; lateral scutes 30, with a median hook only, covering one third of the side, not meeting above or below on the caudal peduncle; maxillary barbel reaching near tip of humeral process.....15. **calderonensis** (Vaillant).

bb. Dorsal spine with series of thorns on the front and on the sides; lateral scutes covering more than half of the sides.

c. Last four or five pairs of scutes separated by the narrow laminate caudal fulcra, the anterior plates of the caudal peduncle not quite meeting above and below; last scute narrow, with one hook; maxillary barbel to the tip of the humeral process; black, a light median line on top of head; a light line from posterior nostril along the edge of the granular part of the head and back along the hooks of the lateral scutes to the caudal; fins barred with black and white.

16. **cataphractus** (Linnæus).

cc. Lateral plates of the caudal peduncle meeting above and below, only the last one separated from its fellow by the caudal fulcra; last plate with several spines above and below the median hook; maxillary barbel reaching to the origin of the humeral process; black, a light line on top of head and a series of light spots along the middle of the back; lateral band similar to that of *cataphractus;* side of head and belly with conspicuous light spots; fins barred or spotted.

17. **spinosissimus** (Eigenmann and Eigenmann).

15. **Acanthodoras calderonensis** (Vaillant).

Plate 10, Fig. 5.

Doras calderonensis Vaillant, Bull. Soc. Philom., Ser. 7, IV, 1880, p. 154 (Calderon); Eigenmann and Eigenmann, Occasional Papers, Cal. Acad. Sci., I, 1890, p. 234; Ribeiro, Peixes, IV, 1911, p. 206, Fig. 97.

Doras (*Rhinodoras*) *depressus* Steindachner, Flussf. Südam., II, 1881, 1, Pl. I, Figs. 3–3*a* (Lago Alexo).

Known from the types of *calderonensis* and *depressus,* the former in the Paris Museum, the latter, one specimen, 103 mm., from Lago Alexo, in the Vienna Museum.

16. **Acanthodoras cataphractus** (Linnæus).

Plate 4, Fig. 5; Plate 10, Figs. 1–4.

Silurus cataphractus Linnæus, Syst. Nat., ed. 10, I, 1758, p. 307; ed. 12, I, 1766, p. 506.

Doras cataphractus Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 276 (?); Kner, Sb. Akad. Wiss. Wien, XVII, 1855, p. 126 (Rio Guaporé; Barra do Rio Negro); Bleeker, Ichth. Arch. Ind. Prodr., I, 1858, p. 54; Günther, Cat. Fish. Brit. Mus., V, 1864, p. 204 (?); Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 234; Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, 393; Ribeiro, Peixes, IV, 1911, p. 208; Fisher, Ann. Carnegie Mus., XI, 1917, p. 419 (Maciél, Rio Guaporé).

Acanthodoras cataphractus Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 17; Silures de Suriname, 1864, p. 40 (Surinam); Eigenmann, Mem. Carnegie Mus., V, 1912, p. 188 (Kangaruma; Georgetown; Gluck Island).

Cataphractus americanus Bloch and Schneider, Syst. Ichth., 1801, p. 107, Pl. 28; Lacépède, Hist. Nat. Poiss., V, 1803, p. 124, 127 (Carolina?).

Doras blochii Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 277 (copied).

? *Doras castaneo-ventris* Schomburgk, Fishes Brit. Guiana, I, 1841, p. 161, Pl. 3 (Rio Pasawiri).

? *Doras brunnescens* Schomburgk, Fishes Brit. Guiana, I, 1841, p. 163 (Upper Essequibo).

Doras polyramma et *polygramma* (Ex Heckel, MS.) Kner, Sb. Akad. Wiss. Wien, XVII, 1855, pp. 126, 127.

Callichthys asper Gronow, Cat. Fish., ed. Gray, 1854, 157.

12032, I. U. M. 5, 58–88 mm. Gluck Island. Eigenmann.

Head 3.8 in the length; D. I, 5; A. 10 or 11; scutes 2 or 3 + 22 to 27. Eye 9 to 10 in head, 2.5 to 3 in snout, 3.5 to 4.5 in interorbital; fontanel subcircular, top of head scarcely arched; nuchal very little steeper; width at the clavicle equals distance from snout to dorsal plate, 2.6 to 2.75 in the length; mouth terminal; teeth numerous, bristle-like, in narrow bands.

Last scute of the caudal peduncle not extending over the entire depth of the peduncle, with a median spine only.

17. **Acanthodoras spinosissimus** (Eigenmann and Eigenmann).

Plate 2, Fig. 3; Plate 11, Figs. 1 to 4; Text-fig. 12.

Doras spinosissimus Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., 1888, p. 161; Occasional Papers Cal. Acad. Sci., I, 1890, p. 235 (Coary); Ribeiro, Peixes, IV, 1911, p. 209; Fisher, Ann. Carnegie Mus., XI, 1917, p. 420 (Maciél, Rio Guaporé). Ahl, Blätter Aquarien und Terrarienkunde 33, 1922, No. 1.

The type, a specimen 150 mm. long, in the Mus. Comp. Zoöl.

7167, C. M., 16005, I. U. M. 6, 50–111 mm. Maciél, Rio Guaporé. Haseman.

Head 3.6; D. I, 5; A. 11; scutes 21 to 23; eye 7.5 in head, 2.33 in snout, 3.5 in interorbital; skull very similar to *cataphractus* but with coarser granulations; jaws equal, mouth terminal, with well-developed teeth; width at clavicle 2.33–2.5 in the length; maxillary barbel about reaching pectoral; the last four scutes extending entirely across the caudal peduncle, with numerous spines; the last two separated above by the fulcra; the next two or three pairs in contact above and below; several pairs in front of the adipose in contact across the back.

Black, a light band along top of head; four light spots between dorsal and caudal along the back; a light band from upper edge of opercle to base of caudal; dorsal and pectoral with white spots or partial bars; ventral surface black, with irregular white spots; barbels all black and white banded.

Genus X. Agamyxis Cope.

Agamyxis Cope, Proc. Am. Philos. Soc., XVII, 1878, p. 101.

Type, *Agamyxis pectinifrons* Cope.

Eye moderate, entirely in anterior half of head, surrounded by serrate ridges; adipose fin not continued forward as a ridge; dorsal spine as long as pectoral spine, serrate in front and on the sides, smooth behind; lateral scutes covering entire side of caudal peduncle and more than upper half of area between anal and tip of humeral process; caudal peduncle with spiniferous plates above and below; nasal erect, pectinate; caudal rounded; breast covered with skin.

This genus differs from *Acanthodoras* largely in having the caudal peduncle covered with plates above and below.

Key to the Species of Agamyxis.

a. Black, lower parts spotted with white, dorsal, anal, ventral and caudal with a yellowish cross bar; head 3.6; pectoral spine 3.2; eye 1.75 in snout, 6 in head; orbital margin raised, granulate, or pectinate; preopercular margin granular, the angle serrate; maxillary barbel not quite reaching base of pectoral spine. Scutes 27; caudal peduncle with spiniferous plates above and below.

18. **pectinifrons** [1] (Cope).

aa. Dark brown with irregular yellow spots scattered over entire body and fins; pectoral spine with cross bars; head 3.2; pectoral spine 3; eye 7.6 in the head, about 2.3 in the snout; orbital margin scarcely raised, prenasal bone moderately so; maxillary barbel extending beyond origin of pectoral; a blunt keel along the middle of the head, strongest on occipital region, fading out and dividing forward; teeth on the anterior edge of the dorsal stronger than those on the sides; 3 + 26 scutes, 4 to 5 spiniferous plates above and below on the caudal peduncle, continuous with the fulcra; types 90 and 110 mm.

19. **flavopictus** (Steindachner).

18. **Agamyxis pectinifrons** (Cope).

Plate 16, Fig. 4.

Doras pectinifrons Cope, Proc. Am. Philos. Soc., XI, 1870, p. 568 (Pebas, Ecuador); Proc. Acad. Nat. Sci. Phila., 1871, Pl. 3; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 241 .

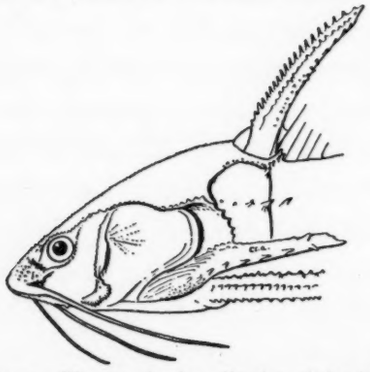


Fig. 19. *Agamyxis pectinifrons* Cope. The type in the collection of the Academy of Sciences, Philadelphia. Sketch of the type.

[1] In the original description Cope states "dorsal and pectoral spines elongate, both serrate before and behind."

Agamyxis pectinifrons Cope, Proc. Am. Philos. Soc., XVII, 1878, p. 679 (new genus).

Known from the type, 76 mm. long, in the collections of the Philadelphia Academy of Sciences.

19. **Agamyxis flavopictus** (Steindachner).

Doras (*Agamyxis*) *flavopictus* Steindachner, Sitzb. K. Akad. Wiss. Wien, 1908, No. VII, p. 3 (Iquitos).

Known from the types, two specimens 90 and 110 mm. long, from Iquitos in the Vienna Museum.

Genus XI. ASTRODORAS Bleeker.

Astrodoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 17.

Type, *Doras asterifrons* Kner.

Width at clavicle 2.6 in the length; lateral plates numerous, covering half or more than half of the sides; caudal fulcra laminate, covering about half the caudal peduncle above and below; adipose fin not continued forward as a keel; head depressed, nuchal region roof-shaped; mouth terminal, equal to interorbital; teeth in bands; serræ on anterior margin of the dorsal spine, none on the sides or behind; skull finely granular or striate; fontanel elongate, continued as an obscure, interrupted groove; eye moderate, in center or a little in front of center of the head; anterior nostril near lip; caudal emarginate; preorbital short, in contact with the anterior *horn of the ethmoid* in front, its expanded posterior edge erect, with about 10 spines; suborbitals narrow, of equal width, the middle one granular. The anterior air-bladder heart- to kidney-shaped, merging into the posterior bladder without constriction. The latter forked to its base, the horns turned laterad, the tips slightly forward in a specimen No. 16003 I. U., from Jutahy; consisting of a short, minute, unforked diverticulum in two specimens No. 7163 C. M., from Santarem.

20. **Astrodoras asterifrons** (Heckel).

Plate 1, Fig. 11; Plate 4, Figs. 2 and 4.

Doras asterifrons Heckel MS., in Kner, Sb. Ak. Wiss. Wien., XVII, 1855, p. 123, Plate II, Fig. 2 (Barra do Rio Negro; Guaporé); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 203 (Rio Cupai); Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., 2d Ser., I, 1890, p. 241 (Jutahy; Teffé; Porto do Moz; Serpa); Ribeiro, Peixes, 1911, p. 202, Fig. 96; Fisher, Ann. Carnegie Mus., XI, 1917, p. 420 (Maciél, Rio Guaporé; Santarem; San Joaquin).

Specimens Examined.

16003, I. U. M. one, 97 mm. to end of longest ray of upper caudal lobe. Jutahy. Thayer Expedition.
7163, C. M. 4, 67 to 89 mm. Santarem. Haseman.

Head 4; D. I, 6; A. 12; scutes 2 or 3 + 24 or 25, over half the depth; the rudimentary scutes on the tympanum above the line of the lateral hooks.

Eye 1.25 in snout, 5 in the head, 1.33 in the interorbital.

Width at clavicle slightly greater than distance from snout to posterior edge of dorsal plate; interorbital concave, the supercilian ridge raised; caudal fulcra laminate, the anterior ones above and below modified into a plate about the size of the eye, near the middle of the caudal peduncle; dorsal spine with two keels on each side; dorsal spine reaching nearly to or beyond origin of adipose; nasal erect, with about 12 spines; last three or four scutes with the median hook only; mouth terminal, teeth in narrow bands, the jaws equal; the abdominal area short; distance between tips of coracoids about 1.33 in distance from symphysis of coracoids to the anus; mottled ashen, darker above than below; fins spotted.

The difference in the air-bladders of the specimens from Jutahy and Santarem is so great that an unusually careful examination was made but yielded only the following differences:

a. Dorsal spine equal to its distance from the anterior edge of the eye; distance between the tips of the coracoids equals the distance between snout and end of dorsal plate; dorsal spine to middle of adipose. Second air-bladder consisting of two sausage-shaped tubes directed sideways. 16003, Jutahy specimen.

aa. Dorsal spine less than its distance from the anterior edge of the eye; distance between the tips of the coracoids equal to less than the distance from the symphysis of the coracoid to the base of the ventrals; greatest width a trifle less than the distance between the snout and the end of the dorsal plate; dorsal to or not quite to the adipose. Second air-bladder consisting of a short median protuberance. 7163 C. M., Santarem specimens.

Dorsal spine equals its distance from the nasal crest, reaching to the adipose according to the figure of the type.

Genus XII. Scorpiodoras Eigenmann, gen. nov.

Type, *Doras heckelii* Kner.

Having the characters of *Astrodoras*, except that the mouth is wider than the interorbital; nasal procumbent, the expanded edge raised but little; with more than 25 spines.

Anterior air-bladder kidney-shaped, large, its length about 1.5 in its width; posterior air-bladder banjo- or scorpion-shaped, the body of it heart-shaped, a posterior horn longer than the main part, recurved like the whip of a scorpion.

21. **Scorpiodoras heckeli** (Kner).

Plate 1, Fig. 30, Text-fig. 12, D.

Doras heckelii Kner, Sb. Akad. Wiss. Wien, XVII, 1855, p. 125, Fig. 4 (Rio Negro); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 204; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 243 (Jutahy; Tonantins; Teffé; Tabatinga); Ribeiro, Peixes, IV, 1911, p. 205, Pl. XXXVIII, Fig. 2.

Habitat: Amazon basin.

16002, I. U. M. one, 161 mm. Teffé. Thayer Expedition.

Head 4.33; width at the clavicle 3.14 in the length, 1.25 in the distance between snout and edge of dorsal plate; D. I, 6; A. 11, scutes 2.5–3 in the depth, 4 + 29, those on the tympanum above the line of the hooks on the scutes; last five scutes without supplementary hooks; mouth terminal, the jaws equal; teeth numerous; nasal suberect, curved so that it seems continuous with the suborbital, with about 25 spines; the superciliary edge not raised; caudal fulcra laminate, the anterior ones not enlarged; dorsal spine with several ridges on the sides, not reaching adipose.

Abdominal area long, the distance between the tips of the coracoid processes 2 in the distance between the coracoid symphysis and the anus. Maxillary barbel extending past tip of humeral process.

Rust colored, the fins spotted; caudal with two longitudinal bands; ventral surface speckled.

Genus XIII. AMBLYDORAS Bleeker.

Amblydoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 86 (*affinis*).
Zathorax Cope, Proc. Acad. Nat. Sci. Phila., 1872, p. 271.

Type, *Doras affinis* Kner = *Doras hancocki* Cuv. and Val.

Width at clavicles 3 in the length; coracoid very much wider than clavicle, the surfaces of the two granular, the space for the ventral part of pectoral muscles reduced to a narrow tube in the anterior part of the thoracic buckler; scutes numerous, covering more than half the sides; caudal fulcra not covering half the caudal peduncle; adipose fin not continued forward as a keel; dorsal spine without hooks or thorns, grooved and ridged on sides and front; head depressed, the skull arched, finely striate-granular, without a median groove; eye in middle of head; anterior nostril near the lip; mouth terminal, the teeth well developed, in narrow bands; fontanel elongate; preorbital in contact with the premaxillary in front, its spines (about 15) covered; suborbitals narrow, of uniform width.

Anterior air-bladder in *hancocki* short, heart-shaped, without cœca. No posterior air-bladder.

Habitat: Guianas; Amazon.

KEY TO THE SPECIES OF AMBLYDORAS.

a. Maxillary barbel to posterior margin of orbit; pectoral spine reaching beyond ventrals; width at the clavicle 2.66 in the length; pale brown, the pectoral spine spotted with dark.....22. **monitor** (Cope).

aa. Maxillary barbel reaching tip of humeral process; width at clavicle 3 in the length; pectoral spines reaching beyond base of ventrals; an irregular black band along sides below the median hooks. Three more or less distinct black blotches on the back, the last across the caudal peduncle.

23. **hancocki** (Cuvier and Valenciennes).

22. **Amblydoras monitor** (Cope).

Plate 16, Fig. 3; Text-fig. 20.

Zathorax monitor Cope, Proc. Acad. Nat. Sci. Phila., 1871, p. 271, Pl. 4, Fig. 1 (Amazon).
Doras monitor Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 245.

Known only from the type in the collection of the Philadelphia Academy of Sciences.

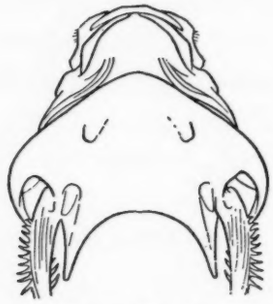


FIG. 20. *Amblydoras monitor* (Cope). Sketch of thorax and lower side of head.

23. **Amblydoras hancocki** (Cuvier and Valenciennes).

Plate 3, Fig. 9; Plate 13, Figs. 1 to 4.

Doras costata Hancock, Zoöl. Journ., IV, p. 242 (Demerara).
Doras hancockii Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 279; Günther, Cat. Fish. Brit. Mus., V, 1864, p. 202 (Demerara; Rio Cupai); Eigenmann, Mem. Carnegie Mus., V, 1912, p. 187 (Lama Stop-Off; Maduni Creek; Wismar; Rockstone; Gluck Island; Tumatumari; Rupununi Pan); Ribeiro, Peixes, IV, 1911, p. 210.
Doras affinis Kner, Sb. Ak. Wiss. Wien, XVII, 1855, 121, Pl. 11, Fig. 1 (Rio Branco; Guaporé); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 202; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 238; Ribeiro, Peixes, IV, 1911, p. 201, Fig. 75.
Amblydoras affinis Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 17.
Doras truncatus Bleeker, l.c., p. 18.

Habitat: Guiana to Guaporé.

Specimens Examined.

12033, I. U. M. 8, largest 110 mm. Lama Stop-Off near Georgetown, Brit. Guiana. Eigenmann.
12034, I. U. M. 2, 103 and 118 mm. Maduni Creek near Georgetown. Eigenmann.
12035, I. U. M. 3, about 37 mm. Rockstone on the Essequibo, Brit. Guiana. Eigenmann.
12036, I. U. M. one, 71 mm. Tumatumari on the Potaro, Brit. Guiana. Eigenmann.
12037, I. U. M. 2, 36 and 40 mm. Rupununi Pan, Brit. Guiana. Eigenmann.
15887, I. U. M. one, 65 mm. Itaya above Iquitos, Peru. Allen.
17040, I. U. M. 17, 60–98 mm. Lagoon near Reyes, Bolivia, Oct. 1921. Pearson.

Head 4 to 4.3; D. I, 6; A. 11 to 13; eye 4.5 in the head, 1.5 in the snout, 1.5 in the interorbital; width of the clavicle 3 in the length.

Maxillary barbel about to tip of humeral process; scutes about half as deep as the body; caudal fulcra laminate, none converted into a plate; caudal slightly emarginate; a light line along the scutes; a black stripe from humeral process to end of anal below the scutes; a black band across caudal behind the last scute, a number of saddle-shaped spots on middle of back; upper parts dark ashy with a few spots smaller than pupil; spots on dorsal and caudal.

Genus XIV. ANADORAS Eigenmann, gen. nov.

Type, *Doras grypus* Cope.

Width between clavicles less than 3 in the length; scutes numerous, covering less than one fourth of the sides; modified fulcra covering much of the caudal peduncle above and below.

Adipose fin short; not continued as a keel; dorsal spine grooved; without serræ; no plates between the dorsal and adipose; granulations on breast inconspicuous, narrow, or confined to the tip of the coracoid. Eye entirely in anterior part of the head; preorbital bone inconspicuous, snout depressed, mouth terminal; teeth minute, in bands; fontanel oval, not continued as a groove; head minutely granular.

First air-bladder heart-shaped, with a median, longitudinal groove, without marginal prolongations; second air-bladder reduced to a minute tubule (*weddellii*) or entirely suppressed (*regani*, *grypus*) (5, Text-fig. 2).

Habitat: Amazon Basin.

KEY TO THE SPECIES OF ANADORAS.

a. Sides with a pale lateral band.

b. Dorsal region copper-colored, with few faint or sepia spots; lateral plates yellow, the region below brownish yellow; caudal with dusky stripes along the upper and lower lobes; lateral plates 3 + 26–27, narrow, leaving wide naked areas above and below. Eye small, 6 in head, 2 in interorbital, 1.5 in snout; interorbital flat; maxillary barbel extending beyond base of pectoral; each lateral plate with one or two points above and below the median hook. Caudal peduncle with 7–10 bony plates, grading into the fulcra above and below; caudal emarginate; caudal lobes with violet bands. 90 mm.

24. **regani** (Steindachner).

bb. Three or more dark blotches, below the band, on a light background; the dark above the band with darker, light-edged, blotches; anterior parts of dorsal, anal and ventrals black, the posterior part light, spotted; middle and edges of caudal light, the base and a stripe along the lobes black; coracoid process flaring backward; distance from union of coracoids to the line uniting the tips one half to one third the distance between the tips; caudal deeply emarginate. . . . . 25. **grypus** (Cope).

bbb. Brown above, with a yellow lateral band; caudal with vertical dark bars. Orbit 1 in interorbital, 3 in head; maxillary barbels reaching end of pectoral spines; caudal truncate, scutes 26, leaving a broad naked band above and below, with only a single median spine. 82 mm.

26. **nauticus** (Cope).

aa. Sides without a lateral stripe.

c. Irregularly spotted; maxillary barbel reaching pectoral spine; opercle granular; eye 3 to 4 in interorbital; nasal without a serrated crest; fontanel oval, with a granular border; coracoid processes flaring but little; the distance between the juncture of the coracoids and a line connecting their tips $\frac{3}{5}$ to $\frac{5}{6}$ of the distance between the tips; outer face of striate portion of the coracoid convex; scutes 25–27; the caudal fulcra more plate-like than in *grypus*, covering most of the caudal peduncle.....27. **weddellii** (Castelnau).

cc. Yellowish with sepia markings, 4 bars on dorsal spine, 5 on pectoral, 4 on posterior part of body; maxillary barbel reaching tip of humeral process; opercle striate; nasal strongly pectinate; opercle reticulate; eye 1.75 in interorbital; scutes 29, with an auxiliary spine above and below; top of head striate; coracoid process reaching middle of pectoral spine; the humeral process beyond the third quarter of that spine; caudal peduncle with six plates...28. **insculptus** (Ribeiro).

24. **Anadoras regani** (Steindachner).

Text-fig. 21.

Doras regani Steindachner, Sitzb. K. Akad. Wiss. Wien, 1908, Nr. XI (Pará).

Known from four specimens, 90 mm. long, in the Vienna Museum, and 16191, I. U. M. 2, 123 and 140 mm. Santarem market. Sept. 1924. C. Ternetz.

FIG. 21. *Anadoras regani* (Steindachner). 140 mm. Photo by N. E. Pearson.

25. **Anadoras grypus** (Cope).

Plate 4, Fig. 7; Plate 15, Figs. 1 to 3.

Doras grypus Cope, Proc. Acad. Nat. Sci. Phila., 1871, p. 270, Pl. XV, Figs. 1–1*a* (Ambyiacu River).
Doras weddellii Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 239 (in part).

Specimens Examined.

4249, I. U. M. one, 129 mm. Teffé. Thayer Expedition.
15849, I. U. M. 50 to 122 mm. to end of middle caudal rays. Lake Cashiboya. Allen.
15961, I. U. M. 3, 80 to about 150 mm. Iquitos. Morris.

Head nearly 4 in the length; D. I, 6; A. about 12; width at clavicle nearly 3 in the length; eye 5.6 in the head, 1.75 in snout, 2.75 in interorbital; maxillary barbel reaching beyond base of pectoral spine; mouth terminal, the teeth in narrow bands;

scutes 26 or 27, narrow, about 3 in the depth; rudimentary scutes feeble, concealed; laminate caudal fulcra covering about half the caudal peduncle above and below; nasal bone not serrate; skull finely granular. For color see the key.

26. **Anadoras nauticus** (Cope).

Zathorax nauticus Cope, Proc. Acad. Nat. Sci. Phila., 1874, p. 133 (Nauta); Proc. Am. Philos. Soc., XVII, 1878, p. 678; Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 246.

Known from the type, 82 mm. long, in the Philadelphia Academy of Natural Sciences.

27. **Anadoras weddellii** (Castelnau).

Plate 4, Fig. 6; Plate 15, Fig. 4.

Doras weddellii Castelnau, Anim. Amer. Sud., 1855, p. 48, Pl. XVII, Fig. 1 (Chiquitos); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 203 (Santarem); Vaillant, Bull. Soc. Philom., Ser. 7, IV, 1880, p. 154 (Calderon); Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, 163 (in part); Occasional Papers Cal. Acad. Sci., I, 1890, p. 239 (the specimens with a restricted coracoidal striation) (Fonteboa; Teffé; Serpa; Porto do Moz; Silva, L. Saraca); Ribeiro, Peixes, IV, 1911, p. 202; Fisher, Ann. Carnegie Mus., XI, 1917 (Manaos; Santarem; Rio Jauru; Bastos, Rio Alegre and Maciél into Guaporé; San Joaquin, Bolivia).

Specimens Examined.

5110, I. U. M. 2, 112 and 113, the smaller a skeleton. Island Marajo. Thayer Exp.
10218, I. U. M. one, about 75 mm. to end of middle caudal rays. Rio Pilcomayo.

Head 3.66; clavicular width about 3 in the length; eye 7 in the head, 2 in the snout, 3 in the interorbital. D. I, 6; A. 12; scutes 3 + 26 or 27, one fourth or one fifth of the depth of the body; caudal fulcra heavy, laminate, reaching to the adipose and nearly to the anal; mouth terminal; teeth well developed, in bands; nasal not serrate.

28. **Anadoras insculptus** (Ribeiro).

Plate 14, Figs. 1 to 3.

Doras insculptus Ribeiro, Comm. Linhas. Telegr. Estrat. Matto Grosso as Amazonas, Annexo 5, 1912, p. 22 (Manaos).

Known from the types, the largest 100 mm., in the Mus. Nacional Rio de Janeiro. I am indebted for the figures to Dr. A. Ribeiro.

Genus XV. Hypodoras Eigenmann, gen. nov.

Type, *Hypodoras forficulatus* Eigenmann.

Adipose fin but little shorter than the anal fin, the anterior two thirds covered by a rhomboidal plate; caudal fulcra laminate, covering the entire caudal peduncle above and below; dorsal spine striate in front and on the side, without serræ; lateral scutes of the caudal peduncle in contact with the plates above and below, with a

single hook; those in front of the caudal peduncle with 2 to 5 hooks above the median one and with 1 to 2 below it; humeral process very strong, with a series of spines; process of coracoid striate; eye immediately in front of the middle of the head; preorbital crest pectinate; suborbital granular.

Anterior air-bladder (Fig. 4, Plate 2) wider than long, kidney-shaped, without marginal tentacles or cœca, with a median longitudinal constriction, directly continued into the posterior air-bladder, which, at the point of union, is about one third as wide as the anterior and forked (divided into two horns) in its posterior three fifths. The horns seem to be continuations of the two lateral lobes of the anterior bladder. One of the horns is turned sidewards.

This genus differs from all the others of the Doradidæ in the plate over the anterior part of the adipose and in the nature of its air-bladder.

29. **Hypodoras forficulatus** Eigenmann, spec. nov.

Plate 4, Fig. 1; Plate 25, Fig. 3; Text-fig. 12, I.

15876, I. U. M. one, *type*, 123 mm., 104 mm. to end of last scute. Iquitos. Allen. Sept. 1920.

This species has the general appearance of *Anadoras grypus* Cope but differs from *grypus* very notably. The data in parentheses refer to a specimen of *D. grypus*, 108 mm. long to the end of the last scute.

Head 3.8; depth 5.3; D. I, 6; A. 11; scutes 2 + 26. Top of head granulose, the fontanel not continued as a groove; prenasal bone deeply pectinate (smooth); humeral region very wide; equal to the distance from snout to dorsal fulcrum; interorbital equals snout (snout and eye); eye 2.66 in interorbital, 6 in the head; teeth minute, in bands; maxillary barbel extending a little beyond coracoid process, banded; outer mental barbel reaching coracoid, inner mental barbel considerably shorter; dorsal spine deeply striate in front and on the sides, smooth behind, a little over half as long as the heavy, serrate, pectoral spine, which reaches the middle of the ventrals; ventrals broad, rounded; anal rounded; caudal truncate; dorsal and pectorals lanceolate; humeral process very strong, reaching to below 4th dorsal ray, with teeth toward its tip equal to those of the pectoral which oppose them; coracoid process striate, narrow, reaching about to third fifth of the pectoral spine; caudal peduncle covered with plates below (naked); a single large plate between the adipose and the numerous caudal fulcra (none); a very large keeled plate over anterior part of the adipose (none); scutes very high (low), covering the entire sides, two or three from the dorsal plate to the humeral process, those of the sides crowded; the median spine moderate, with up to 5 smaller spines above the larger one and two, rarely three, below the main spine in the region of the anal (none); one or none on the caudal peduncle above and

below the central spine; the central spines of nearly uniform size from below the dorsal to the caudal; ashen, a narrow black bar from posterior half of dorsal down and back to a black spot at the end of the humeral process, then downward and forward to the edge of the belly; a wider bar downward and forward from the soft part of the adipose, then to and partly across the base of the anal; a bar across the middle of the caudal peduncle and another across the base of the caudal; dorsal black; the distal part hyaline, spotted with black; caudal, anal, ventrals and pectorals hyaline, with black spots, bases of caudal, anal and ventrals with a larger black spot; ventral surface dark, clouded.

Genus XVI. PHYSOPYXIS Cope.

Physopyxis Cope, Proc. Acad. Nat. Sci. Phila., 1871, pp. 112, 272, 273.

Type, *Physopyxis lyra* Cope.

Eye small, in anterior half of head; no adipose; dorsal spine serrate on lower half of anterior margin; pectoral spine extending beyond origin of anal; caudal truncate; clavicles and coracoid granular, developed into a large armor.

30. **Physopyxis lyra** Cope.

Plate 16, Figs. 1 and 2. Text-fig. 22.

Physopyxis lyra Cope, l.c. (Ambyiacu River).

This species is known from the type 35 mm. long in the collections of the Philadelphia Academy of Sciences.

FIG. 22. Dorsal spine of the type of *Physopyxis lyra*.

Genus XVII. PSEUDODORAS Bleeker.

Pseudodoras Bleeker, Ichthyol. Arch. Indici Siluri, 1858, p. 53 (to include *Doras niger* Valenciennes; *D. brevis* Kner, *D. humeralis* Kner; *D. lipophthalmus* K.).

Oxydoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 14 (type, *Doras niger* Val. = *Pseudodoras niger* Bleeker).

Type, *Doras niger* Valenciennes.

Bleeker in 1858 based his *Pseudodoras* on *Doras niger* Val., *D. brevis* Kner, *D. humeralis* Kner and *D. lipophthalmus* Kner, in the order named. This is a miscel-

laneous collection of fishes, evidently known to Bleeker only as a series of names. There is no evidence that Bleeker considered the *Doras niger* as anything except Valenciennes' species which should remain the type of the name *Pseudodoras*. If there is any doubt on this score it is removed by Bleeker (1863, p. 14) when he distinctly states that his *Pseudodoras niger* is the *Doras niger* of Val. and not the *Doras niger* of Kner.

In Nederl. Tijdsch. Dierk., I, 1863, pp. 12, 14, he endeavored to suppress his *Pseudodoras:* "De plus ma première espèce de *Pseudodoras* est le *Doras niger*, pour lequel le nom d'*Oxydoras* a droit de priorité, et les trois autre espèces, ainsi que le *Hemidoras stenopeltis* sont de vrais Doras, tel que je comprends actuellement le genre Lacépèdien, dont en effet le *Doras carinatus* espèce voisine, est le type."

Bleeker, however, is wrong in assuming that the name *Oxydoras* of Kner has priority to the name *Doras niger* Valenciennes. Kner did not know *Doras niger* of Valenciennes when he framed his name *Oxydoras*, he simply mistook specimens of another species for it. The *Doras niger* Val. cannot be made the type of *Oxydoras*. *Doras niger* of Kner is not the *Doras niger* of Valenciennes, a fact Bleeker had discovered by 1863. On p. 14, 1863, Bleeker makes *Doras niger* Val.—a species not known to Kner—the type of Kner's *Oxydoras*, but he reconsidered this and on page 85, he distinctly makes *Doras* (*Oxydoras*) *niger* Kner (= *Oxydoras kneri* Bleeker) the type of *Oxydoras*. As this was the first and a perfectly legitimate restriction of the genus *Oxydoras*, *O. kneri* should remain the type of *Oxydoras*. Since the *Doras niger* of Kner belongs to a genus distinct from the *Doras niger* of Valenciennes there is no reason why the name *Pseudodoras*, with *Doras niger* of Valenciennes as its type, should be suppressed and it is herewith resurrected.

Width at clavicles less than length of head; coracoid covered with skin; lateral plates few, narrow, 3 + 18 to 23; adipose fin continued forward as a keel; head not depressed, snout conical; mouth inferior; premaxillaries rhomboidal; teeth feeble; serration on posterior margin of dorsal spine weaker than on the anterior; occipital region roof-shaped, granular or striate, without a median groove; eyes lateral, entirely behind the middle of the head; anterior nostril remote from the lips; fontanel double; caudal emarginate; opercle striate; suborbital granular in old. Several tentacles across the roof of the mouth between the second and third pairs of gill-arches, two rows of tentacles converging from in front of the first arch forward, ending in a ridge; smaller ones on the floor of mouth opposite; preorbital comma-shaped, the front incurved, connected with the ethmoid. First suborbital in part under the preorbital, the second large, only half of it forming part of the orbital border, the third very slender (3, Plate 11).

Air-bladders of *P. niger* (Fig. 4, Plate 2) both well developed, with marginal indications of lobules or beginnings of cœca. Anterior bladder elongate, heart-shaped, its width 1.25 in its length. Posterior bladder elliptical, a little more than twice as long as wide; its length less than 2 in the length of the anterior.

The generic description is based on *P. niger*.

KEY TO THE SPECIES OF PSEUDODORAS.

a. Gill-opening extending far below pectoral spine; dorsal spine with feeble teeth on the distal part of the posterior margin, disappearing with age; no conspicuous caudal fulcra; occipital region steeply roof-shaped; eye in posterior half of head, 3–4 in interorbital; highest scutes 1.3–1.5 in interorbital; three small bones in front of the hook-bearing ones; width in front of pectoral equals head less one fifth to three fifths of opercle. (A small granular plate between the upper halves of the first two hook-bearing lateral scutes in the old.).....31. **niger** (Valenciennes).

aa. Gill-openings extending to the base of the pectoral spine; dorsal spine smooth behind (?); caudal fulcra extending halfway to anal and adipose; occipital region obtusely roof-shaped; eye just behind the middle of the head, 3 in the interorbital; scutes 16, high; two small bones in front of the spine-bearing scutes. A golden stripe along middle line from dorsal to end of adipose, an ill-defined dark stripe along the middle of the head; lateral parts of top of head, middle of opercle, lower part of cheeks and lateral plates golden.....32. **huberi** (Steindachner).

31. **Pseudodoras niger** (Valenciennes).

Plate 1, Fig. 16; Plate 17, Figs. 1–4; Plate 23, Fig. 1; Text-figs. 2, 6, 7 and 10.

Doras niger Valenciennes, in Humboldt, Rec. d'Observ. Zoöl. et Anat. Comp., II, 1811, p. 184; Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 291 (?); Schomburgk, Fishes Brit. Guiana, I, 1841, p. 165; Müller and Troschel, in Schomburgk, Reisen, III, 1848, p. 629 (Rivers of Guiana); Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 14 (name only); Eigenmann, Mem. Carnegie Mus., V, 1912, p. 190 (Rupununi).

Rhinodoras niger Günther, Cat. Fish. Brit. Mus., V, 1864, p. 209 (Amazons); Cope, Proc. Am. Philos. Soc., XVII, 1878, 678 (Nauta); Vaillant, Bull. Soc. Philom. (7), IV, 1880, p. 14 (Calderon).

Oxydoras niger Eigenmann and Eigenmann, Proc. Cal. Acad. Sci. (2), I, 1888, p. 159 (Teffé; Gurupa; Manacapuru; Coary; Obidos); Occasional Papers Cal. Acad. Sci., I, 1890, p. 247; Kindle, Ann. N. Y. Acad. Sci., VIII, 1894, p. 251 (Pará); Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 393; Ribeiro, Peixes, IV, 1911, p. 193, Pl. XXXVII; Fisher, Ann. Carnegie Mus., XI, 1917, p. 420 (Santarem; Manaos).

Doras humboldti Agassiz, Selecta Gen. et Spec. Pisc. Bras., 1829, p. 129, Pl. 5 (Rio San Francisco, Brazil); Agassiz, A Journey in Brazil, 1868, ——.

Corydoras edentatus Spix, Selecta Gen. et Spec. Pisc. Bras., 1829, Pl. 5.

Rhinodoras prionomus Cope, Proc. Acad. Nat. Sci. Phila., 1874, p. 134 (Nauta); Proc. Am. Philos. Soc., XVII, 1878, 678 (Nauta).

Rhinodoras teffeanus Steindachner, Sb. Akad. Wiss. Wien, LXXI, 1875, 145, Pl. 3 (Teffé).

This species is known from near sea level to about 3000 feet. In British Guiana, the Amazons from Pará westward to the Marañon, from the Rio San Francisco, and from the following specimens in the collections of Indiana University:

5105, I. U. M. one, 234 mm. to end of middle caudal rays. Pará. Thayer Exp.
15651, I. U. M. 15960, 4, 170–260 mm. Iquitos. Allen, 1920, Morris, 1922.
15450, I. U. M. one, about 250 mm. Yarinacocha. Allen. Sept. 1920.
15845, I. U. M. one, about 200 mm. Lake Cashiboya. Allen. 1920.
15850, I. U. M. one, skin, 202 mm. from snout to end of dorsal plate, along the median line; about 650 mm. over all. La Merced, Peru, 2500 feet. Sr. Ecuador Præli. 1918.
16004, I. U. M. one, 273 mm. to end of the scutes. Rio Pacaya. Allen.
743, T. R. S. one, skin, 965 mm. to end of caudal, Kartabo, Guiana. Beebe.[1]
16169, I. U. M. one, 453 mm. Santarem. 1924. Carl Ternetz.

LOCALITY: Kartabo, Bartica District, British Guiana. Date: 24, VIII, 1922.

Dimensions: From field measurements.	In mm.
Length	840
Head	245
Depth (in front of dorsal fin)	169
Greatest width (anterior to pectoral fins)	175
Depth of caudal peduncle	41
Width of caudal peduncle	53
Eye socket	25.2 x 16.5
Interorbital	77
Eye to posterior nostril	54
Asymmetrical:	
Right	32
Left	21
Interpostnasal	54.5
Dorsal spine	166
Pectoral spine	177
Width of mouth	88
Snout to dorsal	320
Snout to anal	570
Snout to pectoral	210
(The asymmetry in the following four measurements is due to accidental injury.)	
Right maxillary barbel	94
Left maxillary barbel	115
Left inner mental barbel	36
Left outer mental barbel	55
Pectoral	I, 9
Dorsal	I, 6
Ventrals	7
Anal	9
Weight	16 pounds.

Brownish black above, dirty white below. Fins and mouth black. Lateral scutes yellowish horn. Spines ivory white.

FOOD: Flower buds, petals, seeds.

EYE: Sunk beneath surface of head. Iris 15 mm. in diameter. Purplish brown, flecked with yellowish white, the coloration most prominent anteriorly and posteriorly. A narrow pupil rim, yellow, widest posteriorly, and sending downward on lower side of the eye a single irregular line of whitish yellow, passing halfway through the iris. The Creoles call this fish "Fork-snout," and claim that it is an adult *Hemidoras carinatus*, which is also known by that name. This specimen was caught in a drift net in square B7 at Kartabo.

[1] The following notes accompanied the large specimen taken by Dr. W. Beebe.
Specimen Number: T.R.S. 743. Skin only.

D. I, 6; A. 11, scutes 3 + 18 to 23; head 3 to 3.75 in the length to the last scute; eye 8 in the head, 4.5 in the snout, 3 in the interorbital in smallest, respectively 10.5, 5.5, 3.33 in the largest; from snout to dorsal fulcrum 2.4–2.66 in the length to end of scutes. Bones of head striate in young and half grown, becoming striate granular; premaxillaries subrhomboidal, about the size of eye. Teeth very feeble or none.

Dark brown, the fins black.

In the specimen from La Merced (Text-fig. 10), a small plate, similar to the tympanic plates is interpolated between the first two hook-bearing ones and in a line with the distant rudimentary ones. Scutes 3 + 1 + 1 + 17 or 18 (22 in the largest).

32. **Pseudodoras huberi** (Steindachner).

Oxydoras (*Rhinodoras*) *huberi* Steindachner, Sitzb. K. Akad. Wiss. Wien, 1911, CXX, p. 324 (Rio Tocantins at Cameta).

This species is known from the type, 400 mm. long, taken at Cameta by Dr. Snethlage. It is now in the Vienna Museum.

Head 3.4; D. I, 6; A. 12; scutes 2 + 17; eye $8\frac{3}{11}$ in the head, 4 in snout, about 3 in interorbital. Bones of head striate granular; bristle-like teeth in moderate bands; maxillary barbel to tip of humeral process; posterior nostril equidistant from the anterior and the eye; pectoral spine longer than the head.

Black, with a lighter line along the middle of the back, from dorsal to end of adipose, and brown to yellow markings on head and sides.

Genus XVIII. Oxydoras Kner.

Oxydoras Kner, Sb. Ak. Wiss. Wien, XVII, 1855 (various species: *D. stenopeltis* Kner; *carinatus* C. and V.; *niger* Kner, *non* C. and V.; *lipophthalmus* Kner; *d'Orbignyi* Krŏyer).

Oxydoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 85 (reprint, p. 9).

Type, *Oxydoras kneri* Bleeker, based on *Doras* (*Oxydoras*) *niger* Kner, *non Doras niger* Valenciennes.

The status of the name *Oxydoras* as restricted by Bleeker is given under *Pseudodoras*.

In 1863 Bleeker, p. 14, restricted the genus *Oxydoras* of Kner to specimens supposedly belonging to *Doras niger* Valenciennes. But Kner's specimens did not belong to *Doras niger* Val.

Later in the same year, p. 85, Bleeker restricted it by selecting as type the species named *niger* by Kner but which was, in reality, a distinct genus and species named *kneri* by Bleeker.

Similar to *Pseudodoras*, the number of lateral plates much larger, no tentacles on roof of mouth.

Habitat: Paraguay Basin.

33. **Oxydoras kneri** Bleeker.

Plate 1, Fig. 15.

Doras (Oxydoras) niger Kner (*non Doras niger* Val.), Sb. Ak. Wiss. Wien, XVII, 1855, p. 146 (Cujaba).

Oxydoras kneri Bleeker, Nederl. Tijdsch. Dierk., I, pp. 12 and 14 (substitute name for *Doras niger* Kner *non* Valenciennes); Eigenmann, Proc. Acad. Nat. Sci. Phila., 1903, p. 501 (Asuncion); Ribeiro, Peixes, IV, 1911, p. 194; Fisher, Ann. Carnegie Mus., XI, 1917, p. 420 (Corumba).

Rhinodoras knerii Günther, Cat. Fish. Brit. Mus., V, 1864, p. 209.

The type, 15 inches long, is in the Vienna Museum. It has 34 lateral scutes, those on the caudal peduncle deeper.

9840, one, 205 mm. to end of scutes. Asuncion. Ternetz.

Head 3.5; D. I, 6; A. 12; scutes 3 + 34–40; eye 10 in the length of the head, 5.5 in the snout, 3 in interorbital; distance from tip of snout to dorsal fulcrum 2.66 in the length to end of scutes; top of head and opercle striate-granular.

Genus XIX. Rhinodoras Bleeker.

Rhinodoras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 14.

Type, *Doras (Oxydoras) d'Orbignyi* Kröyer MS.

Eye medium, in middle of head; adipose fin prolonged forward as a keel; dorsal spine strongly serrate, the serræ stronger on posterior margin; lateral scutes low, about 30; caudal forked; caudal peduncle covered with plates (modified fulcra) above and below; preorbital plates obscure; head subconical; fontanel continued as an obscure groove to the dorsal. Air-bladders (Fig. 8, Plate 2) similar to those of *Pseudodoras niger* but without indications of cœca, the anterior bladder less depressed; fontanel not divided, not continued as a groove. Mental barbels in two distinct pairs.

34. **Rhinodoras d'orbignyi** (Kröyer) MS.

Plate 1, Fig. 18; Plate 26, Fig. 5.

Doras (Oxydoras) d'orbignyi (Kröyer) Kner, Sb. Ak. Wiss. Wien, XVII, 1855, p. 149, Pl. V, Fig. 9 (Rio Plata).

Doras d'Orbignii Hyrtle, Denk. Ak. Wiss. Wien, XVI, 1859, p. 17 (vertebræ 6 + 4 + 21).

Rhinodoras orbignyi Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 84 (name); Günther, Cat. Fish. Brit. Mus., V, 1864, p. 209 (copied).

Oxydoras d'orbignyi Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 159; Occasional Papers Cal. Acad. Sci., I, 1890, p. 249.

Doras nebulosus Eigenmann and Kennedy, Proc. Acad. Nat. Sci. Phila., July 1903, p. 500 (Asuncion); Ribeiro, Peixes, IV, 1911, p. 213.

9837, I. U. M. one, 160 mm. (154 mm. at present), probably Asuncion. Carlos Ternetz.

Head 3.66; D. I, 6; A. 13; scutes 1 + 29 or 30, width at clavicle 1.15 in head, about 4 in the length; eye 7.5 in the head, 3.5 in snout, 2 in interorbital; posterior

nostril equidistant from anterior and lip; bones of head obscurely striate or granular; mouth inferior, teeth strong, in short bands; maxillary barbel not reaching gill-opening.

Sides and fins sharply clouded.

Genus XX. TRACHYDORAS Eigenmann, gen. nov.

Type, *Trachydoras atripes* Eigenmann.

Snout, preopercle and opercle granular; short and deep; coracoid process partly or wholly granular; first lateral scute very large, connecting the humeral process and the descending process of the dorsal plate; humeral process long and narrow, more or less obliquely truncate; fontanel not continued as a groove; no foramen in the dorsal buckler; snout short, anterior nostrils equidistant from tip of snout and the posterior nostrils or nearer the former; mouth very small; teeth feeble, or none; maxillary barbels just reaching gill-opening or shorter; adipose fin about half the length of the anal, sometimes continued forward as a ridge; depth 3–4.5; one or more pectoral pores.

Habitat: Paraguay and Amazon Basins.

KEY TO THE SPECIES OF TRACHYDORAS.[1]

a. Interopercle covered with skin; hooks of the lateral scutes along the line of the highest point in upper margin of the humeral process; first scute not reaching the lower margin of the humeral process, posterior margin of the humeral process obliquely emarginate.

b. Coracoid process much shorter than the humeral process, granular on part of its length; opercle, suborbital line and preopercular line granular; depth 4 in the length to the end of lateral hooks, equal to the length of the head; dorsal spine equal to head or head and width of humeral granulation; pectoral spine longer; depth of caudal peduncle 4 in the head; eye 1.25 in interorbital, 1.66 in snout, lateral plates .33 in the depth; posterior nostril nearer anterior nostril than eye; dorsal fulcrum dark, middle caudal rays hyaline.....35. **nattereri** (Steindachner).

bb. Coracoid process a trifle longer than the humeral process, striate its entire length; suborbital line covered with skin; opercle and preopercular line striate; depth 3.2 in the length, greater than head and humeral granulations; dorsal spine equals length of head; pectoral spine considerably longer; depth of caudal peduncle 3 in the head; eye 1.4 in interorbital, 3.1 in the head; lateral plates .33 of the depth; posterior nostril nearer eye than to anterior nostril; base of middle caudal rays dusky.....36. **paraguayensis** (Eigenmann and Ward).

aa. Interopercle granular; hooks of the scutes above the line of the upper margin of humeral process; first scute crescent-shaped, extending to the lower margin of the humeral process.

c. Dorsal spine equals head or a little shorter than head, about equal to the pectoral spine; depth of caudal peduncle about 3.8 in the head; eye 1–1.25 in interorbital, equal to snout behind the anterior nostrils; depth of lateral plates about 2.3 to 2.6 in the depth; posterior nostril much nearer eye than to anterior nostril; scutes 35; maxillary barbel with many barblets.....37. **atripes** Eigenmann.

[1] *D. microstomus*, of which only small specimens, 52 mm. and shorter, have been taken, may belong here; in general appearance, mouth, barbels, dorsal buckler, etc., it resembles the species of this group. The preopercle, opercle, etc., are not granular, but the preopercular edge extends as a ridge across the cheek. *D. brevis* also resembles this group.

cc. Dorsal spine equals head and a little more than width of the humeral granulation, about equal to the pectoral spine; depth of lateral plates about half the depth; posterior nostril much nearer eye than to anterior nostrils; D. I, 6; A. 13; scutes 33–34. Maxillary barbel with but one barblet; all fins hyaline; eye equals interorbital.....38. **trachyparia** (Boulenger).

35. **Trachydoras nattereri** (Steindachner).

Plate 18, Figs. 1 and 2; Plate 26, Figs. 2 and 3.

Oxydoras nattereri Steindachner, Flussf. Südam., II, 1881, p. 4, Pl. 2, Figs. 1–1*a*. (Teffé.)

Hemidoras nattereri Eigenmann and Eigenmann, Proc. Cal. Acad. Sci. (2), I, 1888, p. 158 (Coary; Jutahy; Teffé); Occasional Papers Cal. Acad. Sci., I, 1890, p. 253; Fisher (part), Ann. Carnegie Mus., XI, 1917, p. 421 (San Joaquin, Bolivia; Santarem; Maciél; Rio Guaporé).

15967, I. U. M. one, 130 mm. Iquitos. Morris. 1922.

Head 3.6; depth 3.5; D. I, 6; A. 14; scutes 3 + 32; eye 1.5 in snout, 3.5 in head, 1.25 in interorbital.

Snout blunt, occipital region roof-shaped; humeral process more finely striate than in *T. paraguayensis*, the striation not more coarse toward the upper margin; clavicular and coracoid granulation separated in front of the pectoral by an area covered with skin; coracoid process reaching to middle of humeral process; maxillary barbel with about 7 barblets; rudimentary scutes without a hook. Line of hooks on a line with the uppermost point of the humeral process; a dark area about dorsal fulcrum; a round dusky area at the base of each caudal lobe.

Anterior air-bladder with a bunch of finger-like cœca at the outer anterior angles; posterior air-bladder very small, forked, the two branches like a pair of horns, the points of the horns turned ventrad.

36. **Trachydoras paraguayensis** (Eigenmann and Ward).

Plate 1, Fig. 7; Plate 3, Fig. 8; Plate 18, Figs. 3 to 5; Plate 26, Fig. 1.

Hemidoras paraguayensis Eigenmann and Ward, Ann. Carnegie Mus., IV, 1907, p. 116, Pl. XXXIV, Fig. 1 (Corumba).

Hemidoras nattereri Fisher (in part), Ann. Carnegie Mus., XI, 1917, p. 421 (the specimens from San Joaquin, Corumba and Villa Hays).

Specimens Examined.

10127, I. U. M., *type*, 87 mm. to end of last scute. Anisits.

Part of 7201, C. M. 5, 46–88 mm. to end of middle caudal rays. San Joaquin, Bolivia. Haseman, Sept. 1909.

7203, C. M. one, 93 mm. over all. Villa Hays. Haseman, April 13, 1909.

Head 3.33–3.7; depth 3.25 to 3.33 (3 in Villa Hays specimen); D. I, 6; A. 14; scutes 3 + 29 or 30; eye 2 in snout, 4.5 in head, 2 in interorbital. Profile arched, occipital region steeply roof-shaped.

General appearance of *Doras brevis*. Head granular above, except for a space around and between the nostrils and the fontanel; fontanel not continued as a groove; opercle and preopercle granular; suborbitals very narrow, granular; preorbital granular; interopercle covered with skin; humeral process striate, a keel along its lower edge, the ridges near the upper edge stronger than those in the center; posterior edge emarginate, the upper and lower edges parallel; clavicle striate, united with the striate part of the coracoid into a hook-shaped area in front and below the pectoral spine; coracoid process striate, reaching as far as the humeral process; mouth narrow, inferior, maxillary barbels reaching to below some part of the eye, with 3 to 6 barblets; mental membrane narrow, the barbels papillose, without barblets. Teeth, if any, not evident; rudimentary scutes minute but each with a hook; first scute very broad, reaching from the dorsal plate to or near to the lower edge of the humeral process, the part above the hook in the adult with numerous thorns, in several irregular series; fewer in smaller; major hooks below the line of the hooks on the rudimentary scutes of the tympanum, about on a line with the upper margin of the humeral process; succeeding scutes much narrower and shorter, the series graduate to the last.

Dorsal spine shorter than pectoral spine, serrate in front to the upper third; the serræ on the hinder margin with a few minute points; adipose well developed, but usually shorter than the anal; anal more or less rounded; caudal very broad, the fin but slightly emarginate when expanded; pectoral spine reaching just beyond base of ventrals.

No markings; air-bladder heart-shaped; a small diverticulum branched or not at the outer anterior corners; the second air-bladder minute, forked, about equal to the diverticula.

37. **Trachydoras atripes** Eigenmann.

Plate 2, Fig. 5; Plate 18, Figs. 6 to 8; Plate 26, Fig. 4.

Specimens Examined.

15877, I. U. M., *type*, 60 mm. to end of lateral scutes, 72 mm. over all. Brook near R. Itaya, above Iquitos. Allen. Sept. 1920.

7204, C. M. 2, the larger about 80 mm. Berlin, Rio Marmoré. Haseman. Sept. 15, 1909.

7201, C. M., 16006, I. U. M. 9, 73 to 103 mm. to end of middle caudal rays. San Joaquin, Bolivia. Haseman. Sept. 1909.

Head 3.66 to 3.75; depth 4.25 to 5; D. I, 6; A. 14 to 17; scutes 3 + 34 or 35. Eye 1.5 to 2 in snout; 3 to 3.5 in head; 1 to 1.33 in interorbital; head granular, much as in *paraguayensis;* pit of the fontanel much longer, tapering at both ends, continued as a groove to the transverse groove on the occipital; profile not as steep as in *paraguayensis,* the occipital region roof-shaped; suborbital bones stronger and more

prominently granular; nasal granular, its lower margin convex; interopercle and a small angle of lower jaw granular; mouth small, inferior, without evident teeth; maxillary barbel reaching to below the eye, with about 7 barblets; mental barbels united by a broad membrane, without barblets; rudimentary scutes variable, the last smallest, the two anterior linear or with the posterior half wider; coracoid processes and a linear streak of the coracoid, forward of the lower edge of the pectoral spine, granular; granular portion of the clavicle separated from the granular part of the coracoid, covered with skin; angle of the mandible exposed, granular; coracoid process considerably shorter than the humeral process; humeral process narrow, granular, rather than striate; first scute wider below the hook than above it, extending from the dorsal plate to a little below the humeral process, the surface with several sharp pointed granules, many marginal spines; scutes regularly graduate, line of hooks above on middle of the scutes, far above the upper margin of the humeral process; dorsal spine little if any shorter than the pectoral spine; serræ on posterior margin nearly as strong as on the anterior, the serræ antrorse but not as steeply so as on the anterior margin; adipose smaller than the anal; caudal deeply forked; anal rounded, its anterior half dusky.

Several branched cœca along the sides of the anterior air-bladder; posterior air-bladder very short, with branched cœca on each side.

In the type the dorsal fulcrum and base of dorsal rays are dark; anterior half of anal dark; pectoral dusky; middle caudal rays hyaline, a dusky band above and below the hyaline.

38. **Trachydoras trachyparia** (Boulenger).

Oxydoras trachyparia Boulenger, Trans. Zoöl. Soc. London, XIV, 1898, p. 423, Pl. XL, Fig. 2 (Rio Jurua).

This species is known by the two types, 92 mm. long, in the British Museum.

Genus XXI. Doras Lacépède.

Doras Lacépède, Hist. Nat. Poiss., V, 1803, p. 116; 12-mo., IX, 148 (*Silurus carinatus* Linnæus and *Silurus costatus* Linnæus).

Doras Bleeker, Nederl. Tijdsch. Dierk., I, 1863, p. 13 (type *carinatus*).

Oxydoras Kner, Sb. Ak. Wiss. Wien, XVII, 1855 (various species, the first *stenopeltis*, others *carinatus*, *niger* (*non* C. and V.), *lipophthalmus*, *d'Orbigny*, in the order named.

Hemidoras Bleeker, Ichthyol. Arch. Indici Siluri, 1858 (type *stenopeltis*). In 1862 Bleeker abandoned this generic name, placing *stenopeltis* in the present *Doras*.

Hemidoras Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 250.

Oxydoras Günther, Cat. Fish. Brit. Mus., V, 1864, p. 206 (various species, the first one *carinatus*).

Doras Jordan, The Genera of Fishes, 1917, p. 65 (in listing the genera of Lacépède Jordan gives *Silurus carinatus* L. as the type).

Moryrostoma Ribeiro, Arch. Mus. Nac. Rio de Janeiro, 1911, p. 192 (*carinatus*).

Type, *Silurus carinatus* Linnæus.

This is not the *Doras* of Cuvier and Valenciennes, Günther, the Eigenmanns and authors, generally. Lacépède did not specifically state that he selected the *Silurus carinatus* of Linnæus as the type of his *Doras* but placed it first. Neither did Cuvier and Valenciennes specifically select a type for *Doras* but placed *costatus* first. Their restriction is not more specific than is that of Lacépède.

Bleeker specifically selected *carinatus* as the type of *Doras*. This seems to have been the first specific indication of the type. Jordan rules that *carinatus* being the first species mentioned by Lacépède is the type of his genus.

Mandibulary barbels with a basal membrane, with or without barblets; maxillary barbel with barblets; origin of the ventrals nearer the caudal than the snout; nuchal buckler intact, without foramen, without a median groove continued from the fontanel; the back between the dorsals without plates; lateral scutes moderate, the first one connecting the dorsal plate, the first rib and the humeral process; coracoid not granular; preopercle and interopercle covered with skin, not granular; a bony stay from epiotic to the lower end of the process of the dorsal plate, and first scute; adipose fin short, not continued as a keel.

Habitat: Rio Jauru to Guianas and Peru.

The air-bladders in this genus as accepted here differ considerably. In one type the air-bladder is simple without diverticula, excepting that in the old there is developed a minute second air-bladder, *carinatus, micropœus*. In a number of other species a second air-bladder is represented by two divergent recurved tubes. The species having this type are *brevis, punctatus, eigenmanni*, and possibly the adult of *microstomus*. In the third type it is heart-shaped with numerous radial diverticula. They are especially prominent in front.

Key to the Species of Doras.

a. Eye equal to or less than interorbital.

b. First lateral scute unusually large, in contact with the coracoid process; teeth in both jaws; snout covered with skin; eye 1.5 in the snout, 1.25 in the interorbital; maxillary barbels reaching beyond base of pectoral; humeral process broad, rounded; a plate above and below the caudal peduncle; dorsal and pectoral spines of equal length, the latter extending beyond base of ventrals; scutes 29–30. Air-bladder simple, short, broad.....39. **fimbriatus** Kner.

bb. First lateral scute not reaching coracoid process. Humeral process narrow.

c. Dorsal and pectoral spines equal, the former reaching beyond origin of adipose; adipose fin equals anal; no teeth; origin of dorsal nearly midway between tip of snout and end of adipose; eye equals interorbital. Scutes 29–30; unspotted; depth 3.4; maxillary barbel reaching to below eye; mental barbels papillose; eye equals interorbital; air-bladder single, with very numerous diverticula and ending in two recurved horns.....40. **brevis** Heckel.

cc. Dorsal spine shorter than pectoral spine, not reaching the adipose.

d. Maxillary barbel reaching base of pectoral; mental barbels without slender barblets, papillose; snout considerably longer than the eye; scutes 29–30, the hook of the first one distinctly above the line of the rest; the line of hooks on a level with the upper margin of the humeral process; the rudimentary plates feeble, running below the middle of the pseudotympanum, the last one with a thorn, much above the first of the lateral scutes. Eye 1.66 in interorbital; center of opercle granular. Air-bladder ending in two recurved horns...41. **punctatus** Kner.

dd. Maxillary barbel reaching beyond base of pectoral; snout considerably longer than the eye; scutes 26 to 29; the hooks of the lateral scutes just above the line of the upper margin of the humeral process; eye 1.25 in interorbital. Mottled; mental barbels similar to the maxillary, with many barblets. Air-bladder ending in two recurved horns.

42. **eigenmanni** (Boulenger).

aa. Eye greater than interorbital.

e. Lateral scutes 28–32, rarely 26.

f. Maxillary barbel not reaching pectoral; snout less than the eye (in young only?); line of scutes distinctly above the level of the upper margin of the humeral process; the rudimentary plates rather strong, running at or a little above the level of the rest. Scutes 26–32. Interorbital less than vertical diameter of the eye, 1.5 in the longitudinal. Air-bladder in adult probably ending in two recurved horns.....43. **microstomus** (Eigenmann).

ee. Lateral scutes 33–35 (Valenciennes gives 36); orbit elongate; dorsal spine shorter than pectoral spine; snout pointed; lateral plates narrow; interorbital much smaller than eye, 1.6–2 in the length of the eye. Young with a single air-bladder; old with a small second bladder.

g. Scutes well developed.....44. **carinatus** (Linnæus).

gg. Anterior scutes minute; fins pigmented, dorsal dark, without a spot; snout long, subconical; the mouth very large; 4–10 teeth on each side of upper jaw, 2–5 on each side of lower jaw; no definite markings. Similar to *carinatus*.....45. **micropœus** (Eigenmann).

eee. Lateral scutes 37–38, largest on the base of the caudal; orbit much elongate; dorsal spine longer than pectoral spine; snout pointed, no teeth in upper jaw, lower jaw with minute teeth; barbels reaching gill-opening; the mental barbels with barblets similar to those of the maxillary; membrane of the mental barbels very wide; humeral process twice as long as high; interorbital equals vertical diameter of eye. Air-bladder heart-shaped, no second bladder or cœcum, numerous marginal diverticula, especially prominent in front.....46. **lipophthalmus** Kner.

39. **Doras fimbriatus** Kner.

Doras fimbriatus Kner (*Corydoras loricatus* Heckel, MS.), Sb. Akad. Wiss. Wien, XVII, 1855, p. 134, Pl. III, Fig. 5, dorsal view of head (Rio Guaporé).

Oxydoras fimbriatus Günther, Cat. Fish. Brit. Mus., V, 1864, p. 207 (copied).

Hemidoras fimbriatus Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 255.

Known from the types in the Vienna Museum.

40. **Doras brevis** Heckel.

Doras brevis [1] Heckel, MS. in Kner, Sb. Akad. Wiss. Wien, XVII, 1855, p. 138, Pl. VI, Fig. 11 (Barra do Rio Negro).

Known from the types in the Vienna Museum.

[1] The specimens listed as *brevis* by Fisher, Ann. Carnegie Mus., XI, 1917, p. 421, do not belong to this species.

41. Doras punctatus Kner.

Plate 3, Fig. 5; Plate 21, Figs. 1 to 2; Text-fig. 12, E.

Doras punctatus Kner (*Corydoras brevis* Heckel, MS.), Sb. Akad. Wiss. Wien, VII, 1855, p. 136, Pl. VI, Fig. 10 (Matto Grosso; Rio Guaporé).

Corydoras punctatus Hyrtle, Denk. Ak. Wiss. Wien, XVI, 1859, p. 17 (vertebræ 5 + 1 + 21).

Oxydoras punctatus Günther, Cat. Fish. Brit. Mus., V, 1864, copied.

Hemidoras punctatus Eigenmann and Eigenmann, Occasional Papers Cal. Acad. Sci., I, 1890, p. 255.

Hemidoras nattereri Fisher, Ann. Carnegie Mus., XI, 1917, p. 421 (part of No. 7201, San Joaquin, Bolivia).

Hemidoras brevis Fisher, l.c., p. 421, part 7193 only (San Joaquin).

Specimens Examined.

7193, C. M., 96 mm. to end of middle caudal ray, and 7201, C. M. part 2, 69 to end of middle caudal ray. San Joaquin, Bolivia. Sept. 5, 1909. Haseman.

15882, I. U. M. one, 118 mm. R. Paranapura, Yurimaguas. Allen. Nov. 1920.

15883, I. U. M. many, largest 99 mm. Lake Cashiboya. Allen. Aug. 1920.

The specimens enumerated above differ from those described as *punctatus* by Kner in uniformly having longer maxillary barbels. Heckel had 17 specimens between 3 and 5 inches long from the Rio Guaporé. In these he says that the maxillary barbel uniformly fails to reach the gill-opening and has but 3 or 4 barblets.

Head 3.3 to 3.6; depth 3.9 to 4.2; D. I, 6; A. 12–14; scutes 2 + 1 [1] + 29; eye 1.7 in snout, 4 to 5 in the head, 1.6 to 1.75 in the interorbital. Head broad, narrowed to a sharp snout; occiput roofed; mouth subterminal, with teeth in two small patches in each jaw; opercle with a granular patch, cheeks and snout otherwise covered with skin; head above, behind posterior nostril finely granular-striate; fontanel elongate, ending abruptly above the middle of the eye, not continued as a groove; scapular region broad, the humeral process narrow, grooved and striate but not as deeply as in *eigenmanni;* its upper margin emarginate, its lower margin keeled; coracoid process as long as the humeral process; scutes in tympanum imbedded, *the last one with a hook* similar, but smaller than the hooks of the lateral series.

An inconspicuous spot near the center of the dorsal; an obscure dusky band along the middle of the back, from the dorsal to the caudal; a very obscure dusky band across the occipital and continued on the sides just above the hooks of the lateral line and above the center of the upper caudal lobe; a similar band through the center of the lower caudal lobe. In 15882, the bands are obsolete on the back and sides, but well marked on the caudal; the sides and the head in this specimen have rather sharp, small, dark spots. Posterior air-bladder forked.

[1] A rudimentary scute bearing a hook some distance above the hooks of the fully developed series.

42. **Doras eigenmanni** (Boulenger).

P ate 21, Figs. 3 and 4.

Oxydoras eigenmanni Boulenger, Proc. Zoöl. Soc. London, 1895, p. 524; Trans. Zoöl. Soc. London, XIV, 1896, p. 28, Pl. IV, Fig. 3 (Descalvados, Matto Grosso); Eigenmann, McAtee and Ward, Ann. Carnegie Mus., IV, 1907, 116 (Corumba).

Hemidoras brevis Fisher (part), Ann. Carnegie Mus., XI, 1917, p. 421 (Rio Jaurú).

Specimens Examined.

10140, I. U. M. 5, largest 107 mm. to end of middle caudal rays. Corumba, Paraguay. Anisits.
16190, I. U. M. 2, 78 and 85 mm. Santarem market. Carl Ternetz.
7190, C. M. 1, 95 mm. to end of middle caudal rays. Santarem. Dec. 15, 1909. Haseman.
7191, C. M. 1, 80 mm. to end of middle caudal rays. Caceres. May 26, 1909. Haseman.
7192, C. M. 1, 92 mm. Maciél, Rio Guaporé. Aug. 3, 1909. Haseman.
7194, C. M. 3, 75–90 mm. to end of last scute. Rio Jaurú. June 4, 1909. Haseman.

These are some of the specimens identified by Fisher as *brevis*.

Head 3.75 to 4; depth about equal to the head; D. I, 6; A. 12 to 13; scutes 28 to 29 (rarely 26). Eye 2.5 in snout, 4.75 in head, 1.75 in interorbital. Head broad, narrowed to a sharp snout; occiput roofed; mouth subterminal, with two small patches of comparatively strong teeth in both jaws; cheeks and snout in front of the posterior nostrils covered with skin, head above granular-striate from posterior nostril back; fontanel oval, ending abruptly over the pupil, not continued as a groove; scapular region broad, the humeral process deeply grooved and with prominent ridges, leaving a median, terminal depression free from keels; coracoid process heavy, about as long as the humeral process, finely striate from near the suture to the tip; scutes in the tympanum imbedded, scarcely evident; one scute only in contact with the dorsal plate; the series of hooks in a line just above upper margin of the humeral process, the hook of the first scute scarcely above the line of the rest; the scutes between a third and a fourth of the height, the median spine of the last not much smaller than those immediately in front of it; several marginal points above and below the median hook.

Maxillary barbel extending beyond the base of the pectoral spine; with about a dozen or more long, slender barblets in an alternating row on the ventral surface and outer edge; mental barbels and barblets (5–7) similar to the maxillary; basal membrane of mental barbels insignificant, the mental barbels reaching beyond the gill-opening, the outer considerably longer than the inner. Dorsal spine strongly serrate on its anterior margin, less so on its posterior edge, its sides deeply grooved and ridged; pectoral spine usually extending beyond base of ventrals; origin of ventrals midway between snout and end of last scute, or nearer the later point; caudal forked;

anal rounded, adipose about as long as the anal. Everywhere mottled, light below; fins spotted.

This species resembles *punctatus* as figured by Heckel, in which, however, the first thorn of the lateral series lies higher than the rest and the maxillary barbels never reach the gill-opening and have but 3 to 4 barblets.

43. **Doras microstomus** (Eigenmann).

Hemidoras microstomus Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 394 (name only); Mem. Carnegie Mus., V, 1912, p. 193, Pl. XVIII, Fig. 2 (Rockstone and Crab Falls, on the Essequibo, British Guiana); Steindachner, Flussf. Südam., V, 1915, p. 57 (Rio Branco at Boa Vista and Serra Grande; Rio Surumu).

Specimens Examined.

12039, I. U. M. Rockstone. Eigenmann.
12040, I. U. M. Crab Falls. Eigenmann.

Air-bladder broad, simple; an incipient double diverticulum at the end indicates that a forked double cœcum develops with age.

44. **Doras carinatus** (Linnæus).

Plate 1, Figs. 8, 9, 10; Plate 2, Fig. 6; Plate 20, Figs. 1 and 2; Plate 23, Fig. 4; Plate 27, Fig. 8.

Silurus carinatus Linnæus, Syst. Nat., ed. 12, I, 1766, p. 504; Bloch and Schneider, Syst. Ichth., 1801, p. 108.

Doras carinatus Lacépède, Hist. Nat. Poiss., V, 1803, p. 116 (Surinam); Cuvier and Valenciennes, Hist. Nat. Poiss., XV, 1840, p. 288, Pl. 442 (Cayenne); Müller and Troschel, in Schomburgk, Reisen, III, 1848, p. 629 (Essequibo); Bleeker, Ichth. Arch. Ind. Prodr., I, 1858, p. 54; Nederl. Tijdsch. Dierk., I, 1863, p. 13 (name only); Silures de Suriname, 1864, p. 31 (Surinam).

Doras (Oxydoras) carinatus Kner, Sb. Akad. Wiss. Wien, XVII, 1855, p. 144 (Surinam).

Oxydoras carinatus Günther, Catalogue, V, 1864, p. 206 (Surinam; Essequibo River); Vaillant, Bull. Soc. Philom. (7), IV, 1880, p. 154 (Calderon).

Hemidoras carinatus Eigenmann and Eigenmann, Proc. Cal. Acad. Sci. (2), I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 258; Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 394; Mem. Carnegie Mus., V, 1912, p. 194.

Doras oxyrhynchus Valenciennes, in Humboldt, Rec. d'Obs. Zoöl. et Anat. Comp., II, 1833, p. 184 (Tumatumari on the Potaro; Bartica; Rockstone; Crab Falls; Georgetown market).

Specimens Examined.

12026, I. U. M. Rockstone. Eigenmann.
12027, I. U. M. Georgetown. Eigenmann.
12028, I. U. M. Crab Falls. Eigenmann.

45. **Doras micropœus** (Eigenmann).

Plate 1, Fig. 17; Plate 2, Fig. 1; Plate 20, Fig. 3; Plate 24, Figs. 1 and 2; Plate 27, Fig. 5.

Hemidoras micropœus Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 394 (name only); Mem. Carnegie Mus., V, 1912, p. 195 (British Guiana at Lama Stop-Off and Wismar on the Demerara River).

12029, I. U. M. Lama Stop-Off. Eigenmann.

46. **Doras lipophthalmus** Kner.

Doras (*Oxydoras*) *lipophthalmus* Kner (*Corydoras ophthalmus* Heckel, MS.), Sb. Akad. Wiss. Wien, XVII, 1855, p. 147, Pl. 5, Fig. 8 (Rio Negro).

Oxydoras lipophthalmus Günther, Cat. Fish. Brit. Mus., V, 1864, p. 208 (River Capin).

Hemidoras lipophthalmus Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 255.

This species is known from the types, 4 specimens $7\frac{1}{2}$ inches long and a specimen in the British Museum.

Genus XXII. Hemidoras Bleeker.

Hemidoras Bleeker, Ichthyol. Arch. Indici Siluri, 1858.

Type, *Hemidoras* (*Oxydoras*) *stenopeltis* Kner.

Bleeker who created the genus *Hemidoras* in 1858 abandoned it in 1862, placing its type in the present genus *Doras*.

Snout long, subconical, smooth; lateral plates well developed along the entire line; depth more than one fifth, rarely less than one fifth of the length; humeral process longer than deep; mouth small; ventrals behind the middle of the body; maxillary barbel with many barblets, mental barbels united by a basal membrane, fringed with a double row of barblets.

A series of plates along the middle of the back between the dorsals and sometimes between the anus and anal; a small foramen in the dorsal buckler; fontanel continued as a groove.

The air-bladder in *morrisi* is prolonged in a short diverticulum to a point. Numerous branched tufts of diverticula around the entire margins.

Key to the Species of Hemidoras.

a. Premaxillaries and dentaries without teeth or with a small patch of teeth; maxillary barbel extending beyond base of pectoral; scutes 32 to 37. Eye 1.5 in snout, 3–3.4 in head, greater than interorbital; D. I, 6.....47. **stenopeltis** (Heckel).

aa. Eye (orbit) 4.25–5 in head, 2.25–2.75 in snout, 0.8 in interorbital; scutes 33–34.

48. **morrisi** Eigenmann.

47. **Hemidoras stenopeltis** (Heckel).

Corydoras stenopeltis Heckel, MS. as *Doras* (*Oxydoras*) *stenopeltis* in Kner, Sb. K. Akad. Wiss. Wien, XVII, 1855, p. 142, Pl. IV, Fig. 7 (Rio Negro).

Oxydoras stenopeltis Günther, Cat. Fish. Brit. Mus., V, 1864, p. 208 (copied).

Hemidoras stenopeltis Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 256 (Manaos, Rio Negro; Hyavary; Manacapuru; Teffé; Obidos; Tabatinga); Fisher, Ann. Carnegie Mus., XI, 1917, p. 422 (Rio Madeira near San Antonio).

7186, C. M. 2, 107 and 132 mm. San Antonio. Nov. 3, 1909. Haseman.

15907, I. U. M. 1, 80 mm. Pebas, Peru. J. B. Steere.

Head 3.8 to 4; depth 5.2 to 5.33; D. I, 6; A. 13 to 15 (including rudimentary rays).

Eye 1.5 to 1.6 in snout, 3 to 3.4 in the head, about 0.75 in interorbital. Cheeks and snout without granulations; fontanel reaching the posterior edge of the pupil; skull striate; snout curved; mouth inferior; no teeth, or small patches of very minute teeth; anterior nostrils equidistant from tip of snout and eye; mental barbels with about 9 barblets, the basal half of which are scale-like, the posterior slender; maxillary barbel reaching beyond base of pectoral, with about 15 barblets along its outer edge, none on inner edge or surface. Three scutes on the tympanum, the first with downward projecting branch; 32 to 37 hook-bearing scutes, deepest in front, graduate to the last; numerous marginal spines both above and below the median hook; caudal peduncle with inconspicuous fulcra covering about half its length above and below; about ten plates along the back from the tip of the last dorsal ray, graduate to the adipose; dorsal spine as long as, or longer than, the pectoral spine, which reaches the ventrals; adipose fin nearly as long as the anal; caudal deeply forked; origin of ventrals little nearer snout than end of middle caudal rays; humeral process without distinct keel, its lower edge not parallel with the pectoral spine, the entire process below the line of the hooks of the lateral scutes.

A dark area about the dorsal fulcrum; tip of dorsal spine and of first ray black; a pair of dusky streaks on the caudal.

48. **Hemidoras morrisi** Eigenmann, spec. nov.

Plate 27, Fig. 3.

15962, I. U. M. 2, 77 and 147 mm., the latter the type. Iquitos. Morris.

Description of the Type: Head 3.2; depth about 6; D. I, 6; A. 14; scutes 33 or 34; distance between snout and end of dorsal plate $2\frac{3}{11}$ in the length; ventrals nearer tip of middle caudal rays than snout.

Slender, lateral scutes all well developed, about half the depth; the anterior ones with several (up to 6 or 7) marginal spines both above and below the median

hook; about 14 very small scutes in front of the adipose; numerous, narrow, laminate caudal fulcra; a well-defined groove from the fontanel to the dorsal; snout smooth; suborbital line granular, upper posterior part of opercle granular, the rest of the opercle striate; two large granular scutes on the pseudotympanum; humeral spine slender, its dorsal margin emarginate; mouth without teeth (with up to 6 teeth on a side in each jaw in the paratype); snout long, slender; posterior nostril about equidistant from snout and eye; eye (orbit) slightly elongate, $\frac{2}{5}$ of the snout, $\frac{7}{9}$ in the interorbital, 4.33 in the head; maxillary barbel reaching about to base of pectoral; mental barbels short, their connecting membrane narrow; dorsal spine about equal to snout and eye (longer in paratype, equal to pectoral spine); adipose fin not quite as long as anal; pectoral spine almost equal to head, extending past origin of ventrals; a large pectoral pore. No prominent markings.

Genus XXIII. Opsodoras Eigenmann, gen. nov.

Type, *Opsodoras orthacanthus* Eigenmann, spec. nov.

Maxillary barbels with barblets, mental barbels with a double series of papils or barblets; a foramen on each side of the nuchal shield; no plates along the back; origin of the ventrals nearer caudal than snout; lateral scutes well developed, successive ones in contact, or the anterior ones minute, not in contact.

Air-bladder single, blunt or pointed behind, with fine, thread-like, branched diverticula, at least in *hemipeltis*, *parallelus*, *humeralis* and *orthacanthus*. In *leporhinus* of which only young were examined the air-bladder is without diverticula and ends in a blunt double point.

Habitat: Amazon and Guianas.

Key to the Species of Opsodoras.

a. Fontanel not continued as a groove to the dorsal plate; mental barbels papillose; well-developed teeth on the lower jaw; air-bladder profusely provided with thread-like cœca.

b. Anterior scutes minute, successive ones not in contact with each other; interorbital convex; groove from fontanel reaching beyond eye; snout conical; its width in front of the eye little more than its depth; adipose eyelid prominent; maxillary barbel reaching to below posterior border of eye; a dark streak below the lower margin of eye.....49. **hemipeltis** Eigenmann.

bb. Depth of scutes about one half the diameter of the eye; foramen large, triangular; interorbital flat; groove of the fontanel truncate behind; snout depressed, much wider in front of eye than high; depth of humeral process at its widest less than the orbit; maxillary barbel reaching beyond the eye; scutes 0 + 32; edge and upper surface of the air-bladder most profusely supplied with cœca; pectoral spines not reaching ventrals.....50. **parallelus** Eigenmann.

bbb. Depth of scutes at least equal to the diameter of the eye, frequently much higher; foramen moderate, oval; groove of the fontanel pointed behind; depth of the humeral process at its widest equal to the orbit; scutes 3 + 31 or 32; air-bladders with numerous cœca; pectoral spines reaching ventrals. 51. **humeralis** (Kner).

aa. Fontanel continued to the dorsal plate as a well-defined, narrow groove.

c. Keel of humeral process with its point continuous with the line of the hooks of the lateral scutes; scutes 30 to 31; eye 1 to 2 in interorbital; snout long, the distance between the nostrils about equal to the distance between the posterior and the eye; anterior nostril about equidistant from lip and the eye; mental barbels with a double row of barblets; foramen minute.

52. **orthacanthus** Eigenmann.

cc. Keel and point of the humeral process below the line of hooks of the lateral scutes.

d. A black spot on the base of the dorsal and another narrow oblique spot on the base of each caudal lobe.*

e. The spots well defined; the lateral scutes about equal to the vertical diameter of the eye; maxillary barbel reaching the axil, with 3 short graduate barblets on the lower surface at inner edge of the barbel, near its base, with about 20 slender, graduate barblets along its outer edge; scutes 31–32.....53. **trimaculatus** (Boulenger).

ee. Color markings more diffuse; the scutes with a light band, continued to end of middle caudal rays, dark above and below the band, the dark best marked on caudal; lateral scutes less than half the vertical diameter of the eye, 1.66 in its longitudinal; maxillary barbel reaching to pectoral spine, with about 11 flat, fringed barblets along its outer edge; mental barbels with numerous papils; each ramus of the lower jaw with about 5 brown-tipped teeth, those of the two rami forming a median group; lateral plates 33 to 35.

54. **leporhinus** (Eigenmann).

dd. No distinct markings.

f. Lateral scutes 29 to 35.

g. Lower edge of humeral process convex; eye about equal to interorbital.

h. Lateral scutes 35; eye a little greater than interorbital; mental barbels with barblets; the rudimentary scutes large; humeral process long, slender, three times as long as deep; depth of lateral scutes greater than orbit.....55. **boulengeri** (Steindachner).

hh. Lateral scutes 29 or 30, interorbital about equal to the eye; the rudimentary scutes minute; humeral process twice as long as deep; depth of scutes less than orbit.

56. **stübeli** (Steindachner).

gg. Lower edge of humeral process straight; interorbital 1.3 in eye. Lateral scutes 32 to 35; dorsal spine equals pectoral spine, the latter reaching beyond the origin of the ventrals; eye about one fourth larger than the interorbital; scutes 34...57. **morei** (Steindachner).

ff. Lateral scutes 38 to 40; eye three tenths larger than the interorbital.

58. **steindachneri** Eigenmann.

49. **Opsodoras hemipeltis** Eigenmann, spec. nov.

Plate 19, Fig. 2; Plate 24, Fig. 6.

15879, I. U. M., *type*, 143 mm. Rio Ucayali at Contamana. Allen. Sept. 1920.

Head 3.5; depth at occiput 6, at dorsal 5, at caudal peduncle 15.5; D. I, 6; A. 11; scutes about 33.

Sub-spindle shaped, the width everywhere approximating the depth; snout conical, opercles flush with the shoulders; interorbital convex; fontanel not continued

*A new species allied to *trimaculatus* and *leporhinus* has been received; see no. 53a.

as a groove, eye in center of the head, 1.5 in interorbital, 4 in the head; profile gently arched to the posterior nostrils, rapidly descending from the posterior nostrils forward; occipital striate, head otherwise smooth; snout small, width of the lower jaws 2 in the eye; barbels slender, basal membrane connecting the barbels small; the mental barbels simple, the maxillary barbels with a series of barblets along their outer edge, reaching to below posterior part of eye; width of isthmus equals snout; humeral process striate, triangular, its length equals snout, its height 1.5 in the eye; first eleven scutes rudimentary but increasing slightly, those above anal graduate, the most fully developed ones above end of anal; dorsal spine equals head behind posterior nostril, basal half of its anterior edge rough, its entire posterior edge serrate; adipose fin minute, half the length of the anal; caudal forked, its lobes equal; head behind posterior nostrils; anal short, truncate, its base less than snout; ventrals a little longer than snout, reaching half way to middle of anal; pectoral spines broken, the first ray equals head less half the snout; dusky above the lateral scutes, light below; lower border of orbit dark, no other markings.

Air-bladder prolonged, with many thin, branched diverticula.

50. **Opsodoras parallelus** Eigenmann, spec. nov.

Plate 19, Fig. 3; Plate 23, Fig. 3.

15964, I. U. M., *type*, 151 mm. Iquitos. Morris. 1922.

This species is evidently closely related, if distinct from *O. hemipeltis*. The lateral scutes are much better developed, the top of the head and nasal bones are granular, the opercle is striate; teeth of the lower jaw well developed but very fine, in two patches; mental barbels papillose, the maxillary barbels reaching to below middle of opercle; pectoral spines equal to the head behind the anterior nostrils, with small serræ on both surfaces to near the tip.

Profile less arched; interorbital flat.

Air-bladder prolonged into a point; with many thin, branched diverticula.

51. **Opsodoras humeralis** (Kner).

Plate 1, Fig. 6; Plate 2, Fig. 4; Plate 18, Figs. 9 and 10; Plate 19, Fig. 1.

Doras humeralis (*Corydoras humeralis* Heckel, MS.) Kner, Sb. Akad. Wiss. Wien, XVII, 1855, p. 140, Pl. IV, Fig. 6 (Barra do Rio Negro).

Oxydoras humeralis Günther, Cat. Fish. Brit. Mus., V, 1864, p. 206.

Hemidoras humeral's Eigenmann and Eigenmann, l.c., 1888, p. 158; 1890, p. 258.

Hitherto known only from the types.

Specimens Examined.

15880, I. U. M. one, about 154 mm. Rio Ucayali at Contamana. Allen. July 1920.
15881, I. U. M. 22, largest 145 mm. Specimen drawn 138 mm. long. Iquitos. Allen.
7189, C. M. 4, largest 122 mm. San Joaquin, Bolivia. Sept. 5 and 6, 1909. Haseman.

Resembling *brevis*, *leporhinus* and *microstomus*.

Head 3.5, depth at occiput 4.5–4.8; at dorsal spine 4–5; at caudal peduncle 14.5–15.5; D. I, 6; A. 10 to 12; scutes 0 to 4 + 29 to 32.

Heavy in front of dorsal; depth distinctly greater than the width except at caudal peduncle; snout subconical; opercles flush with the shoulders; interorbital slightly convex, fontanel not continued as a groove on the occipital; profile gently arched to posterior nostrils; the snout rapidly pointed in front of this point; distance between snout and anterior nostril about equal to the distance between the posterior nostril and eye, less than the distance between the anterior and posterior nostrils; top of head granular to posterior nostrils; opercle striate, head otherwise smooth; width of lower jaw a little more than half the length of the eyes; upper jaw with rarely a tooth; lower jaw with one or two series of teeth, those of the outer series larger, about 10 on each side; humeral process rough, very wide, its greatest width equal to the eye, 0.4–3 in its length, the margin from the point to its upper anterior angle either convex or with a distinct angle; mental barbels papillose, the maxillary barbel reaching to some point below the eye, with a lobe at its base on the inner margin, with about 7 slender barblets along its outer margin; scutes all well developed, graduated from the first back, about 0.4 in the height, with a median spine and a serrated margin; dorsal spine a little longer or shorter than the head; basal six tenths of anterior margin serrate, its entire posterior edge serrate; adipose fin half the length of the anal; caudal lobes about equal to the head behind the anterior nostrils; anal short, truncate, its base less than the snout; ventrals about equal to the snout; pectoral spines extending beyond origin of the ventrals; pectoral spine much longer than the dorsal spine, about equal to its distance from the snout to the upper angle of the humeral process; coracoid process covered.

Ashen above, light below; shadowy bands on the caudal lobes.

Air-bladder rounded behind, with many thin, branched marginal diverticula.

52. **Opsodoras orthacanthus** Eigenmann, spec. nov.

Plate 22, Fig. 3; Plate 23, Fig. 2.

Specimens Examined.

15884, I. U. M. 1, 133 mm. type. Iquitos. Morris. 1922.
15885, I. U. M. 4, 69–82 mm. Lake Cashiboya. Allen. Aug. 1920.
15886, I. U. M. 3, 77–98 mm. Mouth of Rio Pacaya. Allen. 1920.

Distinguished by the fact that the hooks on the lateral plates form a continuous, straight or nearly straight line with the ridge and tip of the humeral process.

Head 3.25; depth 5; D. I, 6; A. 12; scutes 30–31. Eye 2.33–2.66 in snout, 4.5–5 in the head, 1–2 in interorbital; snout acutely pointed; no teeth, width of lower jaw about 1.5 in the eye.

Width at the first dorsal and first anal rays equal to the depth at the same points; head wider than deep; head granular to the posterior nostrils; the foramen between dorsal plate and occipital minute; upper part of opercle granular; mental barbels with barblets; snout and cheeks covered with skin; humeral process triangular, the lower edge sloping, with a distinct ridge parallel with the depressed pectoral spine, its tip beyond the middle of the pectoral spine; coracoid process covered, reaching a little beyond middle of humeral process; dorsal spine serrate in front, roughened behind, about equal to snout and eye; pectoral spine equals length of head, reaching beyond origin of ventrals; base of adipose almost equal to base of anal; caudal forked, the upper lobe about equal to snout and eye; anal and ventrals rounded; two rudimentary scutes at beginning of lateral line at a much higher level than the fully developed ones; lateral scutes equally developed, the hooks under the dorsal much below the middle, behind the dorsal in the middle of the sides; each scute with a median hook and marginal serræ; sides above the hooks dusky, below them light, parallel dusky shades on the caudal lobes.

Air-bladder with many marginal diverticula.

53. **Opsodoras trimaculatus** (Boulenger).

Oxydoras trimaculatus Boulenger, Trans. Zoöl. Soc., XIV, 1898, p. 422, Pl. XL, Fig. 1 (Rio Jurua, 62 mm.).
Leptodoras trimaculatus Fowler, Proc. Acad. Nat. Sci. Phila., 1914, p. 264, Fig. 14 (Rupununi River, British Guiana, 77 mm.).
Hemidoras lipophthalmus (*non* Kner) Fisher, Ann. Carnegie Mus., XI, 1917, p. 421 (Santarem, 58 mm.).

Fowler proposed his *L. trimaculatus* as a new species but it is very probably identical with the *O. trimaculatus* of Boulenger.

7200, C. M. one, 58 mm. to end of middle caudal rays. Santarem. Haseman.

Head 3.66 in the length, caudal depth 4.6; D. I, 6; A. 14; scutes 2 or 3 + 30; eye 1 in snout, 2.8 in head, 0.5 in interorbital.

Snout very narrow, sharply decurved, pointed; mouth inferior, below the level of the eye, teeth very few, minute; cheeks and snout covered with skin, the skull striate, granular; a well-developed foramen on each side of the occiput; anterior nostril equidistant from snout and eye; fontanel very long, a bridge behind the pupil, continued as a groove to the dorsal plate; mental barbels very short, covered with

wart-like papillæ; maxillary barbels reaching to some point on the base of the pectoral. four short graduate barblets on its inner lower surface, about 20 longer, more slender barblets along its outer edge; anterior scutes about 0.4 of the height, with about 4 marginal lobes below and one above the median hook; caudal fulcra numerous, laminate; lower edge of humeral process without a keel, its point below the line of the hooks; dorsal spine shorter than pectoral spine, longer than head; pectoral spine extending beyond the base of the ventrals; origin of ventrals a little nearer end of middle caudal rays than the snout; caudal deeply forked; a triangular black spot on base of anterior half of the dorsal, converging black stripes on base of caudal lobes.

This specimen is undoubtedly young. Its head is striate. It differs from the *trimaculatus* of Boulenger (62 mm.) in that the latter does not have the fontanel continued as a groove. It is possible that both are juvenile forms of a species resembling *O. steindachneri*.

53a. **Opsodoras ternetzi** Eigenmann, spec. nov.

16173, I. U. M. one specimen, type 128 mm. Tapajos at Santarem. 1924. Carl Ternetz.

Head 4; depth 4.75; D. I, 6; A. 10; plates 4 + 33.

Ventral profile very slightly curved; dorsal profile very convex to the dorsal, thence nearly straight but downward to the caudal peduncle. Snout conical; depth at dorsal fulcrum equal to length of snout and eye; eye 3.33 in head; eye behind middle of head; interocular 2 in length of eye; snout decurved; maxillary reaching axil, with about 17 barblets; fontanel continued to the dorsal plate; dorsal spine equal to length of head, with numerous spines, the antrorse spines extending the basal sixth of the spines, the retrorse spines extending about the basal six-sevenths of the posterior face of the spine; pectoral spine about equal to the dorsal spine, the hooks extending along the entire spine; caudal distinctly forked; depth of lateral scutes above the ventrals about 2 in length of eye. Upper surface of tip of snout dark; dorsal fulcrum dark; a narrow dusky streak on middle of back; a dark band above the scutes, a dusky streak below the scutes; a pair of dusky bands above and below the middle caudal rays. Lower margin and keel of humeral spine far below the line of the lateral spines; the small spines in the tympanum above the lateral hooks. Numerous and conspicuous mucous pores. Teeth not evident.

Air-bladder with irregular series of partly branched, lateral diverticula, a bunch of diverticula at middle of sides and another at the outer, anterior angle of the bladder; no posterior air-bladder, but an irregular protuberance from which diverticula radiate.

54. **Opsodoras leporhinus** (Eigenmann).

Hemidoras leporhinus Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 394 (name only;) Mem. Carnegie Mus., V, 1912, p. 195, Pl. XIX, Fig. 1 (British Guiana at Tumatumari, Potaro River, and Crab Falls, Essequibo River); Steindachner, Flussf. Südam., V, 1815, p. 58, Pl. IX, Figs. 1–4 (Rio Surumu; Rio Branco at Serra Grande and Boa Vista).

I. U. M. 12021, 2 young, about 52 mm. Crab Falls. Eigenmann.

55. **Opsodoras boulengeri** (Steindachner).

Hemidoras (*Leptodoras*) *boulengeri* Steindachner, Flussf. Südam., V, 1915, p. 63, Pl. VIII, Figs. 1–3 (mouth of Rio Negro).

Known from the types.

56. **Opsodoras stübeli** (Steindachner).

Plate 27, Figs. 6 and 7.

Oxydoras stübeli Steindachner, Flussf. Südam., IV, 1882, p. 5, Pl. III, Figs. 1–1*b* (Rio Huallaga).

Hemidoras stübeli Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 257.

Known from the types, three specimens 80–120 mm. long, in the Vienna Museum.

57. **Opsodoras morei** (Steindachner).

Plate 27, Figs. 1 and 2.

Oxydoras morei Steindachner, Flussf. Südam., II, 1881, p. 6, Pl. I, Figs. 2–2*a* (Rio Negro).

Hemidoras morei Eigenmann and Eigenmann, l.c., 1888, p. 158; 1890, p. 257.

Known from the type, a specimen 125 mm. long, in the Vienna Museum.

58. **Opsodoras steindachneri** Eigenmann, spec. nov.

Hemidoras carinatus non Linnæus, Steindachner, Flussf. Südam., V, 1915, p. 55 (mouth of Rio Negro).

Based on a specimen 197 mm. to base of caudal, 235 mm. over all, in the Vienna Museum.

Head about $2\frac{5}{6}$; depth $4\frac{2}{7}$; scutes 38–40; D. I, 6; A. 3, 9.

Foramen tolerably large; snout long, compressed; maxillary barbels not quite reaching pectoral; mental barbels connected for half their length by membrane; mouth inferior, a few feeble teeth in each jaw; lateral plates low, of about uniform height to the middle of the body, then increasing in height to the middle of the caudal peduncle; depth about $4\frac{2}{7}$; distance between snout and dorsal 2.33 in the length; humeral process oblique, with convex posterior margin, 3 times as long as high; distance between tip of snout and ventrals $1\frac{7}{8}$ in the length.

Width of head 2 in its length; length of eye 4 in the length of the head, 2.35 in snout; height of eye equals interorbital, 5.66 in length of head; dorsal spine $1\frac{1}{6}$,

pectoral $1\frac{5}{7}$, ventral 1.8, depth of caudal peduncle 6, its length 2.4 in the length of the head.

The above is an abstract of Steindachner's description of *Doras carinatus* mentioned above. But *Doras carinatus* does not have a foramen except in the very young. It is therefore very probable that the specimen recorded by Steindachner, which has a large foramen, is not specifically or even generically identical with those recorded by Eigenmann from Guiana. Guiana specimens have about 33 scutes. Steindachner records 38 on one side and 40 on the other of his specimen. This makes Steindachner's identification doubly doubtful.

Genus XXIV. HASSAR Eigenmann and Eigenmann.

Hassar Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158.

Type, *Oxydoras orestis* Steindachner.

Snout, preopercle, and interopercle covered with skin. Anterior scutes of the lateral line very small; humeral process blunt, rounded, about twice as long as broad, extending past middle of pectoral spine, without keel; pectoral spine barely reaching ventrals or shorter; mouth large; a small patch of minute teeth on the lower jaw; orbit elongate; barbels slender, with barblets, with a narrow connecting membrane; ventrals behind the middle of the length; snout decurved, prolonged, pointed; first nostril equidistant from tip of snout and eye or nearer the latter; numerous small pectoral pores. A foramen on each side of x in the dorsal buckler.

Habitat: Amazon, Paranahyba, Guiana.

KEY TO THE SPECIES OF HASSAR.

a. Fontanel not continued as a narrow groove to the dorsal.

b. Dorsal plain; eye mostly in posterior part of head; adipose fin continued forward as a ridge, longer than anal.....59. **affinis** (Steindachner).

bb. Dorsal with a black spot.

c. The black spot on the first three rays, not extending to their tips; eye extremely elongate; depth at base of pectoral equal to snout or snout and 0.4 of the eye; depth below dorsal spine 5.33–6 in the length to the last scute; head in the scapular region covered with skin, not very rough; distance between snout and dorsal about 2.5 in the length to the end of the lateral plates.....60. **orestis** (Steindachner).

cc. The black spot on the first two rays, extending to their tip;[1] eye moderately elongate; depth at base of pectoral at least equal to snout and half the eye; depth below the pectoral spine about 4.66 in the length to the last scute; head in the scapular region more rugose; distance between snout and dorsal spine 2.5–2.6 in the length to the end of the lateral scutes.

61. **wilderi** Kindle.

aa. Fontanel continued as a narrow groove to the tip of the occipital process. A black spot on the dorsal; snout conical; dorsal spine with hooks on the basal part of the anterior margin; sides shaded toward the back; lateral plates 33.....62. **notospilus** (Eigenmann).

[1] The dark area in Plate 29, Fig. 4 should be continued to the tip of the rays.

59. **Hassar affinis** (Steindachner).

Plate 27, Fig. 4.

Oxydoras affinis Steindachner, Flussf. Südam., II, 1881, p. 7, Pl. I, Fig. 1 (Rio Puty, tributary of Paranahyba north of Therezina).

Hemidoras affinis Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 258 (Rio Puty); Steindachner, Flussf. Südam., V, 1915, p. 59 (Itapicurú at Caxias; Paranahyba at Engenho da Agua).

60. **Hassar orestis** (Steindachner).

Oxydoras orestis Steindachner, Sb. Akad. Wiss. Wien, LXXI, 1875, p. 138, Pl. I (Xingu).

Hemidoras orestes Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158 (Huytahy); Occasional Papers, I, 1890, p. 258; Kindle, Ann. N. Y. Acad. Sci., VII, 1894, p. 251 (Itaituba, Brazil); Fisher, Ann. Carnegie Mus., XI, 1917, p. 422 (Santarem).

5123, I. U. M. three. Itaituba, Brazil. Hartt.

16172, I. U. M. nine, 143–255 mm. R. Tapajos, Santarem. C. Ternetz.

16188, I. U. M. four, 133–137 mm. Amazon, Santarem. C. Ternetz.

61. **Hassar wilderi** (Kindle).

Plate 22, Fig. 2.

Hassar wilderi Kindle, Ann. N. Y. Acad. Sci., VII, 1894, p. 251 (Troceras on Rio Tocantins).

Known from the types only, in the collections of Indiana University and of Cornell University.

5120 I. U. M., *types*, 162 and 207 mm. Troceras, Rio Tocantins.

62. **Hassar notospilus** (Eigenmann).

Hemidoras notospilus Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 394 (name only); Mem. Carnegie Mus., V, 1912, p. 196, Pl. XIX, Fig. 2 (Crab Falls).

Known from the type, a specimen 70 mm. long, in the Carnegie Museum.

Genus XXV. LEPTODORAS Boulenger.

Leptodoras Boulenger, Ann. and Mag. Nat. Hist. (7), II, 1898, p. 478.

Type, *Oxydoras acipenserinus* Günther.

Air-bladder very short, its anterior end fitting into a cup-shaped downward process of the united vertebræ, two diverticula at posterior end, a pair of diverticula at the outer anterior corners.

Snout, preopercle and interopercle covered with skin. Slender, depth 5–9; humeral process not half as long as the pectoral spine, about as deep as long; mouth large, no teeth; barbels short, width of the membrane connecting the bases of the

mental barbels more than half their length; maxillary barbels divided beyond the bone into an outer lateral fimbriated part and an inner part coterminous with the mental barbels; outer mental barbel divided distally into two; a small foramen in the dorsal buckler (or none?), a narrow groove extending back from the fontanel tending to disappear; lateral plates 36–44, the first scute extending down to the lower edge of the humeral process; in all but *hasemani* ventrals in front of the middle of the length; adipose sharply defined behind, continued as a low crest to near the tip of the depressed dorsal spine; snout decurved, the bones smooth; first nostril nearer the second than to the snout; one pectoral pore.

Habitat: Amazon, Guiana.

KEY TO THE SPECIES OF LEPTODORAS.

a. Dorsal plain.

b. Dorsal spine with its membranous extension but little, if any, longer than the first ray; lateral plates covering half or more than half of the sides.

c. Eye about 3.5–6 in the head; interorbital about half the length of the eye; distance between tip of snout and dorsal 3 or less than 3 in the length to the end of the lateral line; distance between the dorsals about equal to the length of the head; 38 or 39 lateral scutes; pectoral reaching middle of ventrals; depth 6–7; a pair of parallel, dusky bands on the caudal, the middle rays light.....63. **linnelli** Eigenmann.

cc. Eye 4.8–5 in the head, greater than interorbital; distance between tip of snout and dorsal more than 3 in the length to the end of the lateral line; distance between the dorsals much longer than the head; 42 lateral scutes; pectoral reaching base of ventral; depth 7.5.

64. **acipenserinus** (Günther).

bb. Dorsal spine much produced beyond the soft rays, twice as long as the head; lateral plates covering less than half of the sides; eye 3 in snout, 1 in interorbital, 6.5 in head; maxillary barbel not reaching gill-opening; 44 lateral scutes; pectoral reaching base of ventral; depth 9.

65. **juruensis** Boulenger.

aa. Dorsal with a small round spot. Dorsal spine not longer than the following ray; lateral plates covering 0.4–0.5 of the body; eye a little more than 3.5 in the head; distance between snout and dorsal 2.75 in the length; distance between the dorsals equals length of head; 36–41 lateral scutes; pectoral not reaching middle of ventrals; depth 5–5.5; plain.....66. **hasemani** (Steindachner).

63. **Leptodoras linnelli** Eigenmann.

Plate 2, Fig. 2; Plate 5, Figs. 3 and 4; Plate 20, Fig. 4; Plate 24, Figs. 3 to 5.

Leptodoras linnelli Eigenmann, Repts. Princeton Univ. Exp. Patagonia, III, 1910, p. 395 (name only); Mem. Carnegie Mus., V, 1912, p. 191, Pl. XVII, Fig. 1 and Pl. XVIII, Fig. 1 (British Guiana at Tumatumari on the Potaro; Rockstone and Crab Falls on the Essequibo River).

Leptodoras acipenserinus non Günther, Fisher, Ann. Carnegie Mus., XI, 1917, p. 422 (Maciél, Rio Guaporé).

Hemidoras (*Leptodoras*) *linnelli* Steindachner, Flussf. Südam., V, 1915, p. 65 (Rio Branco at Boa Vista and Serra Grande).

Specimens Examined.

7188, C. M. 3. Maciél, Rio Guaporé. Haseman.
12022, I. U. M. Tumatumari. Eigenmann.
12023, I. U. M. Crab Falls. Eigenmann.
12024, I. U. M. Rockstone. Eigenmann.

64. **Leptodoras acipenserinus** (Günther).

Oxydoras acipenserinus Günther, Proc. Zoöl. Soc. London, 1868, p. 230, Pl. XX (Xeberos); Steindachner, Flussf. Südam., II, 1881, p. 8 (Xeberos).

Hemidoras acipenserinus Eigenmann and Eigenmann, Proc. Cal. Acad. Sci., 2d Ser., I, 1888, p. 158; Occasional Papers Cal. Acad. Sci., I, 1890, p. 255.

Leptodoras acipenserinus Boulenger, Ann. and Mag. Nat. Hist. (7), II, 1898, p. 478. (New genus.)

15878, one, 110 mm. Rio Paranapura at Yurimaguas. Allen. Nov. 1920.

Head 3.75; depth 8.5; width at origin of ventrals greater than depth at same place; depth of caudal peduncle 2 in its width with the spines; D. I, 6; A. 14; scutes 4 + 42.

Width of head equals the length of the snout, greater than its depth; eye 2.5 in snout, 4.8 in the head; interorbital 1.4 in the eye; lower profile straight; dorsal profile arched, descending from the nostrils to the sharp snout; fontanel continued as a narrow groove to the dorsal; top of head behind the eye granular; sides of head and snout with pearl-organ like papillæ; mouth inferior, narrow; width of lower jaw about 1.5 in the eye; snout projecting for nearly half the length of the eye; no teeth; bases of barbels connected by a broad membrane; maxillary barbel reaching below posterior part of eye; gill-openings wide, the isthmus equals length of eye; dorsal spine equals snout and half the eye, with antrorse spines on the basal half of its anterior edge, posterior margins roughened; adipose fin much shorter than the anal; caudal deeply forked, the lobes sharp pointed, a little greater than snout and eye; pectoral spines equal to snout and two thirds the eye; reaching past origin of ventrals, serrate on both margins, those on the posterior edge largest at the tip; scute less than half the depth, with a central spine and a serrated posterior edge; humeral process *half as broad as long*, reaching to middle of pectoral spines; coracoid process short, covered. Back dusky; sides and lower parts lighter. No distinct color markings.

Both this species and *linnelli* have a foramen where dorsal plate and occipital process meet.

65. **Leptodoras juruensis** Boulenger.

Leptodoras juruensis Boulenger, Ann. and Mag. Nat. Hist. (7), II, 1898, p. 478 (Jurua).

Known from the type only, a specimen 235 mm. long, in the British Museum.

66. **Leptodoras hasemani** (Steindachner).

Hemidoras hasemani Steindachner, Flussf. Südam., V, 1915, p. 61, X, Figs. 4–7 (Rio Negro at its mouth; Rio Branco at Boa Vista and Serra Grande).

Known from the types, the largest 132 mm. long, in the Vienna Museum.

Genus XXVI. NEMADORAS Eigenmann, gen. nov.

Type, *Oxydoras elongatus* Boulenger.

As far as known with the characters of *Opsodoras* but the maxillary barbels simple. Fontanel not continued as a groove.

KEY TO THE SPECIES.

a. Depth 5 in the length; posterior nostril twice as distant from the anterior as from the eye; eye 4.5 in the head, 1.4 in interorbital; humeral process tapering to a point; pectoral spine as long as dorsal, nearly as long as head; D. I, 6; A. 12; 33 lateral scutes; olive above, white below; lateral shields and fins orange.....67. **elongatus** (Boulenger).

aa. Depth 3.66 in the length; posterior nostril slightly nearer the eye than the anterior nostril; eye 2 in the snout, 4 in the head, 1.5 in interorbital; humeral process obliquely truncate posteriorly; pectoral spine longer than dorsal, a little longer than head; D. I, 5; A. 11; 30 lateral scutes; pale olive above, white below; fins white.....68. **bachi** (Boulenger).

67. **Nemadoras elongatus** (Boulenger).

Plate XXI, Fig. 6.

Oxydoras elongatus Boulenger, Trans. Zoöl. Soc. London, XIV, 1898, p. 424, Pl. XL, Fig. 4 (Rio Jurua).

This species is known from the type, 105 mm. long, in the British Museum.

68. **Nemadoras bachi** (Boulenger).

Plate XXI, Fig. 5.

Oxydoras bachi Boulenger, l.c., p. 423, Pl. XL, Fig. 3 (Rio Jurua).

This species is known from the type, 90 mm. long, in the British Museum.

EXPLANATION OF PLATES.

General Notations on the Various Plates and Figures.—Notations peculiar to some one plate are given in the legend of the particular plate. For special notations for the auditory apparatus see legend of plate.

1 = Premaxillary; 2 = ethmoid; 3 = lateral ethmoid; 4 = preorbital (laminate); 5 = frontal; 6 = sphenotic; 7 = pterotic; 8 = supra-occipital; 8*a* = basi-occipital; 8*b* = ex-occipital; 9 = epiotic; 9*a* = process of the epiotic; 10 = supraclavicle; 11 = parapophysis of coalesced vertebræ; 12 = maxillary; 13 = palatine; 14 = mesopterygoid; 14a = metapterogoid; 15 = quadrate; 16 = preopercle; 17 = interopercle; 18 = opercle; 19 = hyomandibular; 20 = mandible; 21 = parasphenoid; 22 = prootic; 23 = orbitosphenoid; 23*a* = alisphenoid; *A. B. C.* = suborbitals; *D.* = nasal.

X, *Y*, *Z* = Elements of the dorsal plate.

A. IV = Anterior process of the fourth vertebra.

a = Anterior mesial articular process at base of dorsal spine.

b = Anterior articular surface of the first interneural.

c = Articular surface of the second interneural.

D. S. = Dorsal spine.

d = Opening through the base of the dorsal spine, through which the ring formed by the union of the prolongations of the first and second interneural passes.

E = Modified first dorsal spine.

e = Opening admitting blood vessels, etc., into the interior of the dorsal spine.

f = Flanges or lateral processes of the interneurals.

K = Articular surface at the tip of the lateral flange of the second interneural.

l = Lateral articular surface at the base of the dorsal spine.

O = Modified first dorsal spine, see also *E*.

P. IV = Posterior process of the fourth vertebra.

Pr. IV = Process of the fourth vertebra.

tr = Tripus.

Plate I.

Air-bladders of:

1. *Pterodoras granulosus* (Valenciennes), the ventral wall cut away exposing the inner surface of the dorsal wall.

2. The inner surface of the ventral wall, cut away from 1.

3. Dorsal view of the bladder of *Pterodoras granulosus*.

4. *Megalodoras irwini* Eigenmann. Dorsal view of the bladder of the adult.

5. *Platydoras costatus* (Linnæus), front view of air-bladder showing indentations into which the process of the fourth vertebra fits.

6. *Opsodoras humeralis* (Kner).

7. *Trachydoras paraguayensis* (Eigenmann and Ward).

8. *Doras carinatus* (Linnæus). Note the minute diverticulum at the middle of the lower end of the figure, the beginning of the second air-bladder.

9. *Doras carinatus* (Linnæus). The interior of the anterior end of the air-bladder, seen from immediately behind the cross-partition.

10. *Doras carinatus* (Linnæus). Longitudinal median section showing particularly the ridges and tentacles in the gullet and the air-bladder. The air-bladder shows the median partition "*m*," in the posterior part, the partial cross-partition "*c*," separating the posterior from the anterior part, the flexion over the fourth vertebra "*f*," and "*i*," the intruded end piece of the transverse process of the fourth vertebra.

11. *Astrodoras asterifrons* (Heckel).

12. *Centromochlus heckelii* Filippi, and

13. *Trachycorystes insignis* of the Auchenipteridæ.

14. *Platydoras costatus* (Linnæus).

15. *Oxydoras kneri* Bleeker.

16. *Pseudodoras niger* (Valenciennes).

17. *Doras micropœus* (Eigenmann).

18. *Rhinodoras d'orbignyi* (Kröyer).

Plate II.

Fig. 1. *Doras micropœus* (Eigenmann). I. U. M., 12029, 245 mm., Lama Stop-Off.

Fig. 2. *Leptodoras linnelli* Eigenmann. I. U. M., 12022, about 180 mm., Tumatumari.

Fig. 3. *Acanthodoras spinosissimus* (Eigenmann and Eigenmann). I. U. M., 16005, 63 mm., Maciél.

Fig. 4. *Opsodoras humeralis* (Kner). I. U. M., 15881, about 120 mm., Iquitos.

Fig. 5. *Trachydoras atripes* Eigenmann. I. U. M., 16006, about 100 mm., S. Joaquin.

Fig. 6. *Doras carinatus* (Linnæus). I. U. M., 12026, about 185 mm., Rockstone.

Fig. 7. *Lithodoras dorsalis* (Valenciennes). I. U. M., 4248, about 165 mm., Pará.

Fig. 8. *Centromochlus heckelii* Filippi. Auchenipteridæ.

Fig. 9. *Trachycorystes insignis* (Steindachner). Auchenipteridæ.

Plate III.

Fig. 1. *Platydoras costatus* (Linnæus). I. U. M., 77 mm., Rio Morona.

Fig. 2. *Platydoras costatus* (Linnæus). I. U. M., 17041, 185 mm., Lake Rogoagua.

Fig. 3. *Megalodoras irwini* Eigenmann. I. U. M., 15427, about 300 mm., now shrunken, Iquitos.

Fig. 4. *Centrochir crocodili* (Humboldt). I. U. M., 13557, about 175 mm., Soplaviento.

Fig. 5. *Doras punctatus* Kner. I. U. M., 15883, about 95 mm., Lake Cashiboya.

Fig. 6. *Pterodoras granulosus* (Valenciennes). I. U. M., 15847, 165 mm. to end of middle caudal ray, Iquitos.

Fig. 7. *Franciscodoras marmoratus* (Reinhardt). After Steindachner.

Fig. 8. *Trachydoras paraguayensis* (Eigenmann and Ward). I. U. M., 16009, 81 mm. to end of middle caudal rays, S. Joaquin.

Fig. 9. *Amblydoras hancocki* (Cuvier and Valenciennes). I. U. M., 12034, about 95 mm., Maduni Creek.

Plate IV.

Fig. 1. *Hypodoras forficulatus* Eigenmann. I. U. M., 15876, *type*, 123 mm., Iquitos.

Fig. 2. *Astrodoras asterifrons* (Heckel). I. U. M., 16003, 97 mm., Jutahy.

Fig. 3. *Scorpiodoras heckeli* (Kner). I. U. M., 16002, 161 mm., Teffé.

Fig. 4. *Astrodoras asterifrons* (Heckel). I. U. M., 16035, 80 mm., Santarem.

Fig. 5. *Acanthodoras cataphractus* (Linnæus). I. U. M., 12032, about 82 mm., Gluck Island.

Fig. 6. *Anadoras weddelii* (Castelnau). I. U. M., 5110, 113 mm., Marajo.

Fig. 7. *Anadoras grypus* (Cope). I. U. M., 15849, about 120 mm., Lake Cashiboya.

PLATE V.

Radiographs by Dr. J. E. P. Holland.

FIGS. 1 AND 2. *Platydoras costatus* (Linnæus).
FIGS. 3 AND 4. *Leptodoras linnelli* Eigenmann.
FIGS. 5 AND 6. *Centrochir crocodili* (Humboldt).

PLATE VI.

Photographs of *Megalodoras irwini* Eigenmann. I. U. M., 15427, about 300 mm.

FIG. 1. Membrane bones of left side from without.
FIG. 2. Dorsal view.
FIG. 3. Suborbitals and preorbital.
FIG. 4. Ventral view of the skull and membrane bones.

PLATE VII.

Megalodoras libertatis (Ribeiro).

Drawings of the type by E. Cruzlima. Loaned by Dr. A. Ribeiro.

PLATE VIII.

Photographs of *Pterodoras granulosus* (Valenciennes).

FIG. 1. Side view. I. U. M., 15848, 180 mm., R. Puinahua.
FIG. 2. Dorsal view of the same.
FIG. 3. Side view of another specimen, skeletonized.
FIG. 4. Ventral view of skull and membrane bones.

PLATE IX.

FIG. 1. Side view of a young specimen of *Platydoras costatus* (Linnæus). I. U. M., 15874, 82 mm., Lake Yarinococha.

FIG. 2. Dorsal view of an older specimen.

FIG. 3. Side view of skeleton. I. U. M., 17041, Lake Rogoagua.

FIG. 4. Ventral view of *Centrochir crocodili* (Humboldt). 13557, about 175 mm., Soplaviento. See Plate III, Fig. 4.

FIG. 5. *Centrochir crocodili* (Humboldt). From Steindachner.

FIG. 6. *Lithodoras dorsalis* (Valenciennes). I. U. M., 4248 mm., about 165 mm., Pará. See Plate II, Fig. 7.

PLATE X.

FIGS. 1 AND 2. *Acanthodoras cataphractus* (Linnæus). I. U. M., 12032, 86 mm., Gluck Island.
FIGS. 3 AND 4. The same from a specimen 80 mm.
FIG. 5. *Acanthodoras calderonensis* (Vaillant). After Steindachner.

PLATE XI.

FIGS. 1 AND 2. *Acanthodoras spinosissimus* Eigenmann and Eigenmann. C. M., 7167, Maciél.
FIGS. 3 AND 4. The same. I. U. M., 16005, 93 mm., Maciél.

PLATE XII.

FIG. 1. *Megalodoras uranoscopus* Eigenmann and Eigenmann. M. C. Z. type, 530 mm. E. N. Fischer, del.

FIG. 2. Dorsal view of the same specimen.

PLATE XIII.

Amblydoras hancockii (Cuvier and Valenciennes).

FIGS. 1 AND 2. I. U. M., 12034, 118 mm., Maduni Creek.

FIG. 3. I. U. M., 12033, 112 mm., Lama Stop-Off.

FIG. 4. I. U. M., 12034, 95 mm., Maduni Creek.

PLATE XIV.

Anadoras insculptus (Ribeiro).

Drawings by E. Cruzlima, based on the type in the Museu Nacional, Rio de Janeiro. Dr. A. Ribeiro has kindly loaned these drawings.

PLATE XV.

FIGS. 1–3. *Anadoras grypus* (Cope). 1 and 2, I. U. M., 15849, 139 mm., Lake Cashiboya. 3, 118 mm., Lake Cashiboya.

FIG. 4. *Anadoras weddelii* (Castelnau). I. U. M., 5110, 113 mm., Marajo.

PLATE XVI.

FIG. 1. Ventral view, 2. dorsal view of *Physopyxis lyra* Cope. *Type* in Mus. Phila. Acad. Sci.

FIG. 3. *Amblydoras monitor* (Cope). *Type* in Mus. Phila. Acad. Sci.

FIG. 4. *Agamyxis pectinifrons* (Cope). *Type* in Mus. Phila. Acad. Sci.

PLATE XVII.

FIGS. 1–4. *Pseudodoras niger* (Valenciennes). Figs. 1 and 2, side and dorsal views of a specimen I. U. M. 16004, 273 mm. to end of the scutes, Rio Pacaya. Allen. Figs. 3 and 4, I. U. M., 15651, about 237 mm., Iquitos.

PLATE XVIII.

FIGS. 1 AND 2. *Trachydoras nattereri* (Steindachner). I. U. M., 15967, 131 mm., Iquitos.

FIGS. 3–5. *Trachydoras paraguayensis* (Eigenmann and Ward). Fig. 3, C. M., part of 7201, San Joaquin. Figs. 4 and 5, I. U. M., 16009, 81 mm. to end of middle caudal rays, San Joaquin.

FIGS. 6–8. *Trachydoras atripes* Eigenmann. Figs. 6 and 7, I. U. M., 16006, about 100 mm., San Joaquin. Fig. 8, C. M., part of 1201, San Joaquin.

FIGS. 9–10. *Opsodoras humeralis* (Kner). I. U. M., 15881, about 120 mm., Iquitos.

PLATE XIX.

FIG. 1. *Opsodoras humeralis* (Kner). I. U. M., 15881, 138 mm., Iquitos. W. S. Atkinson, del.

FIG. 2. *Opsodoras hemipeltis* Eigenmann. I. U. M., 50879, *type*, 143 mm., Contamana. W. S. Atkinson, del.

FIG. 3. *Opsodoras parallelus* Eigenmann. I. U. M., 15964, *type*, 151 mm., Iquitos. W. S. Atkinson, del.

PLATE XX.

FIGS. 1–2. *Doras carinatus* (Linnæus). I. U. M., 12026, about 185 mm., Rockstone.

FIG. 3. *Doras micropœus* (Eigenmann). I. U. M., 12029, 245 mm., Lama Stop-Off.

FIG. 4. *Leptodoras linnelli* Eigenmann. I. U. M., 12022, about 180 mm., Tumatumari.

PLATE XXI.

FIGS. 1 AND 2. *Doras punctatus* Kner. I. U. M., 15882, 118 mm., R. Paranapura, Yurimaguas.

FIGS. 3 AND 4. *Doras eigenmanni* (Boulenger). After Boulenger.

FIG. 5. *Nemadoras bachi* (Boulenger). After Boulenger.

FIG. 6. *Nemadoras elongatus* (Boulenger). After Boulenger.

PLATE XXII.

FIG. 1. *Franciscodoras marmoratus* (Reinhardt). After Lütken.

FIG. 2. *Hassar wilderi* Kindle. I. U. M., 5120, *type*, about 165 mm., Troceras. W. S. Atkinson, del.

FIG. 3. *Opsodoras orthacanthus* Eigenmann. I. U. M., 15884, *type*, 133 mm., Iquitos. W. S. Atkinson, del.

PLATE XXIII.

FIG. 1. *Pseudodoras niger* (Valenciennes). T. R. S., No. 743, 840 mm., Koriabo, Guiana. Photo by John Tee-Van of a giant specimen from Kartabo.

FIG. 2. *Opsodoras orthacanthus* Eigenmann. I. U. M., 15884, *type*, 133 mm., Iquitos. See Plate 22, Fig. 3.

FIG. 3. *Opsodoras parallelus* Eigenmann. I. U. M., 15964, *type*, 151 mm., Iquitos. See Plate 19, Fig. 3.

FIG. 4. *Doras carinatus* (Linnæus). I. U. M., 12026, 233 mm. to end of middle caudal rays, Rockstone.

PLATE XXIV.

FIGS. 1 AND 2. *Doras micropœus* (Eigenmann). Fig. 2, I. U. M., 12029, 245 mm., Lama Stop-Off.

FIGS. 3–5. *Leptodoras linnelli* Eigenmann. Fig. 3, I. U. M., 12022, 167 mm., Tumatumari. Fig. 4, The same specimen from below. Fig. 5, I. U. M., 12022, about 180 mm., Tumatumari. See Plate 2, Fig. 2.

FIG. 6. *Opsodoras hemipeltis* Eigenmann. I. U. M., 15964, *type*, 151 mm., Iquitos.

PLATE XXV.

FIG. 1. *Megalodoras irwini* Eigenmann. T. R. S., 2567, 612 mm., Kartabo.

FIG. 2. *Megalodoras irwini* Eigenmann. I. U. M., 15427, *type*, 278 mm., Iquitos. W. S. Atkinson, del.

FIG. 3. *Hypodoras forficulatus* Eigenmann. I. U. M., 15876, *type*, 123 mm., Iquitos. W. S. Atkinson, del.

PLATE XXVI.

FIG. 1. *Trachydoras paraguayensis* (Eigenmann and Ward). I. U. M., 10127, *type*, 87 mm., Corumba. W. S. Atkinson, del.

FIGS. 2 AND 3. *Trachydoras nattereri* (Steindachner). After Steindachner.

FIG. 4. *Trachydoras atripes* Eigenmann. I. U. M., 15877, *type*, 60 mm. to end of lateral scutes, Iquitos. W. S. Atkinson, del.

FIG. 5. *Rhinodoras d'orbignyi* (Kröyer). I. U. M., 9837, *type* of *Doras nebulosus* Eigenmann and Ward, 160 mm., Asuncion. W. S. Atkinson, del.

PLATE XXVII.

FIGS. 1 AND 2. *Opsodoras morei* (Steindachner). After Steindachner.

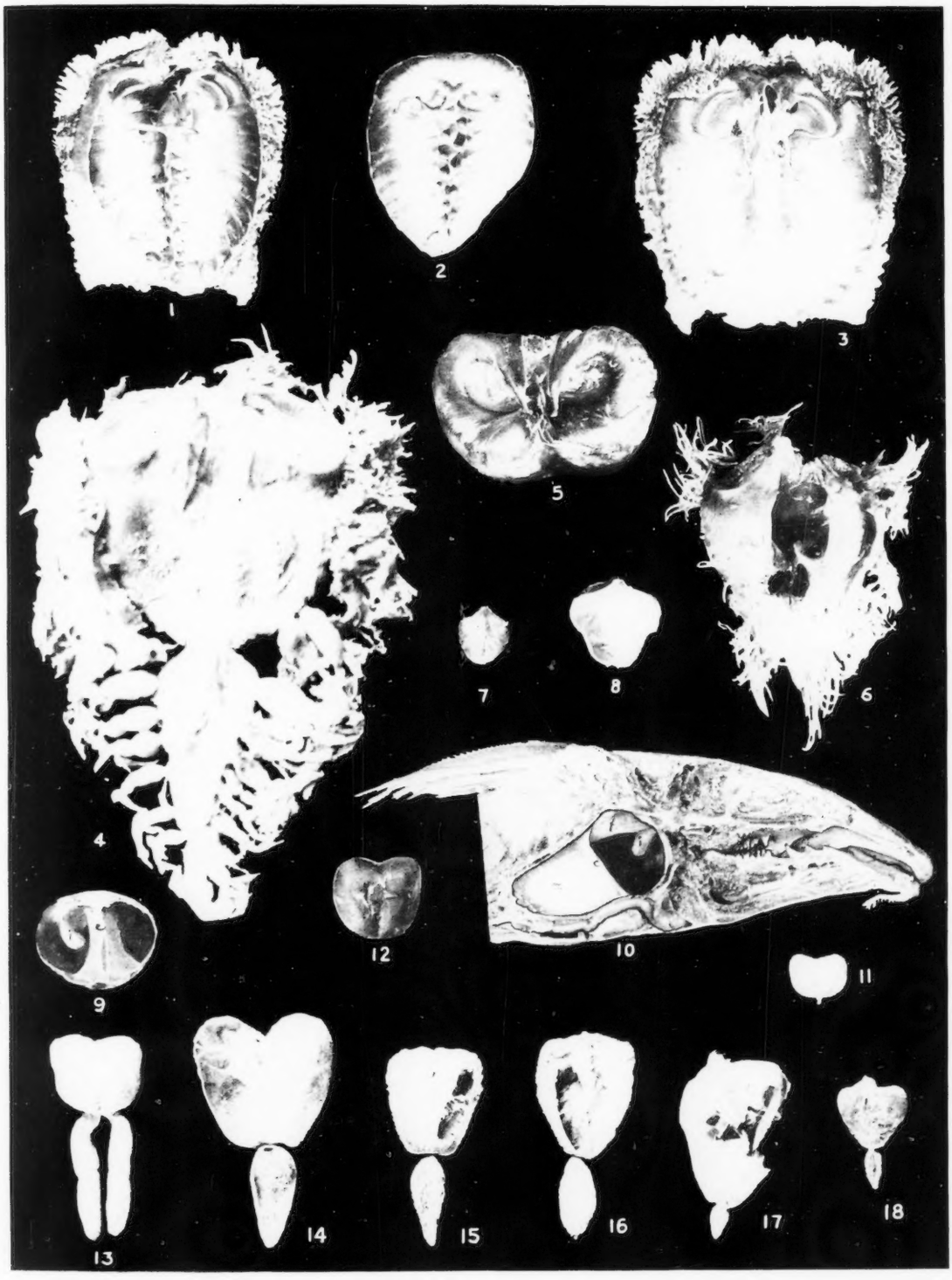
FIG. 3. *Hemidoras morrisi* Eigenmann. I. U. M., 15962, *type*, 147 mm., Iquitos. W. S. Atkinson, del.

FIG. 4. *Hassar affinis* (Steindachner). After Steindachner.

FIG. 5. *Doras micropæus* (Eigenmann). I. U. M., 12029, about 355 mm., Lama Stop-Off. W. S. Atkinson, del.

FIGS. 6–7. *Opsodoras stübeli* (Steindachner). After Steindachner.

FIG. 8. *Doras carinatus* (Linnæus). I. U. M., 12025, about 235 mm., Tumatumari. W. S. Atkinson, del.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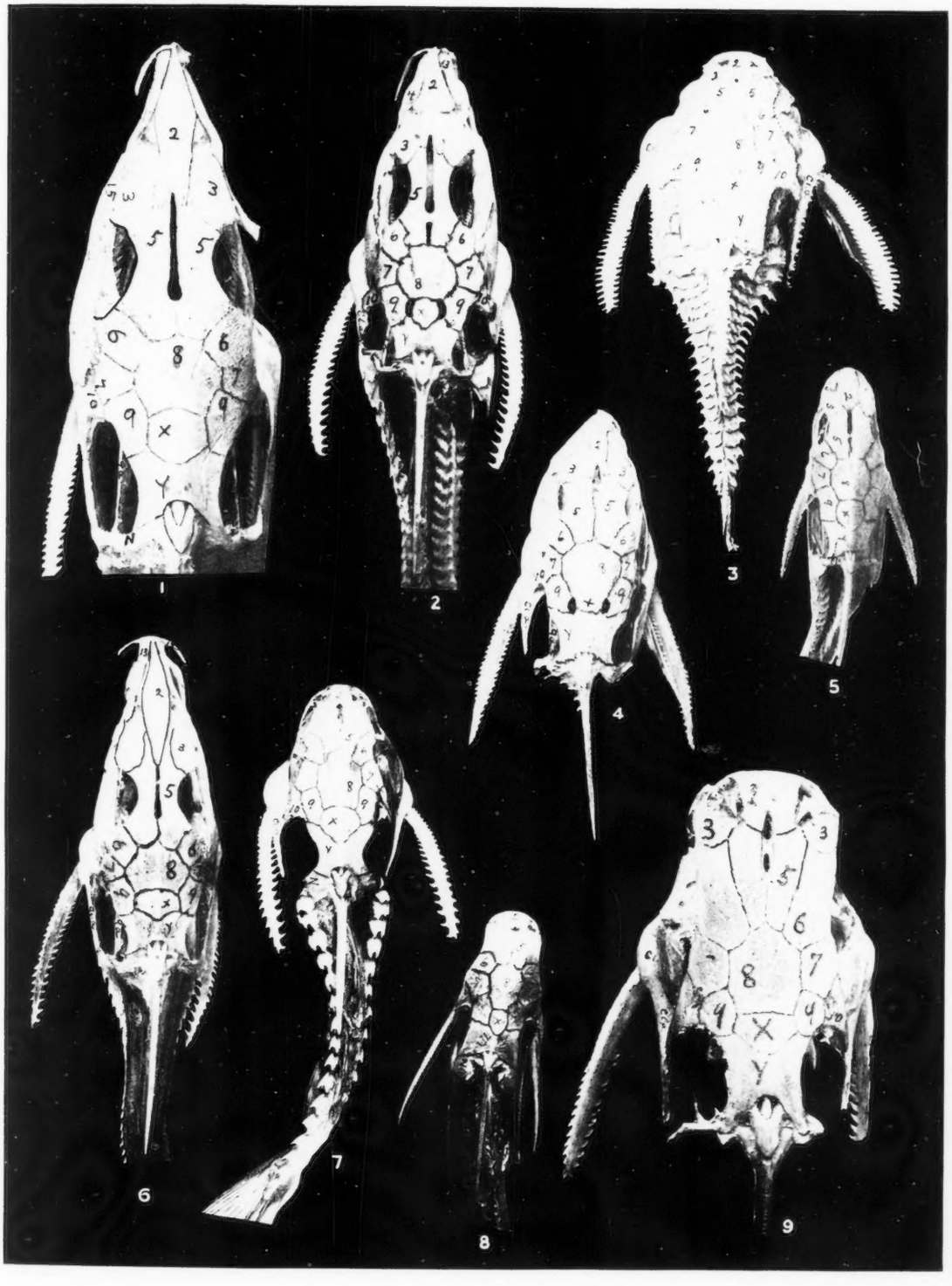
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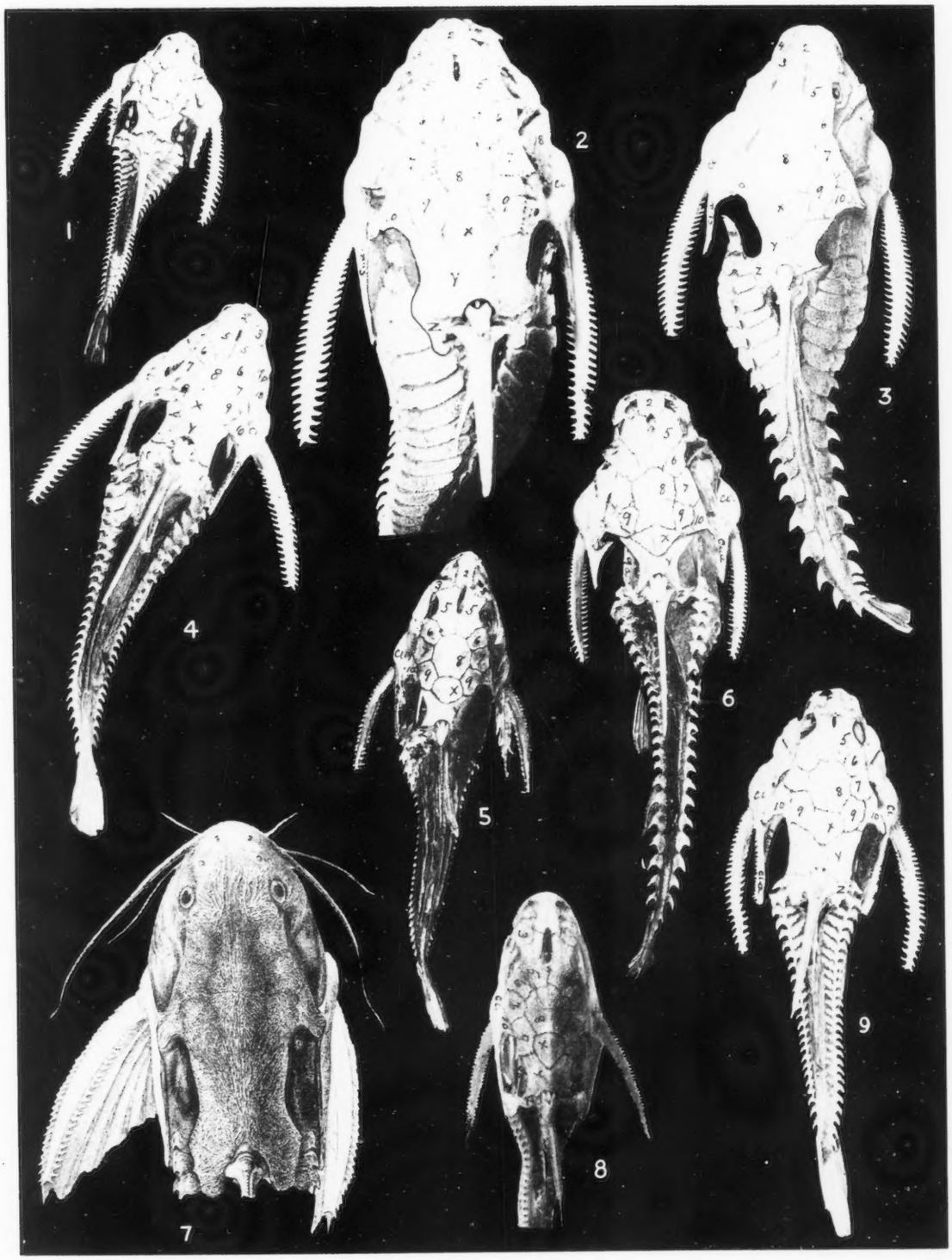
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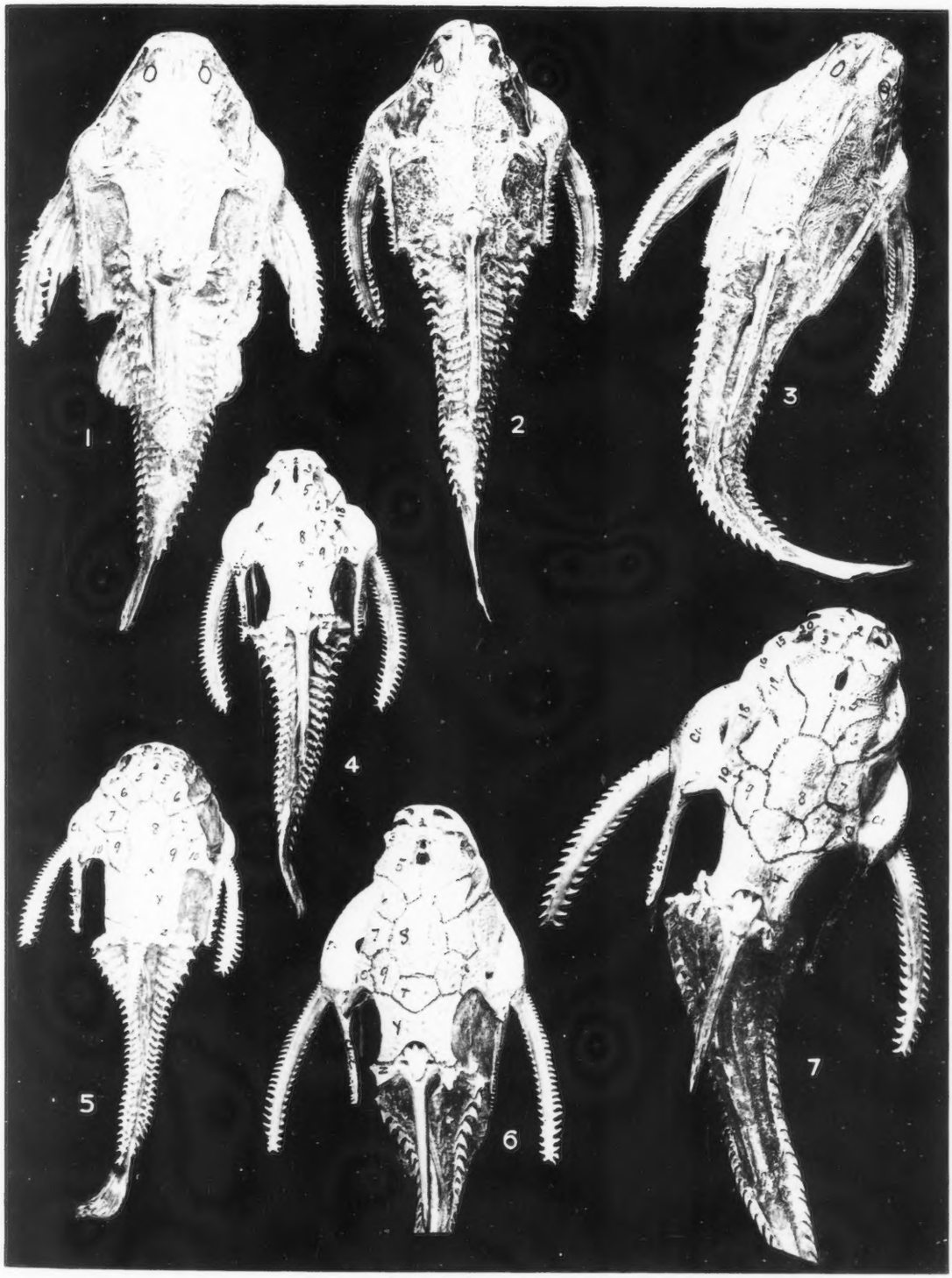
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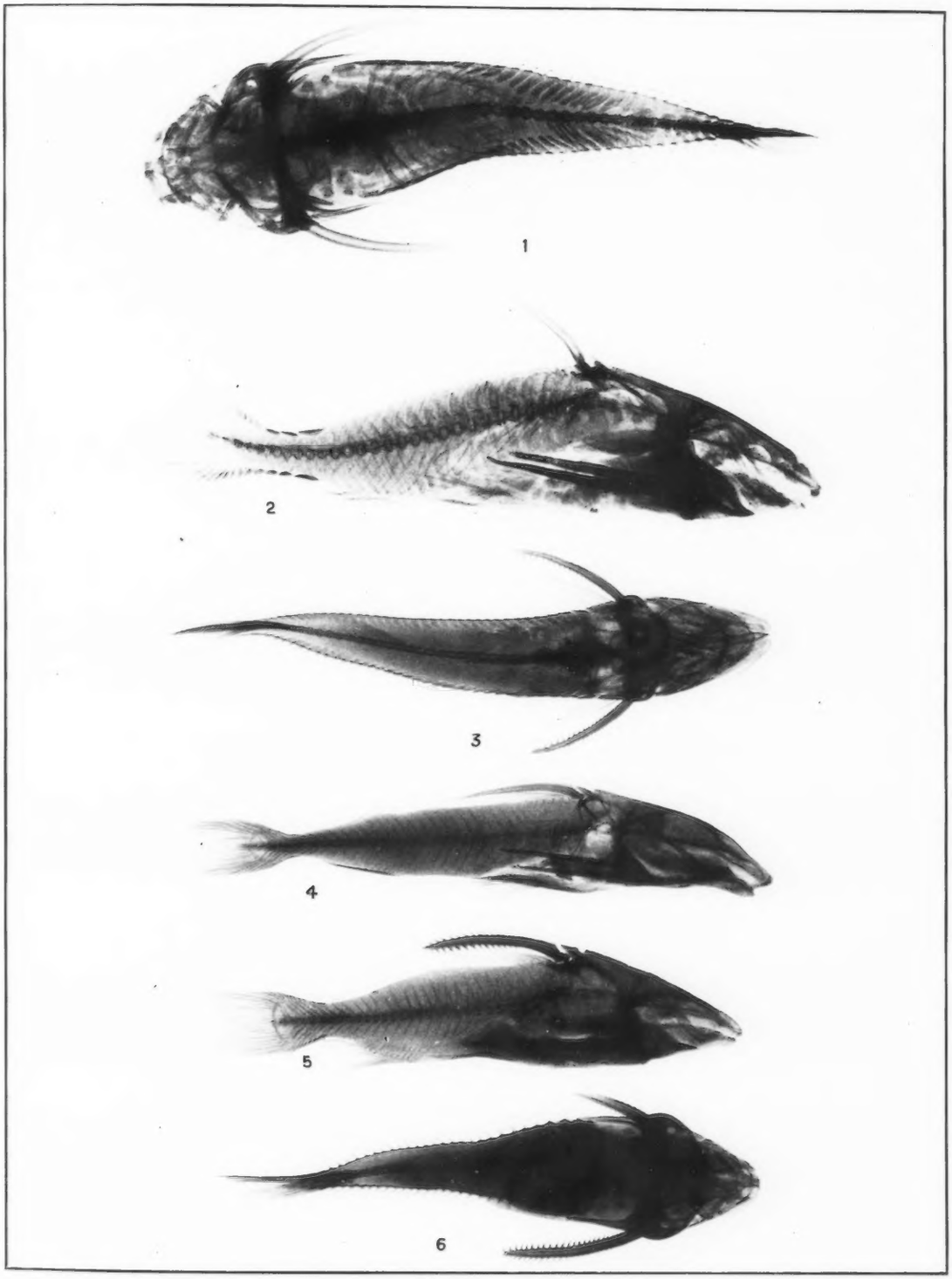
PLATE V.



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3
4
A
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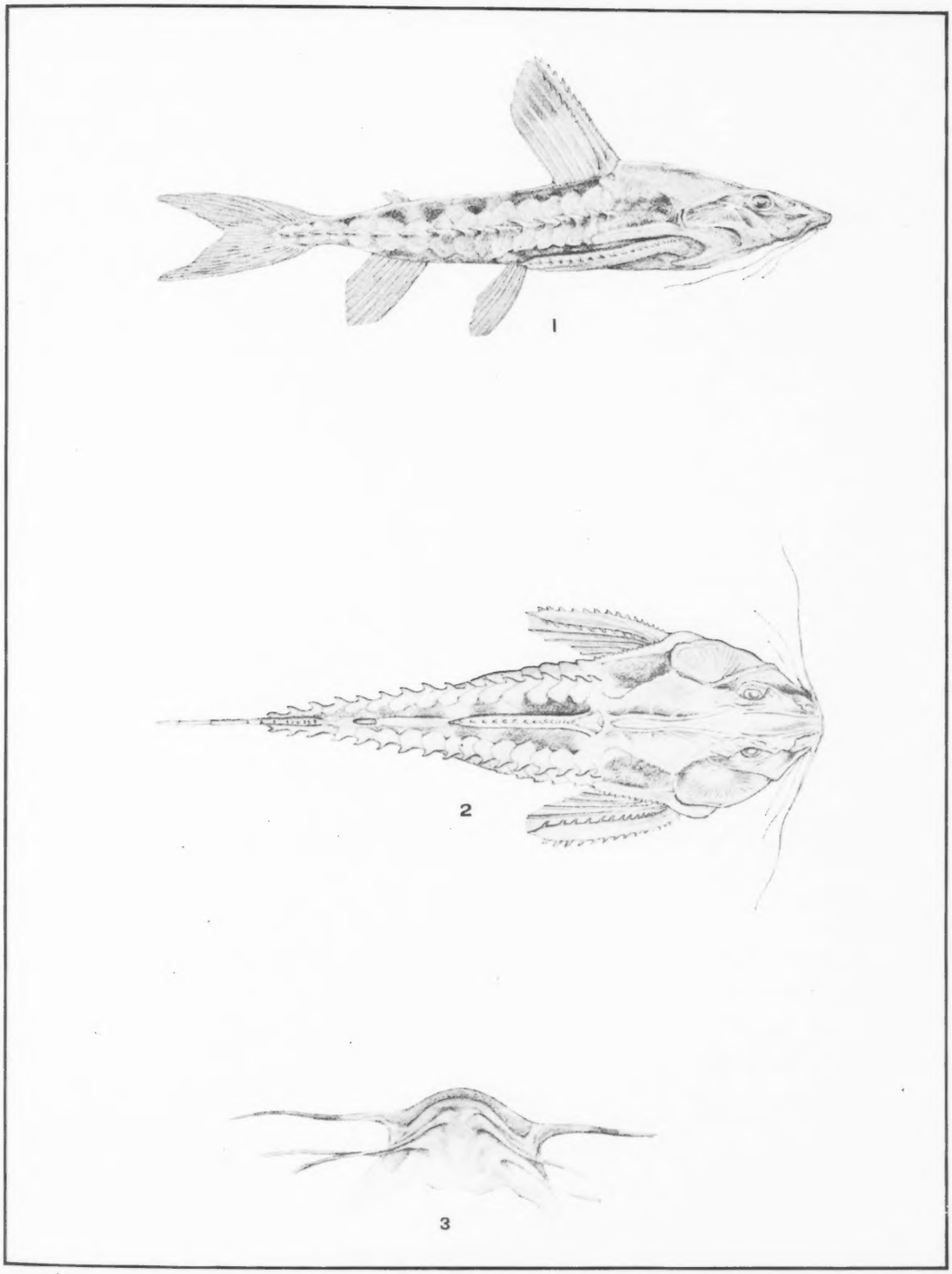
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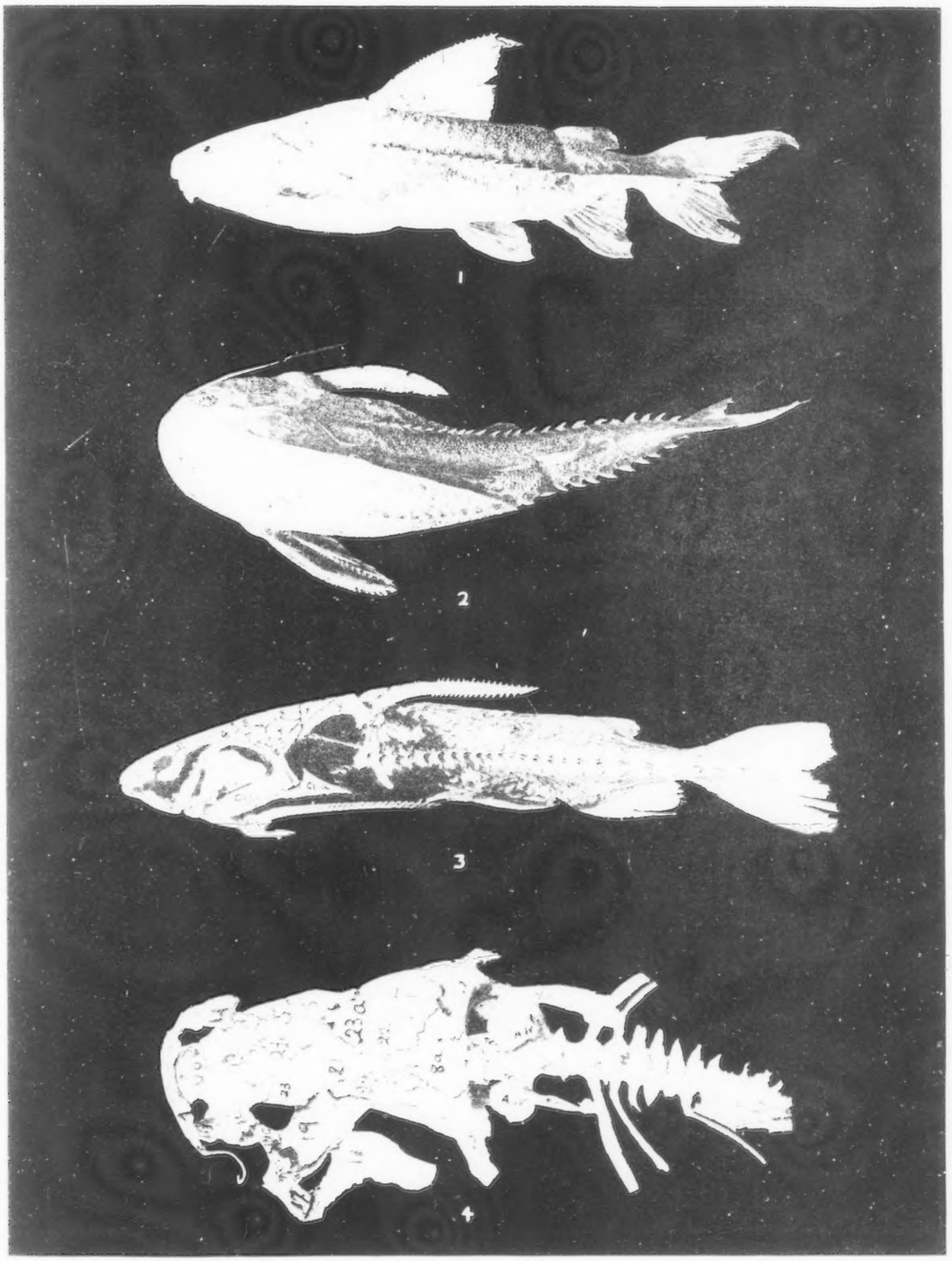
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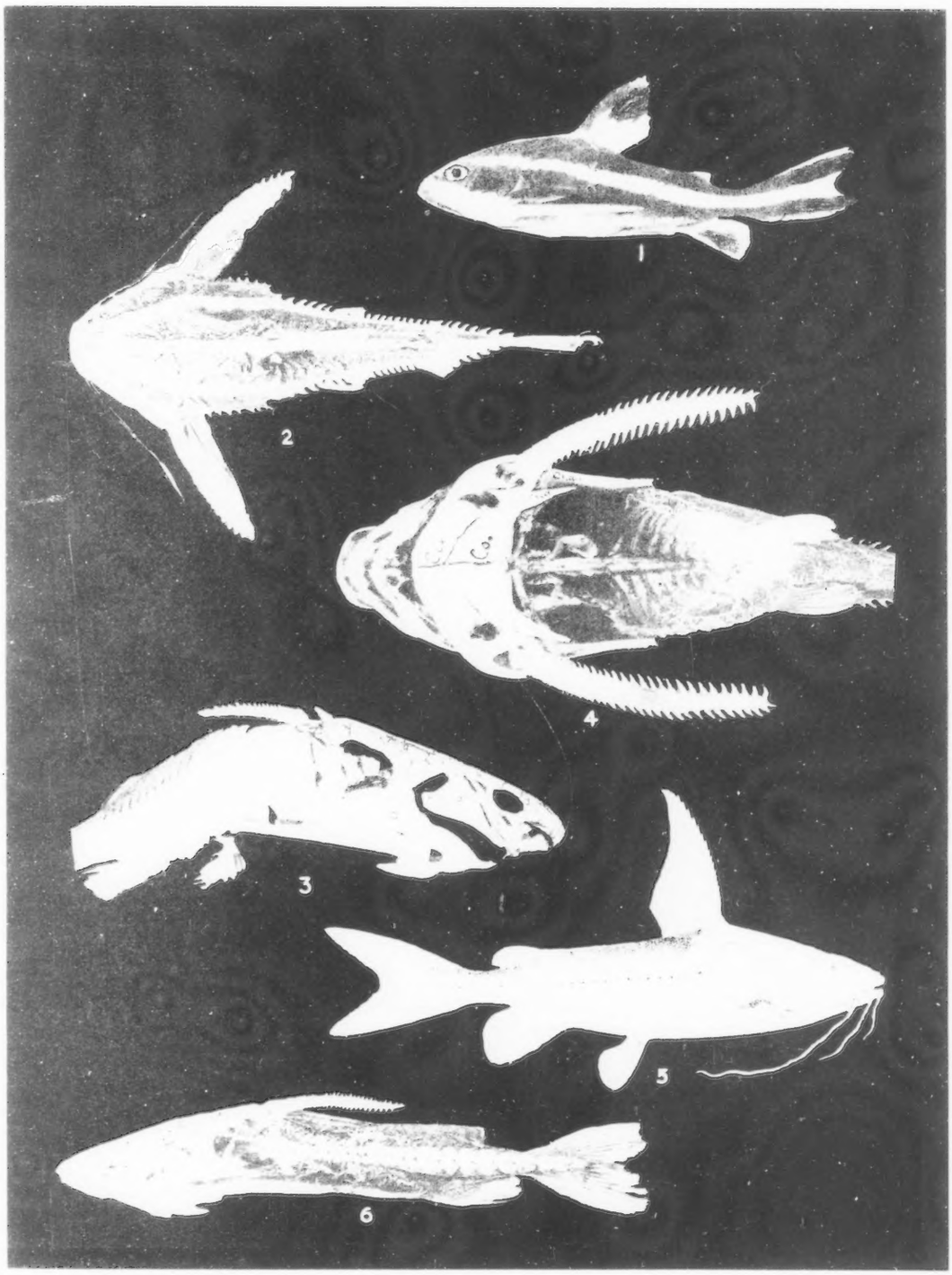
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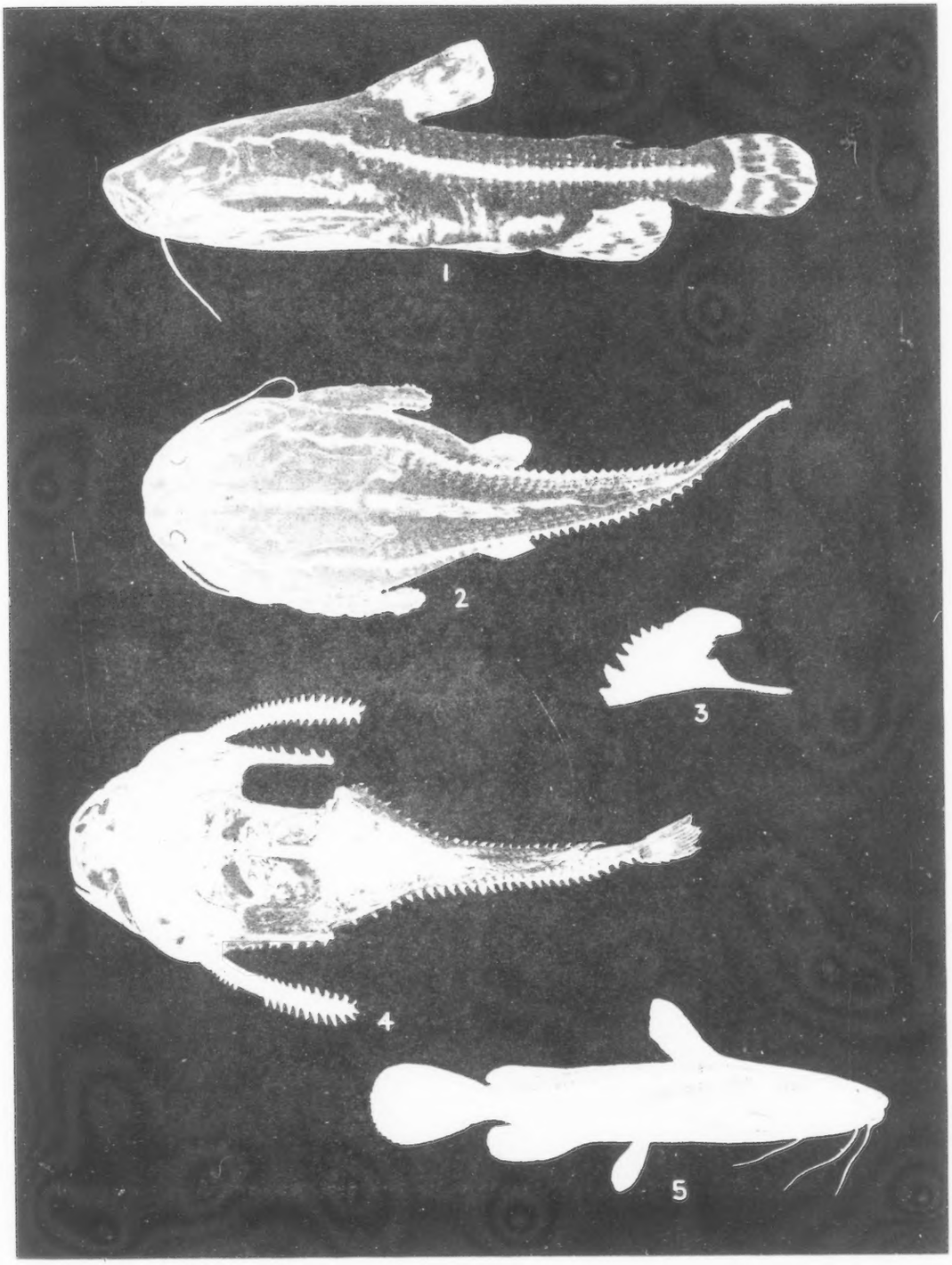
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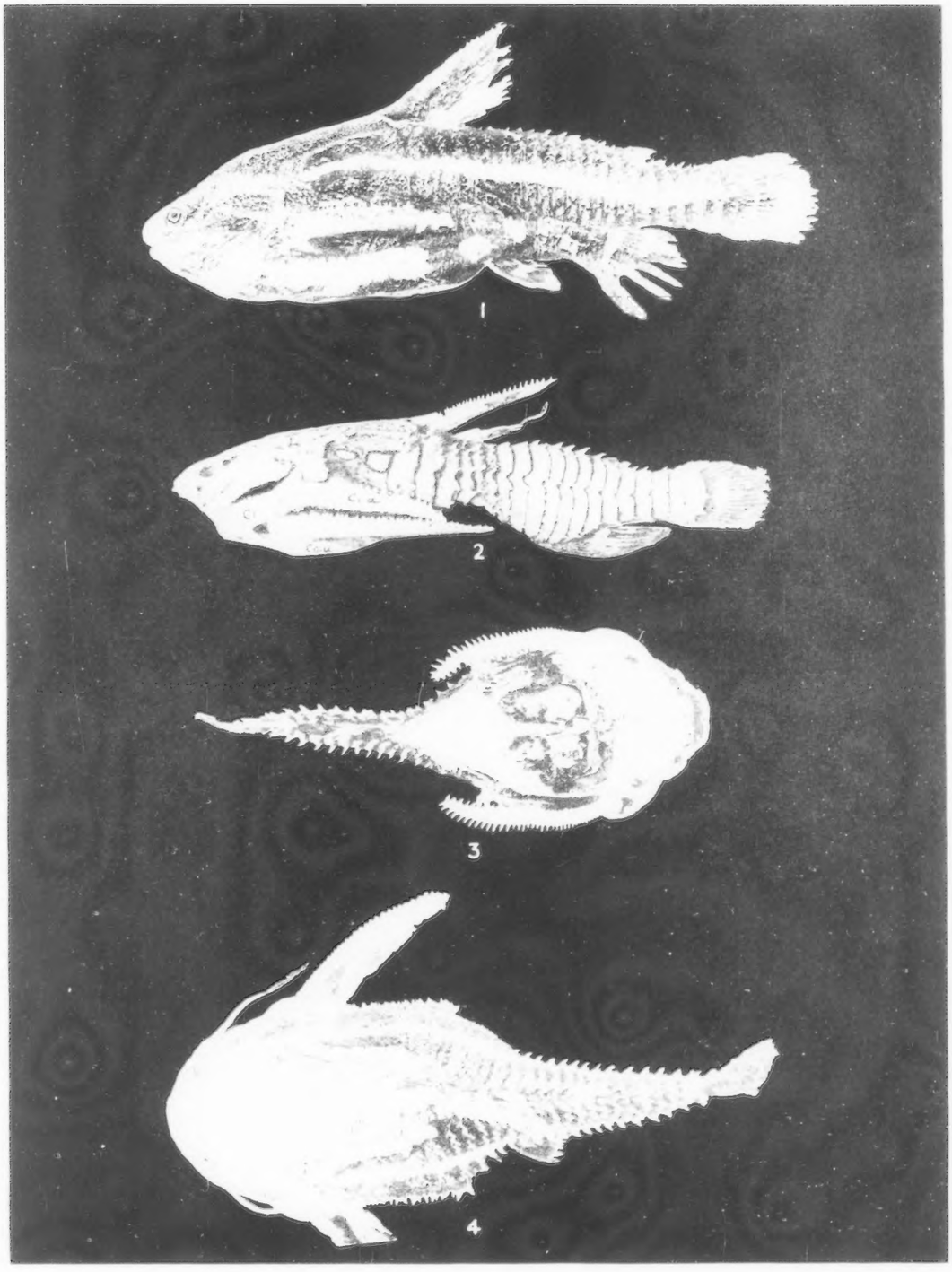
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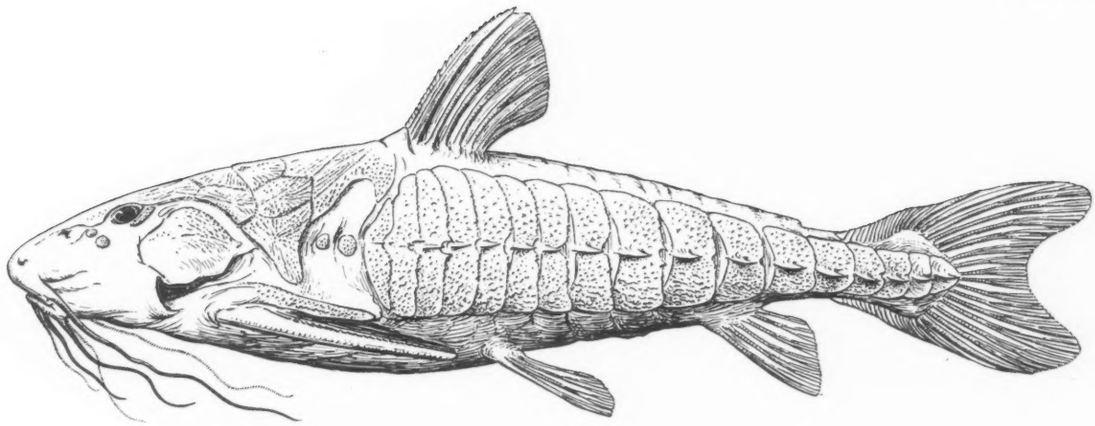

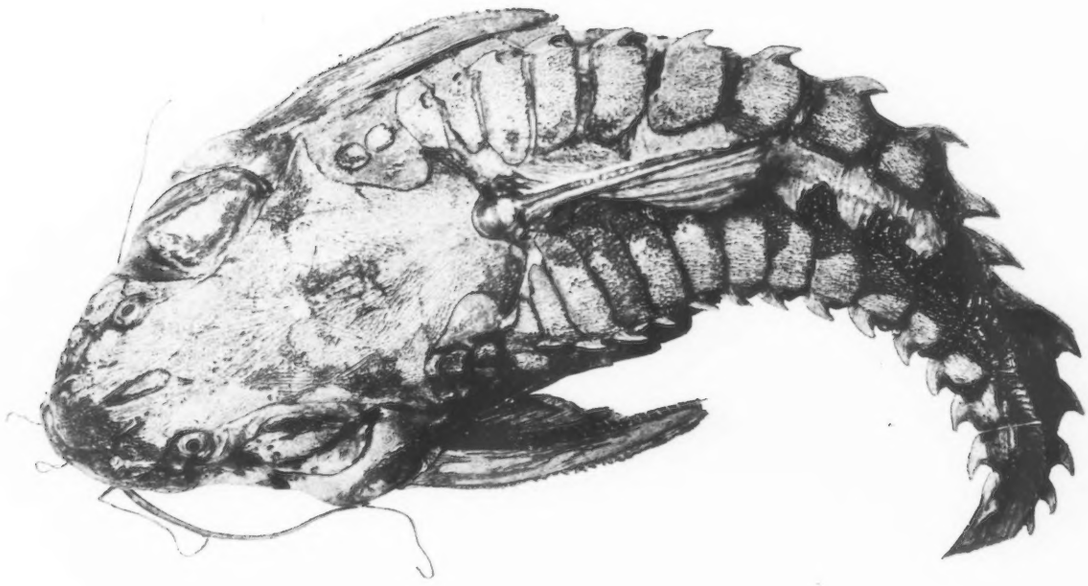

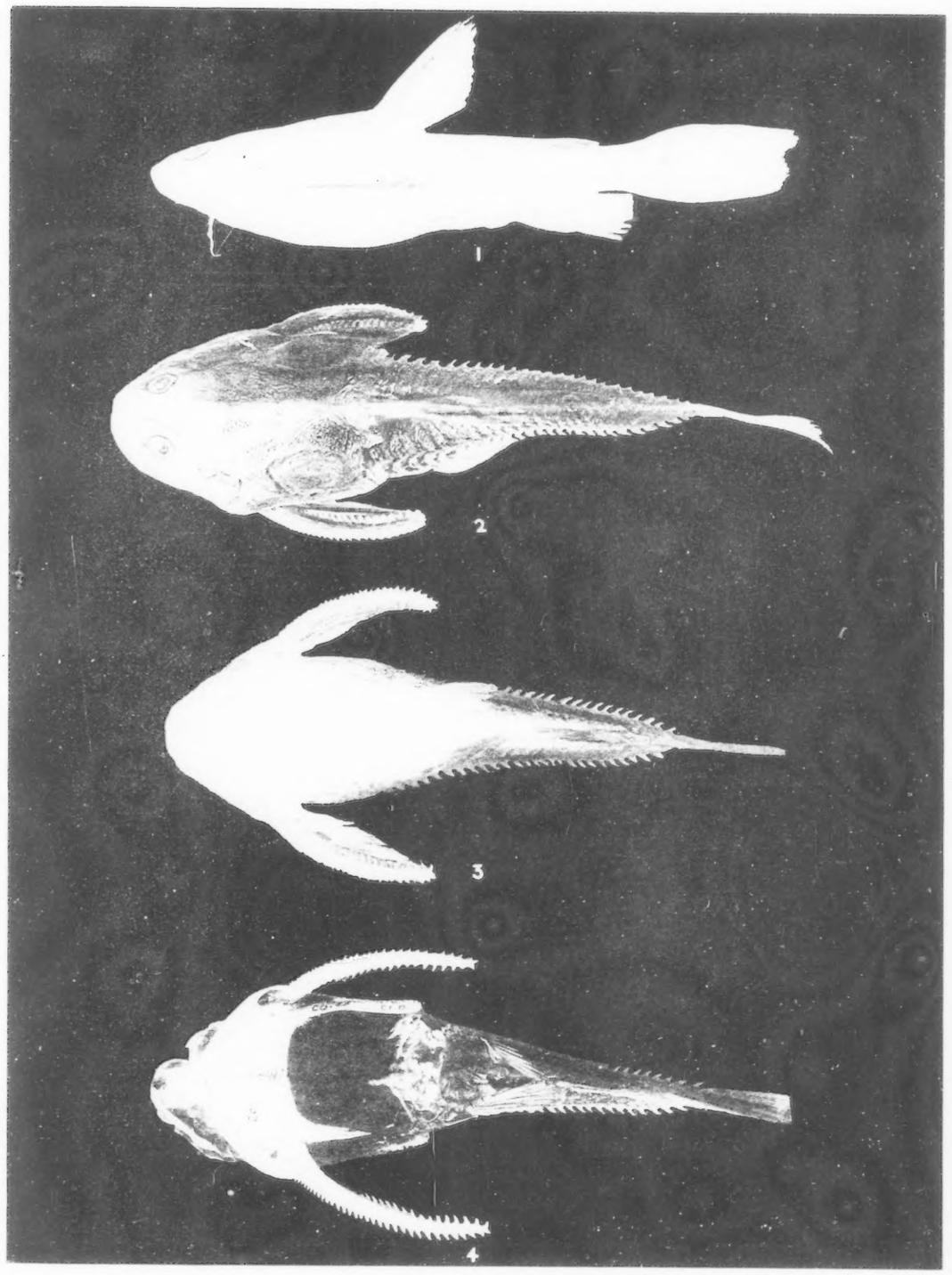
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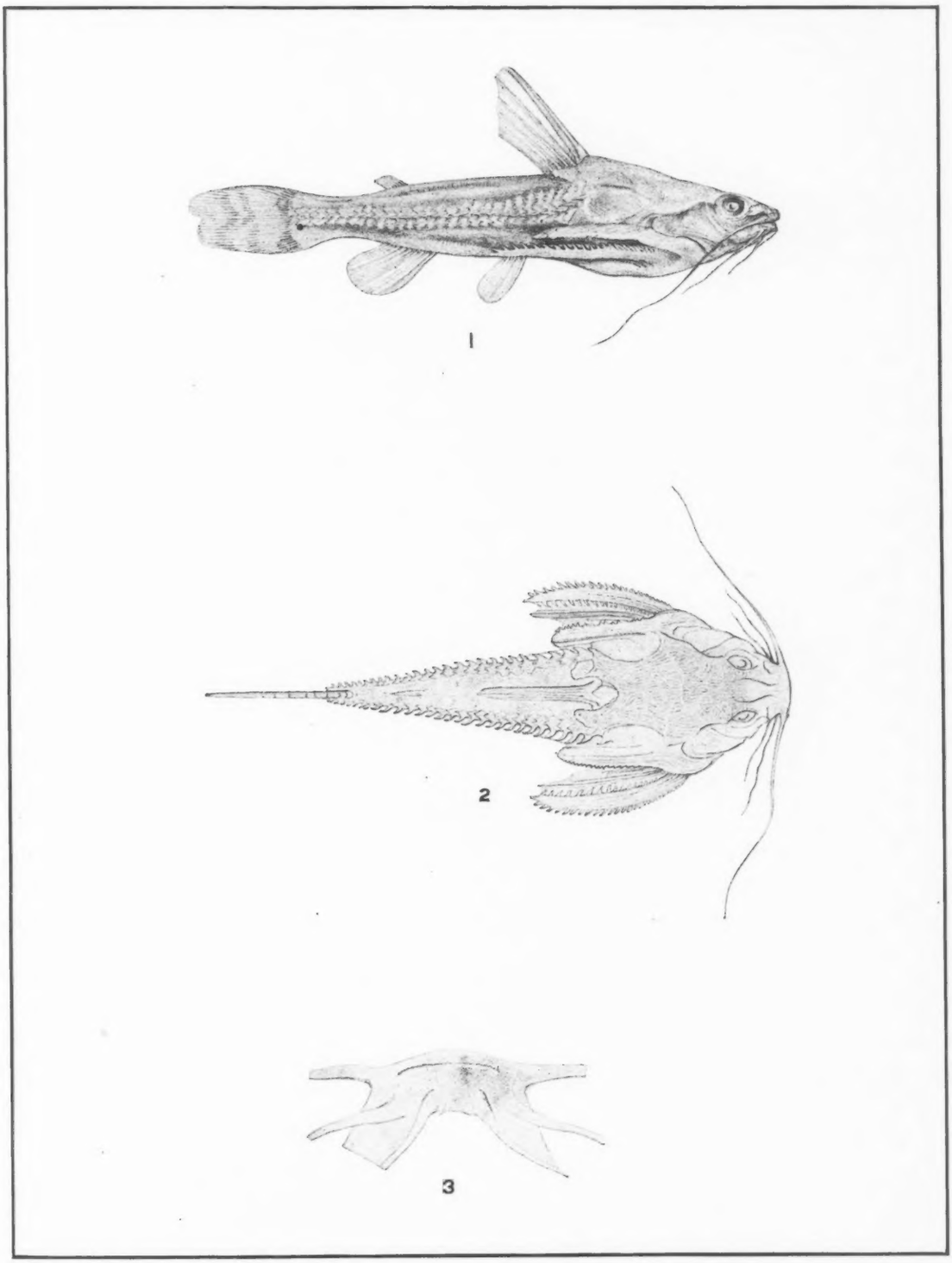

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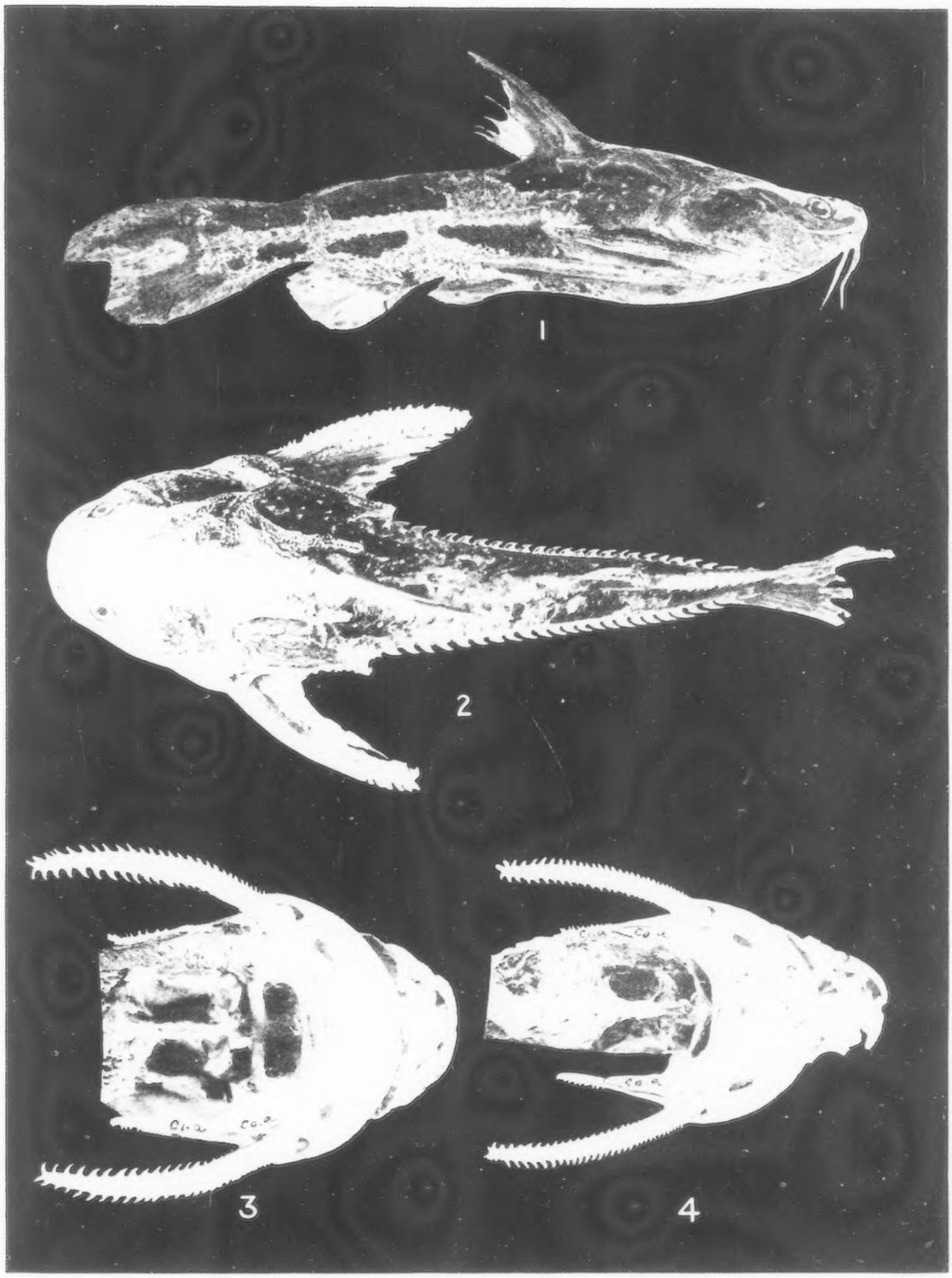
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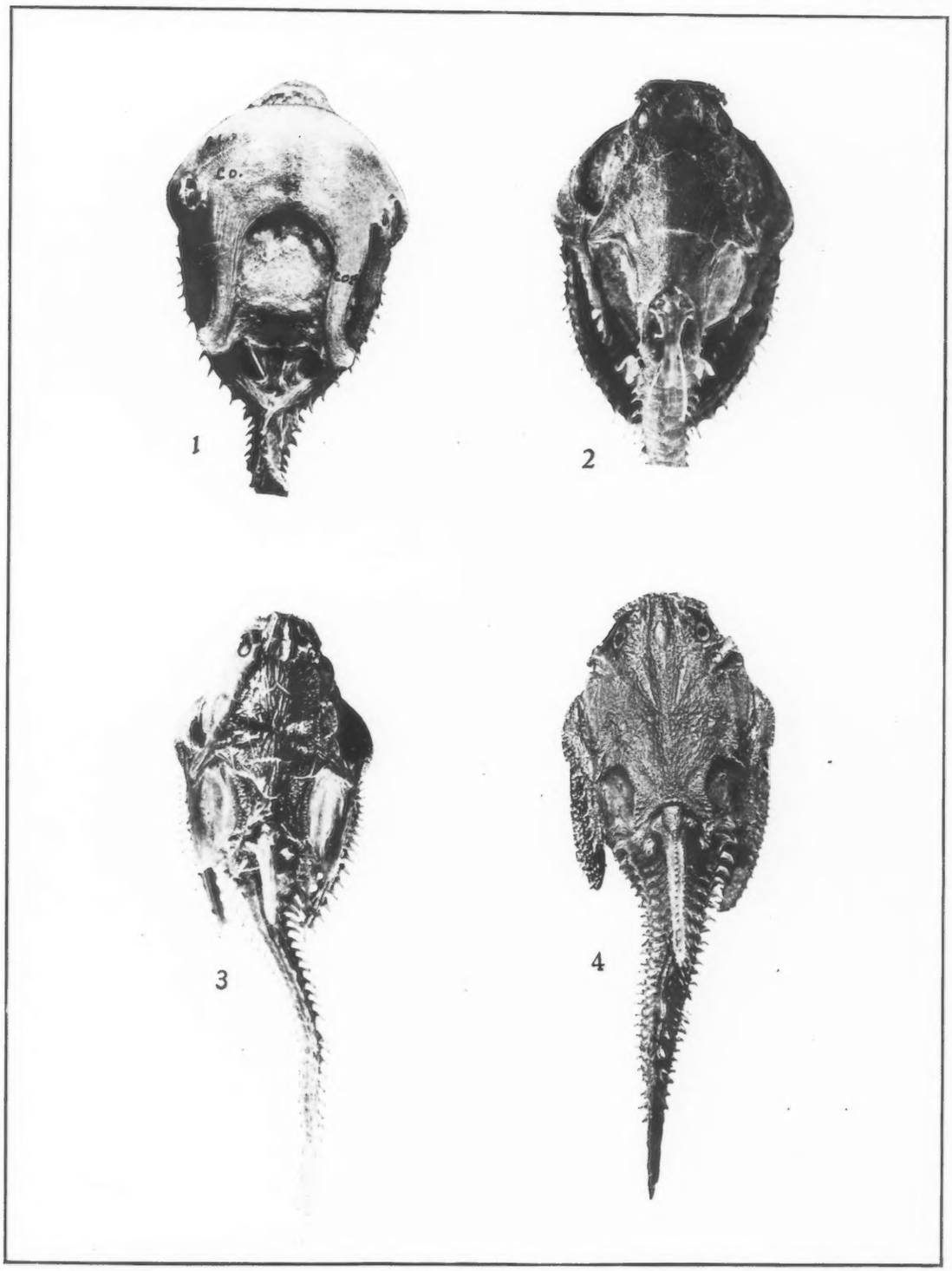
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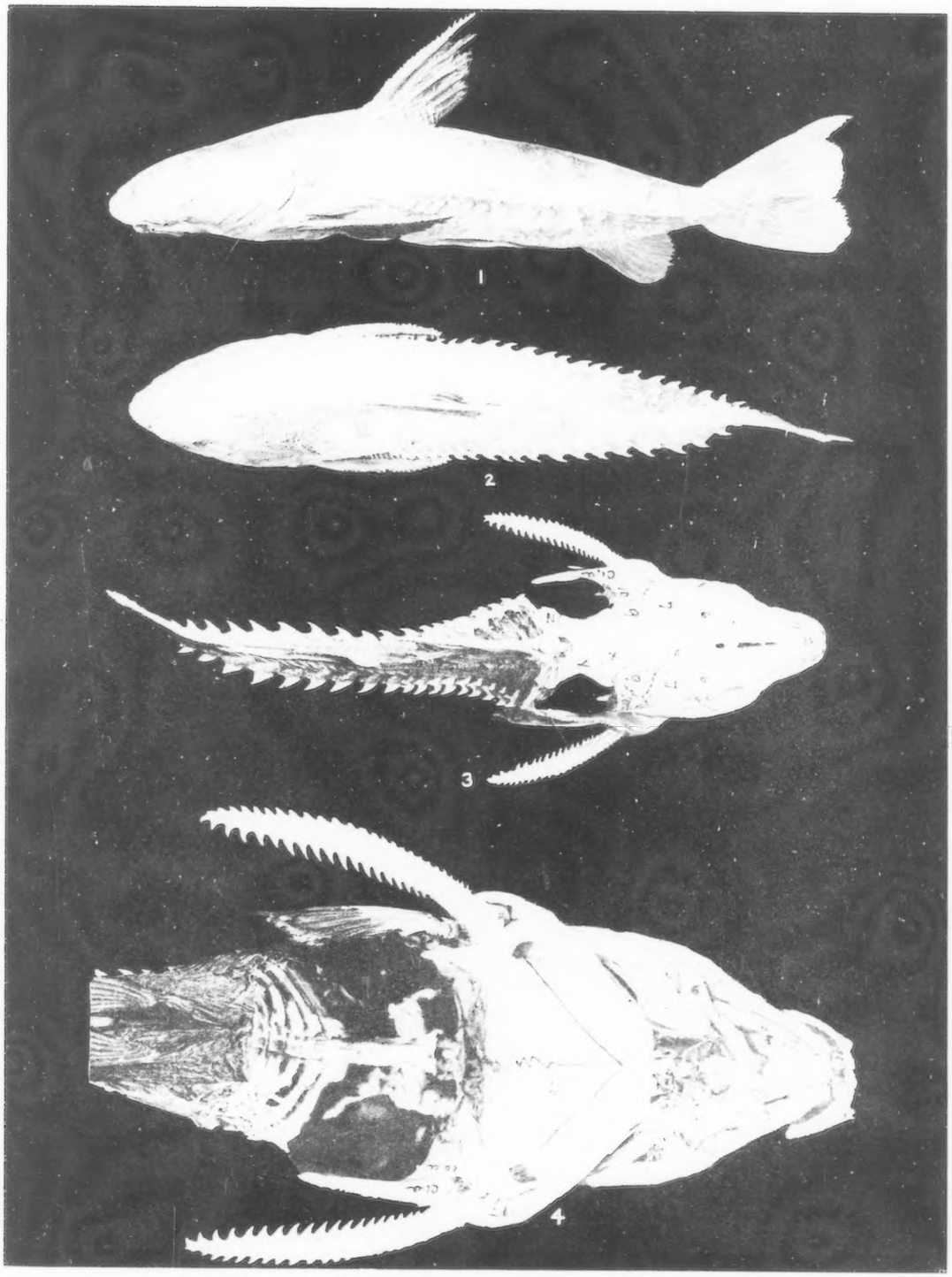
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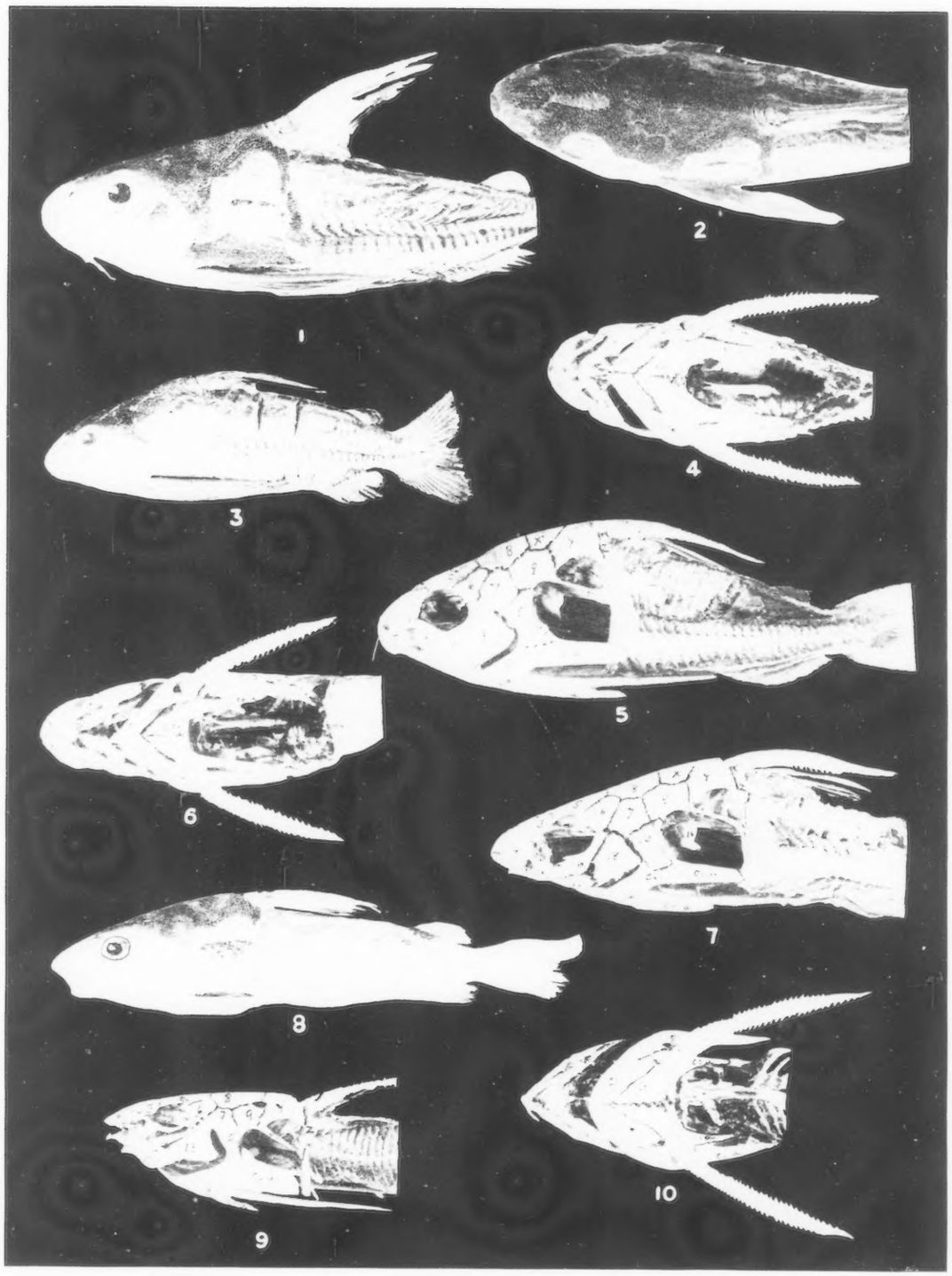
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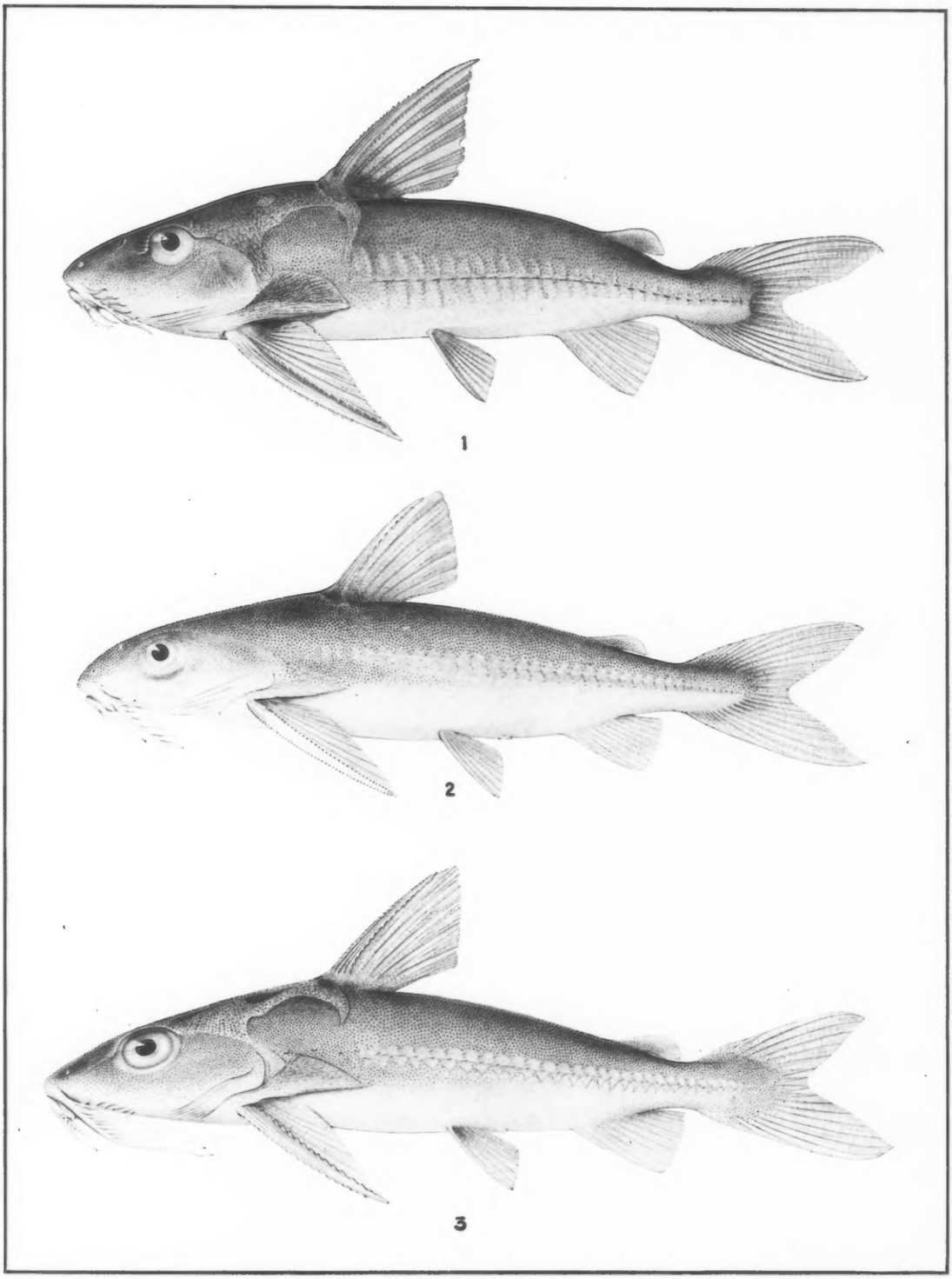
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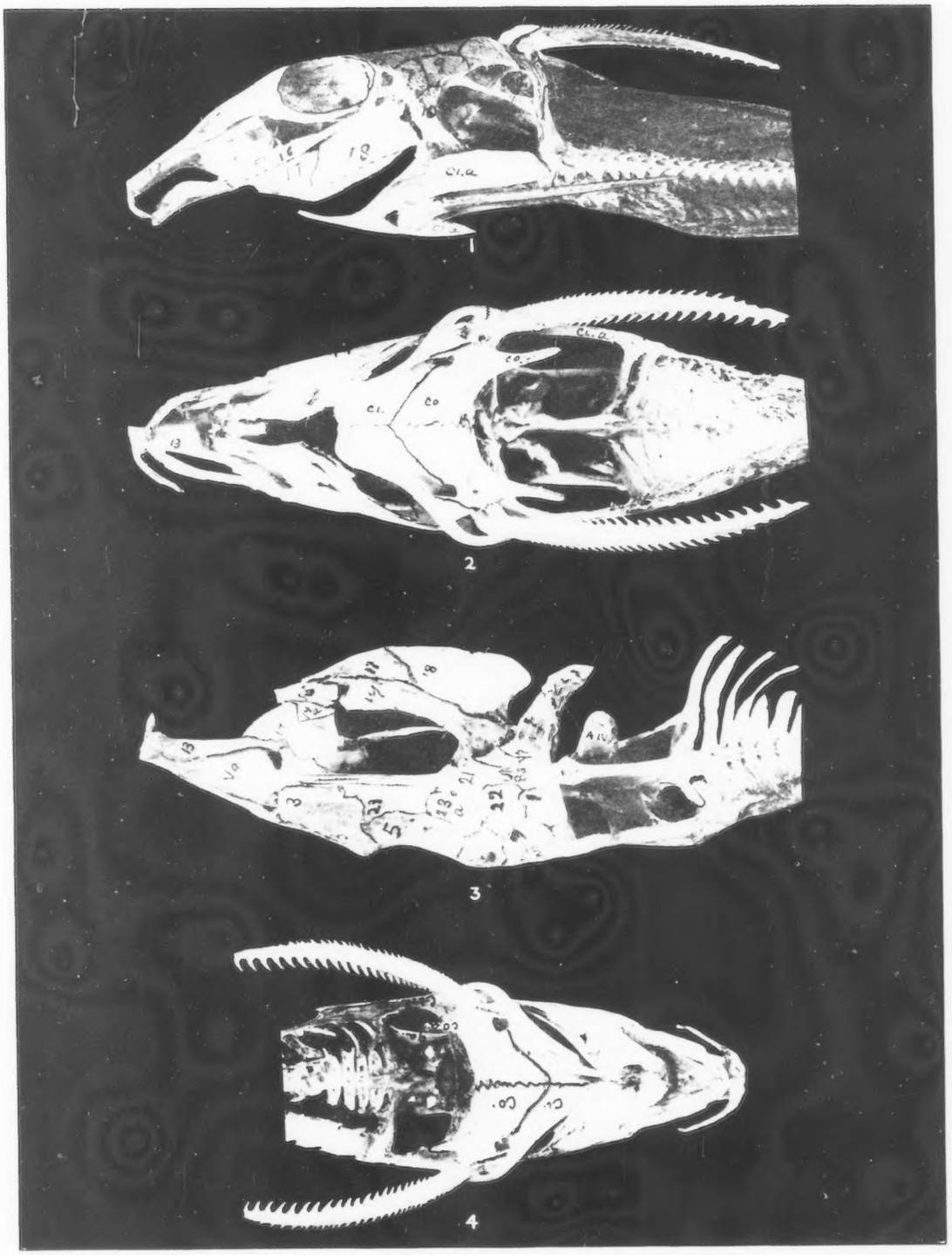
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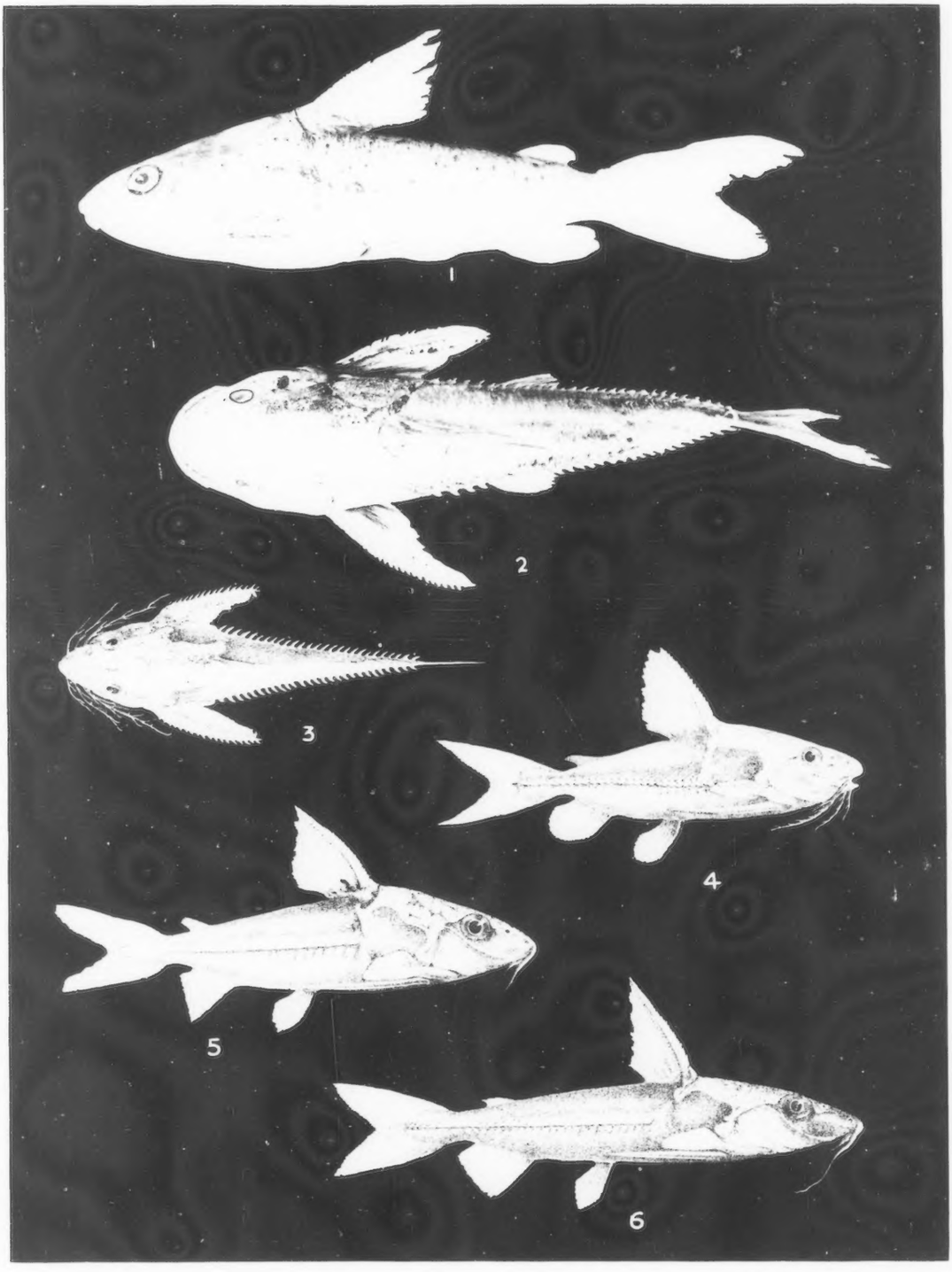
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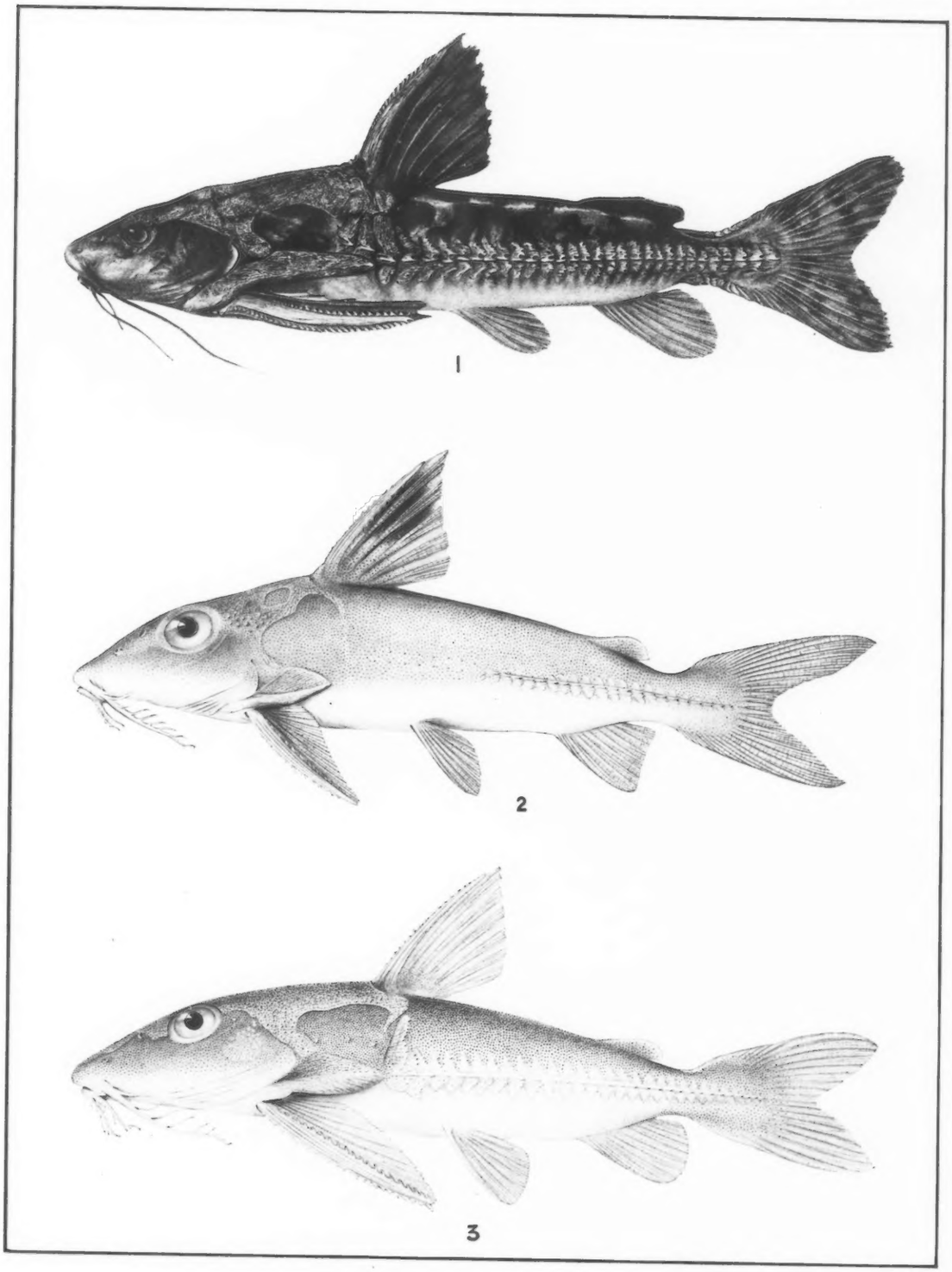
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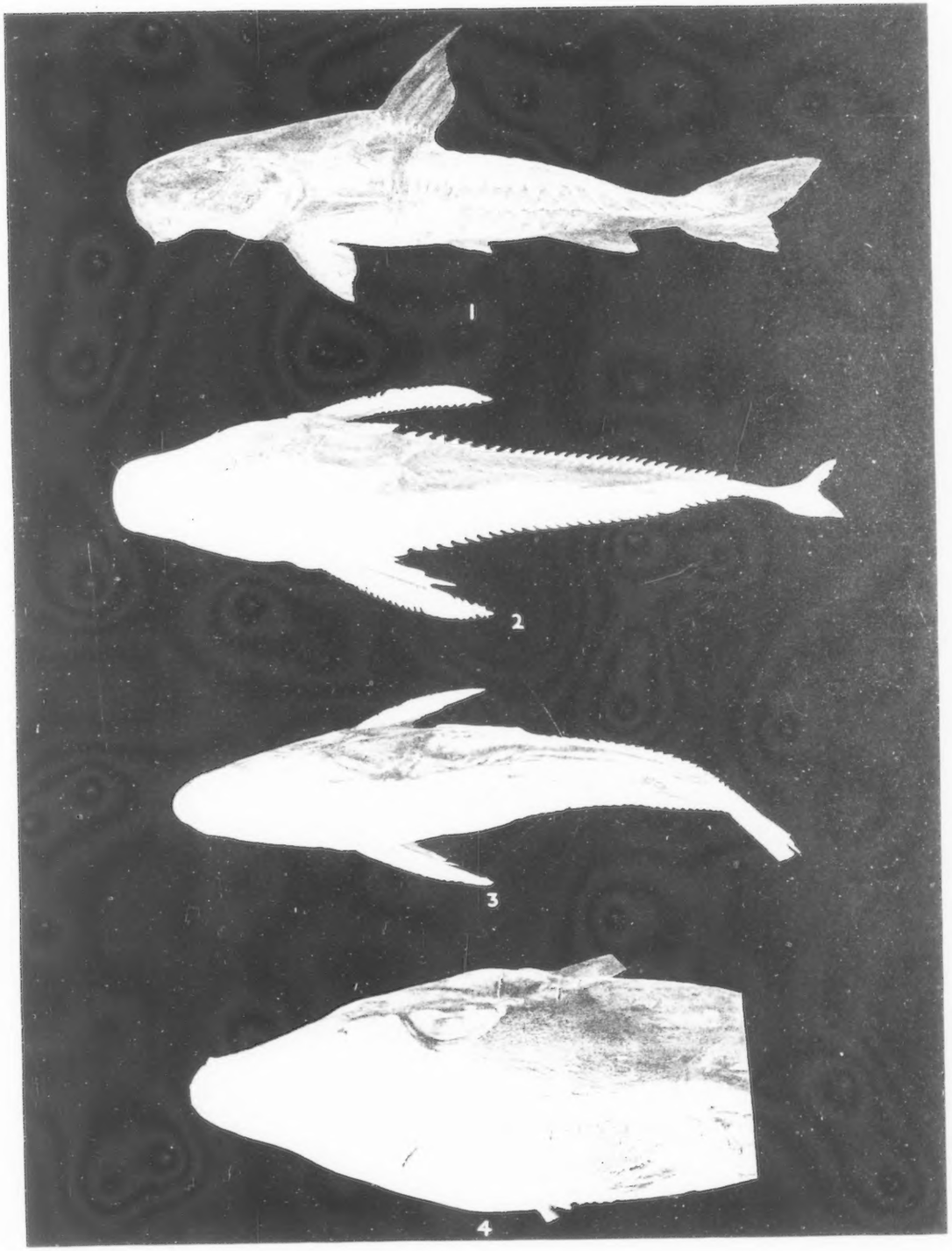
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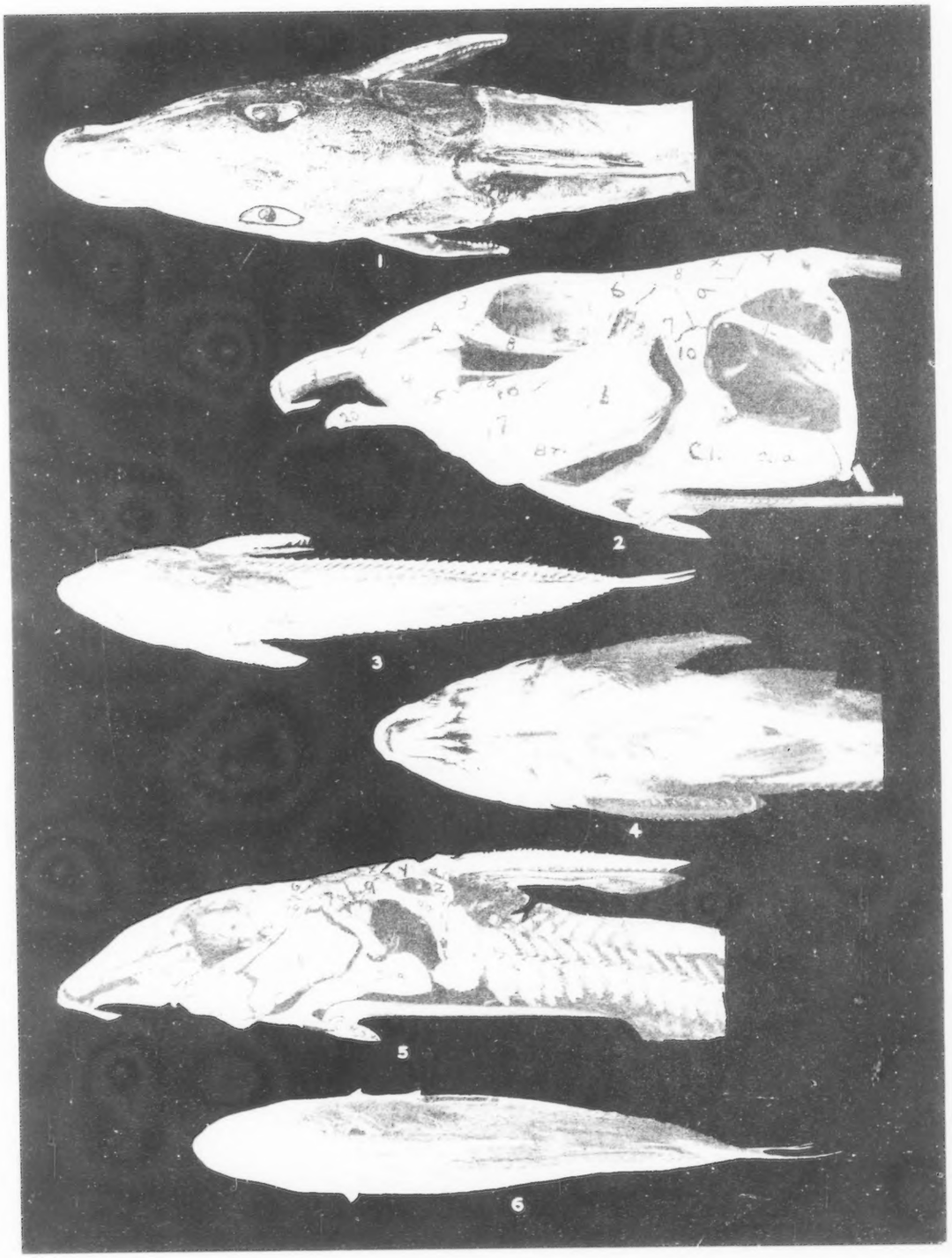
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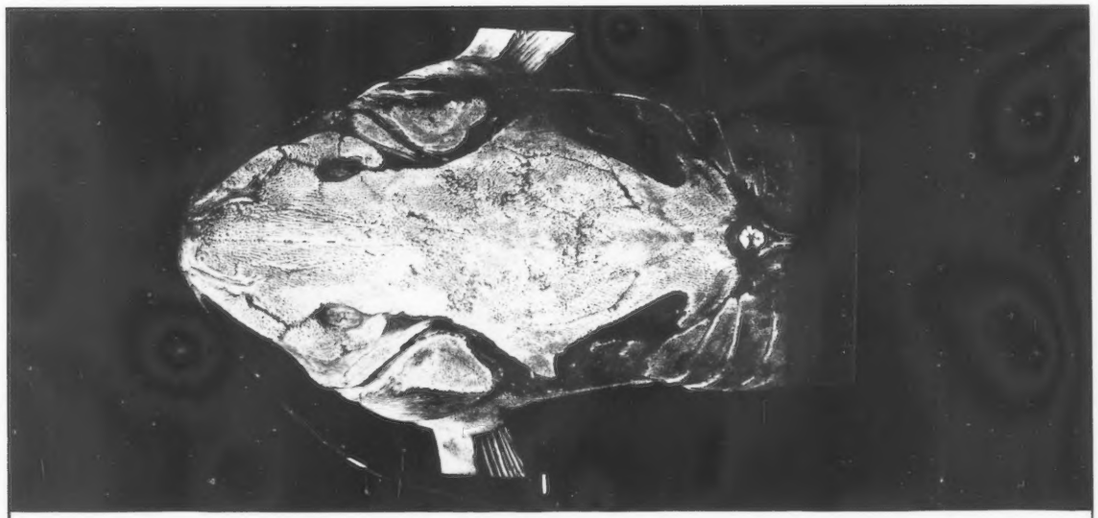

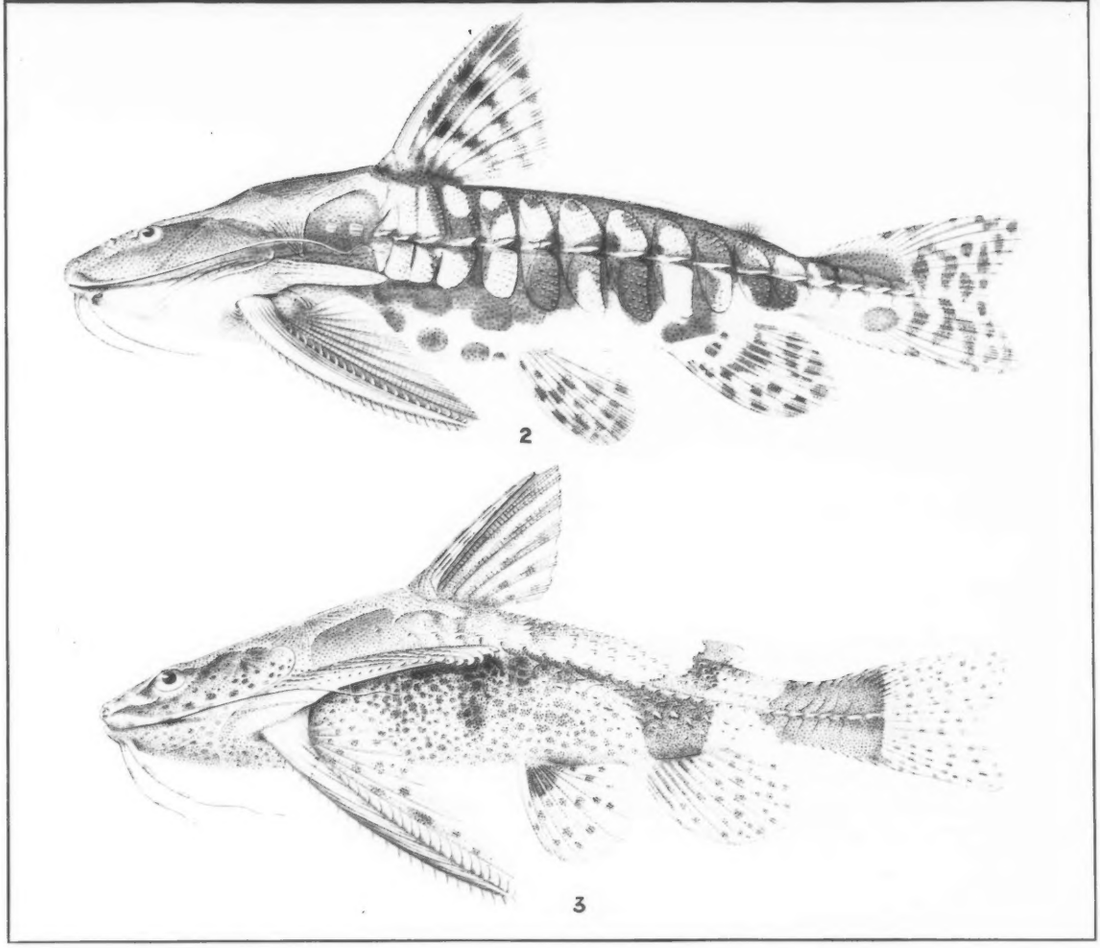
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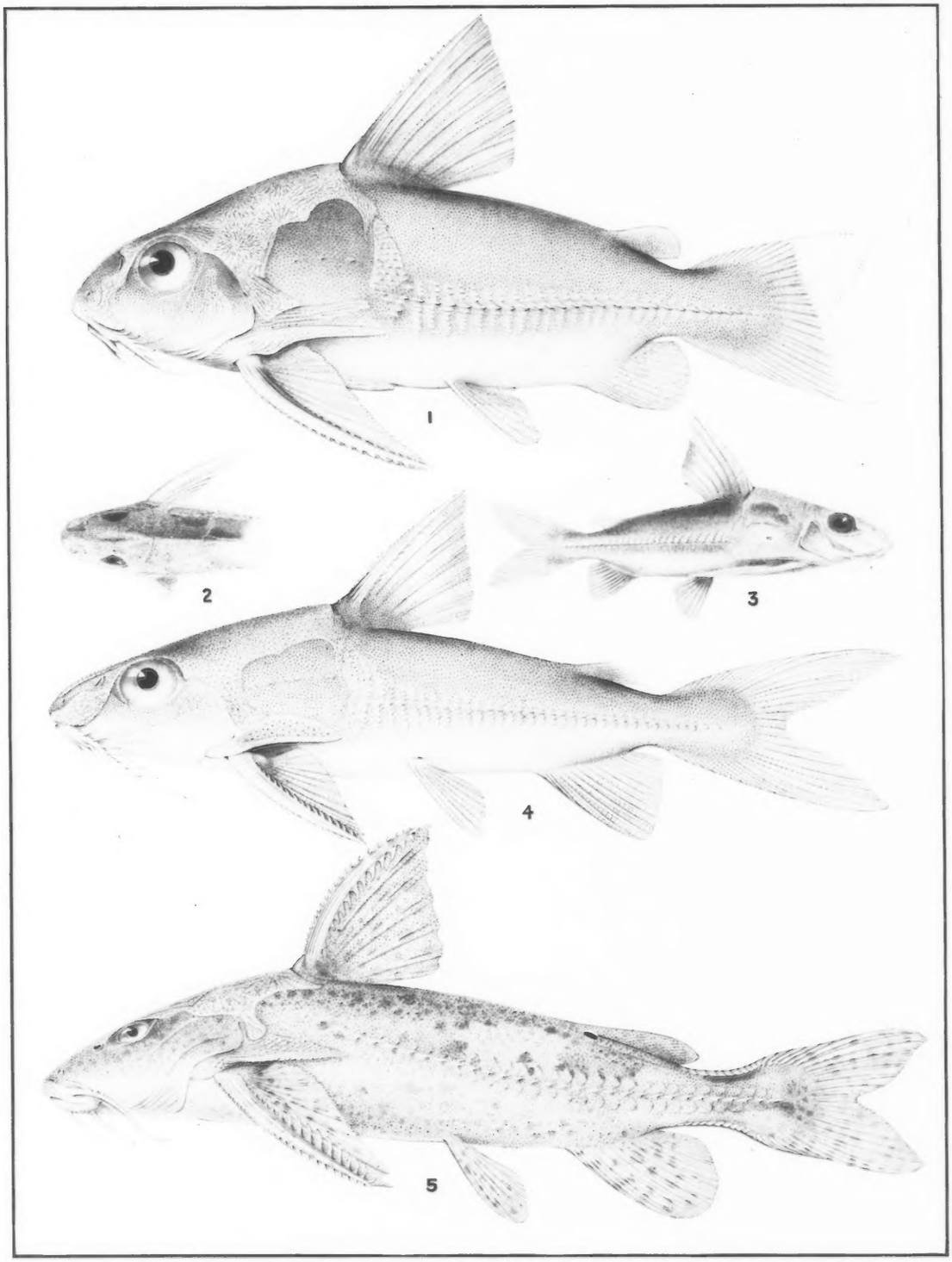
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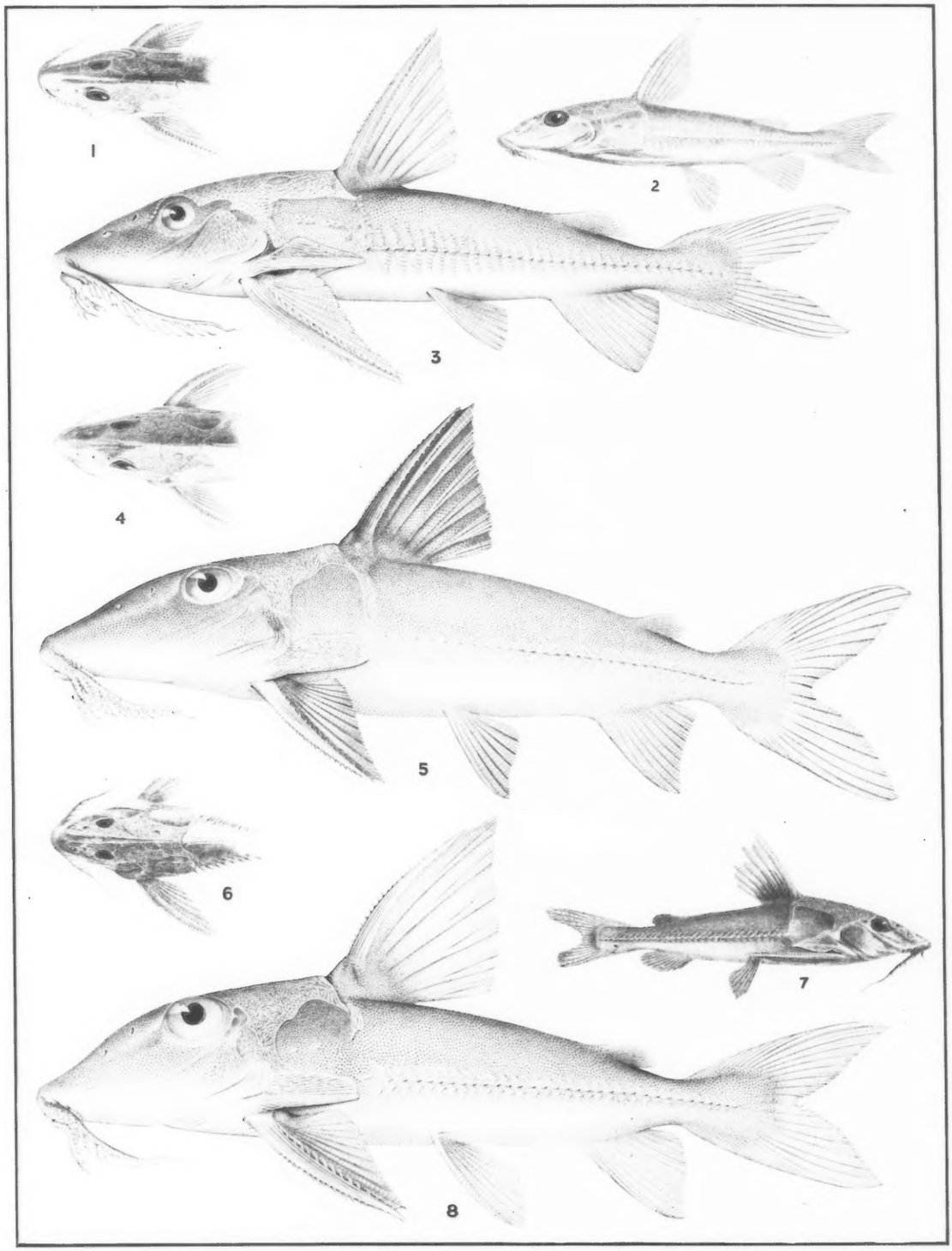
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